Polarimetric methods for tomographic imaging of natural volumetric media

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Polarization Coherence Tomography [Cloude 2006, 2007]

Principle

- Born’s approximation (Common to most SARTOM techniques)

\[ \gamma(k_z) = \frac{\int_{z_0}^{z_0+h_v} f(z) \, dz}{\int_{z_0}^{z_0+h_v} f(z) \, dz} \]

- Decomposition of \( f(z) \) over Legendre Polynomials

\[ f(z') = \sum_{i=0}^{N_o} a_i P_i(z') \Rightarrow \gamma(k_z) = \sum_{i=0}^{N_o} a_i g_i(z_0, h_v, k_z) \]

- Reconstruction \( \hat{\gamma}(k_z) \rightarrow \hat{a}_i \rightarrow \hat{f}(z') = \sum_{i=0}^{N_o} \hat{a}_i P_i(z') \)
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Basic steps

1. Set volume limits: \( z_0, z_0 + h_v \)
2. Compute polynomial integrals at order \( N_{opct} \)
3. Select a polarization channel
4. Invert linear relation set
Outline

Tomography using Orthogonal Polynomials (TOP)

Fully Polarimetric TOP

Perspectives for Adaptive TOP
Proposed TOP LS approach (example with Legendre polynomials)

MB-inSAR coherence decomposition using OP

Normalization: \( \gamma(k_j) = e^{jk_z} \int_{-1}^{1} f'(z) e^{j k_j h \nu / 2} z^m \, dz, \quad \int_{-1}^{1} f'(z) \, dz = 1 \)
Proposed TOP LS approach (example with Legendre polynomials)

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Normalization: \[ \gamma(k_{z_j}) = e^{jk_{z_j}z_{m}} \int_{-1}^{1} f'(z) e^{jk_{z_j}hv} z \, dz, \quad \int_{-1}^{1} f'(z) \, dz = 1 \]

Decomposition over \( N_0 + 1 \) OP and integration

\[ f'(z) = \sum_{i=0}^{N_0} a_i P_i(z) + r(z) \quad \text{and} \quad g_{ij} = e^{jk_{z_j}z_{m}} \int_{-1}^{1} P_i(z) e^{jk_{z_j}hv} z \, dz \]

Coherence: \[ \gamma(k_{z_j}) = \sum_{i=0}^{N_0} a_i g_{ij} + \tilde{r}(k_{z_j}) \]
Proposed TOP LS approach (example with Legendre polynomials)

MB-inSAR coherence decomposition using OP

Normalization: \[ \gamma(k_z) = e^{jk_z z_m} \frac{1}{-1} \int f'(z) e^{j \frac{k_z h_v}{2} z} \, dz, \quad \int_{-1}^{1} f'(z) \, dz = 1 \]

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Vertical profile reconstruction using \( M \) inSAR images, \( N_0^{th} \) order OP

From \( \hat{\gamma} \in \mathbb{C}^{M-1} \) estimate \( \hat{a} \in \mathbb{R}^{N_0+1} \)

Reconstruct estimated profile \( \hat{f}'(z) = \sum_{i=0}^{N_0} \hat{a}_i P_i(z) \)
Proposed TOP LS approach (example with Legendre polynomials)

Least-Square TOP compact solution (generalized order PCT)

- LS criterion at arbitrary order $N_o$ : $c = ||\hat{\gamma} - Ga||^2$, $a \in \mathbb{R}^{N_o+1}$, $[G]_{ij} = g_{ij}$
- Compact LS solution : $\hat{a} = \text{arg min}_a c = [\hat{a}_0, \hat{a}'^T]^T$

\[
\hat{a}_0 = 0.5 \Rightarrow \int_{-1}^{1} f'(z)dz = 1, \quad \hat{a}' = (X + X^T)^{-1}(v + v^*)
\]

\[
G = [g_0G_r], \quad X = G_r^\dagger G_r, \quad v = G_r^\dagger (\gamma - 0.5g_o)
\]
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Selecting polynomial order

- $M$ MB-inSAR images : $N_{opct} = 2(M - 1)$ DoF
- $N_o = N_{opct}$ : PCT "inversion". $N_o < N_{opct}$ : LS fitting
Proposed TOP LS approach (example with Legendre polynomials)

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Selecting polynomial order

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Reasons for choosing $N_o < N_{opct}$

- Finite $N_o$ values may not explain the whole tomographic content
  \[
  \hat{\gamma}(k_z) = \sum_{i=0}^{N_o} a_i g_{ij} + \tilde{r}(k_z) + \hat{\gamma}_n
  \]
- High $N_o$ value (PCT inversion) sensitive to mismodeling
- Volume limits, $z_0, z_0 + h_v$ need to be estimated

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TOP LS estimator: influence of the selected order $N_o$

**Configuration**
- $M = 6$ images
- $N_{opt} = 10$
- $N_{TRUE} = 3$

**Behavior w.r.t. $N_o$**
- $N_o < N_{true}$: underfitting
- $N_{TRUE} \leq N_o \leq N_{opt} - 2$: correct fit
- $N_{opt} - 1 \leq N_o \leq N_{opt}$: severe instability
TOP LS estimator: influence of orthogonal terms

- $M = 3$ images, $N_{opct} = 4$
- $N_{true} = 4 \rightarrow$ estimate $\hat{a}$
- Add terms at orders 5 and 6 $\rightarrow \hat{a}_{new}$
- RMSE for lower order terms
TOP LS estimator: influence of orthogonal terms

- $M = 3$ images, $N_{OPCT} = 4$
- $N_{TRUE} = 4 \rightarrow$ estimate $\hat{a}$
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Conclusions on TOP order selection

- $N_{OPCT}$: maximal order for a unique solution
- $N_o = N_{OPCT} \Rightarrow$ severe instabilities
- $N_o < N_{OPCT} - 2$: stable estimates, low sensitivity to higher order terms
- Number of images:
  - $M = 2 \rightarrow N_o = 1$: phase center under sinc approx.
  - $M > 2$: profile parameters may be estimated, with $N_o \ll 2(M - 1)$
- Volume limits, $z_0, z_0 + h_v$ need to be estimated
TOP LS estimator: influence of volume limit selection

- $M = 6$, $dk_z = 0.2$
- TOP solutions for various $N_o$
- Adaptive solution
  \[
  \hat{a}, z_0, h_v = \arg \min ||\hat{\gamma} - G\hat{a}||^2
  \]

Conclusions

- Volume extent underestimation $\rightarrow$ severe instabilities
- Volume extent overestimation $\rightarrow$ consistent estimates
- Adaptive nonlinear Least Squares:
  - Slightly improve estimation performance
  - Correct solution may be "missed" + high complexity
TOP LS estimator: application to real data

- TROPISAR Campaign over French Guyana
- ONERA/SETHI MB-POLinSAR data at P band
- Resolutions: $\delta r = 1m, \delta az = 1.245m$
- $M = 6$ images
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Estimation of volume limits

- Capon HH tomogram
- SB-PolinSAR volume bounds
  - $h_\nu$ may be underestimated (tunable)
  - $z_0$ overestimated
- POLTOMSAR bounds
  see Huang et al. IGARSS 2011
TOP LS estimator: application to real data

TOP LS, HH

- $M = 6$
- $N_o = 6 \ll N_{op\ CT}$
- Fixed bounds, $-40m \leq z \leq 40m$

Capon tomogram

- Similar global features
- Similar z-resolution (color coding)
- Legendre polynomials
  $\Rightarrow$ high intensity at estimation bounds

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TOP LS estimator: application to real data

- $N_o = 6$
- $-40m \leq z \leq 40m$
- $N_o = 6$
- $-60m \leq z \leq 60m$
- $N_o = 4$
- POLTOMSAR bounds

High sensitivity to the volume bounds, possibly unstable
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Perspectives for Adaptive TOP
Full rank POLTOM : FR-P-CAPON

Coherence tomography and spectral estimators

$$\gamma(kz) = \frac{\int_{z_0}^{z_0+h_v} f(z)e^{jkz}dz}{\int_{z_0}^{z_0+h_v} f(z)dz} = e^{jkm} \int_{-1}^{1} f'(z) e^{j\frac{kz,h_v}{2}}dz$$

▸ Equivalent to a normalized, or "whitened" covariance matrix
▸ $f(z) = a\delta(z - z_1)$, optimal estimation by BF
▸ $f(z) = \sum_i a_i\delta(z - z_i)$, optimal estimation by CAPON
▸ ...
Full rank POLTOM : FR-P-CAPON

Coherence tomography and spectral estimators

$$\gamma(k_z) = \int_{z_0}^{z_0+h_v} f(z) e^{j k_z z} \, dz / \int_{z_0}^{z_0+h_v} f(z) \, dz = e^{j k_z z_0} \int_{-1}^{1} f'(z) e^{j \frac{k_z h_v}{2} z} \, dz$$

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- …

Classical CAPON estimators

- SP-CAPON : $\hat{f}(z) = (a(z)^\dagger R^{-1} a(z))^{-1}$
- P-CAPON : $\hat{f}(z), \hat{\omega}_{opt}(z)$ : limited rank-1 POLSAR information ($H = 0$)
Full rank POLTOM : FR-P-CAPON

Coherence tomography and spectral estimators

\[
\gamma(k_z) = \frac{z_0 + h_v}{z_0} \int_{z_0}^{z_0 + h_v} f(z) e^{i k_z z} \, dz / \int_{z_0}^{z_0 + h_v} f(z) \, dz = e^{i k_z z_m} \int_{-1}^{1} f'(z) e^{i k_z h_v z} \, dz
\]

- Equivalent to a normalized, or "whitened" covariance matrix
- \( f(z) = a \delta(z - z_1) \), optimal estimation by BF
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Classical CAPON estimators

- SP-CAPON : \( \hat{f}(z) = (a(z)^\dagger R^{-1} a(z))^{-1} \)
- P-CAPON : \( \hat{f}(z), \hat{\omega}_{opt}(z) \): limited rank-1 POLSAR information (\( H = 0 \))

Full-rank CAPON estimator

- Full rank POLSAR coherency matrix : \( \hat{T}(z) (H \neq 0) \)
- 3-D POLSAR MLC information : \( \hat{T}(az, rg, z) \)
  - Decompositions, classifications, ... usual POLSAR tools
  - Undercover parameter estimation
FR-P-CAPON over the Paracou tropical forest at P band

SPAN (norm)

PAULI

T11 (norm)

T33 (norm)
FR-P-CAPON over the Paracou tropical forest at P band

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FR-P-CAPON over the Dornstetten temperate forest at L band
FR-P-CAPON over the Dornstetten temperate forest at L band

SPAN (norm)

mean alpha

H

A

range bins

range bins

range bins

range bins

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FR-P-CAPON over the Dornstetten temperate forest at L band

SPAN (norm)

H-alpha classification

Wishart classification
FR-P-CAPON over the Dornstetten temperate forest at L band

Pauli Van Zyl

Volume Van zyl

Pauli Yamaguchi

Volume Yamaguchi
Full rank polarimetric tomography: FP-TOP

Classical TOP

- TOP (and PCT): single channel technique, e.g. $f_{HH}(z)$
- Pol. basis are not equivalent: $f_{HH}(z), f_{HV}(z), f_{VV}(z) \neq f_{P1}(z), f_{P2}(z), f_{P3}(z)$

FP-TOP using MB-ESM optimization

- Joint diag. (Ferro-Famil et al. 2009, 2010)
- Basis Maximizing the MB-POLinSAR information,
  $$\omega_1, \omega_2, \omega_3 = \arg \max_{\omega_1, \omega_2, \omega_3} \sum_{i,j} |\gamma(\omega_i, k_{z_j})|^2$$
- Estimate $f_{\omega_1}(z), f_{\omega_2}(z), f_{\omega_3}(z)$
- Compute $f_{pq}(z)$ from $f_{\omega_i}, i = 1 \ldots 3$, using $U_3 = [\omega_1 \omega_2 \omega_3]$
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Adaptive TOP

$$\gamma(kz_j) = \frac{z_0 + hv}{z_0} \int_{z_0}^{z_0 + hv} f(z) e^{jkz_j z} \, dz / \int_{z_0}^{z_0 + hv} f(z) \, dz = e^{jkz_j z_m} \int_{-1}^{1} f'(z) e^{jkz_j hv} \, dz$$

Principle of adaptivity

- Classical spectral estimator tracks $z_m$, OP set characterizes the structure of the volume
- Joint OP-spectral cost function: fast semi-analytical solutions
- $z_0$, $h_v$ and $N_0$ are adaptively estimated
Conclusion

Tomography using Orthogonal Polynomials (TOP)

▷ LS fit using any polynomial order $N_o$ : analytical compact solution
▷ Stability and robustness assessment :
  ▷ Order selection, Volume bounds
  ▷ $N_o \ll N_{OPCT}$

Fully Polarimetric TOP

▷ Full Rank P-CAPON : $T(az, rg, z)$
▷ 3-D polarimetry, under-cover soil moisture . . .
▷ FP-TOP

Perspectives for Adaptive TOP

▷ OP based spectral estimators
▷ Adaptive order and volume bound selection