Minimum endmember-wise Distance Constrained NMF for SMA of Hyperspectral Images

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Minimum endmember-wise Distance Constrained NMF
1. Introduction

Generally, spectral sensors are deployed on either aircrafts or satellites and images are acquired in low spatial resolution. As a result, pixels in a spectral remote sensing image often contain more than one type of ground objects and are known as mixed pixels or mixtures. The existence of mixed pixels not only influences the performance of image classification and target recognition, but also is an obstacle to quantitative analysis of spectral remote sensing images.
(1) Endmember Extraction/Detection

- Spectral Library
  - Spectral EE algorithm
    - Geometric algorithms (N-FINDR, VCA, SGA, ...)
    - Least unmixing error (IEA, UFCLS, UGDME)
    - Machine learning approach (self-organizing neural networks)
    - Morphological algorithms

- Spatial-Spectral EE algorithm
  - Automated Morphological Endmember Extraction (AMEE)
  - Spatial-spectral EE tools
  - Spatial Purity based EE (SPEE) (IEEE TGRS, Vol.48, No.9, 2010)
(2) **Abundance Estimation** -- Optimization Problem

- Nonnegative least square method
- Gradient Descent Maximum Entropy
- augmented Lagrangian approach
- Multi-channel Hopfield Neural Network *(IEEE GRSL, Vol.7, No.3, 2010)*
(3) EE + AE simultaneously (Unsupervised)
-- Blind Signal Decomposition Problem

➤ Independent component analysis
  -- independent set
➤ Nonnegative matrix factorization
  -- constraints
➤ ......

UFCLS, GDME
Problems and Motivation

- The Linear Mixture Model (LMM) has been widely utilized in SMA due to its effectiveness and simplicity. However, endmember must be determined previously.

- Recently, unsupervised SMA has been proposed to simultaneously extract endmembers and estimate their corresponding fractional abundance.—UFCLS, GDME

However, pure pixels for each endmember are assumed to present in the data and the strong requirement does not hold in many cases.

Therefore, a robust unsupervised SMA algorithm must be capable to handle highly mixed hyperspectral data.
Many nonnegative matrix factorization (NMF) based algorithms have been proposed for this highly mixed situation:

- **MiniDisCo-NMF** *(IEEE TGRS, Vol.48, No.6, 2010)*: minimum the variance of each endmember spectra -- over-smooth the spectra

- **MVC-NMF** *(IEEE TGRS, Vol.45 No.3, 2007)*: minimum the volume of simplex determined by endmembers, however:
  - DR is required
  - numerical instability

Minimum endmember-wise distance constrained NMF(\textbf{MewDC-NMF})

endmember-wise distance \(\leftrightarrow\) volume of simplex

Triangular (2D): Perimeter \(\leftrightarrow\) area
2. MewDC-NMF

Nonnegative Matrix Factorization (NMF):

\[ R \approx M \cdot A \quad s.t. \quad M \succeq 0, A \succeq 0 \]

- Linear mixture model: \[ R = M \cdot A + N \]
- Abundance Non-negative constraint: \[ A \succeq 0 \]
- Abundance Sum-to-one constraint – pseudo band:

\[ \bar{R} = \begin{pmatrix} R \\ \delta \mathbb{1}_{1o} \end{pmatrix}, \quad \bar{M} = \begin{pmatrix} M \\ \delta \mathbb{1}_{1c} \end{pmatrix} \]
However, minimizing the representation error in LMM by NMF is not sufficient for SMA since the unmixing result of NMF is not unique. Therefore, extra constraint must be imposed on NMF to ensure satisfying unmixing results of hyperspectral images:

\[
f(M, A) = \frac{1}{2} \| R - M \cdot A \|_2 + \lambda_1 J_1(M) + \lambda_2 J_2(A)
\]

\[
s.t. \quad M > 0, A > 0
\]

It is very difficult to enforce a constraint on \( A \) since this may be problem-dependent. Therefore, in this paper, we enforce extra constraint on \( M \) only.
An endmember-wise distance constraint (enforce compactness of endmembers) is proposed

\[ J(M) = \frac{1}{2} \sum_{i=1}^{c} \sum_{j=1}^{c} (m_i - m_j)^T (m_i - m_j) \]

or in matrix form

\[ J(M) = \frac{1}{2} \sum_{i=1}^{c} \text{trace} \left( (M - MD_i)^T (M - MD_i) \right) \]

in which

\[ D_i = e_i 1_c^T \]
Compared with MVC:

- Volume of simplex determined by endmembers (area)
- Cumulative distance between endmembers (perimeter)

- Convex constraint
- DR is not necessary
- Numeric instability can be avoided
Alternating optimization algorithm

\[ A^{k+1} = \arg \min_{A \geq 0} f(M^k, A^k), \]

\[ M^{k+1} = \arg \min_{M \geq 0} f(M^k, A^{k+1}) \]

Therefore

\[ A^{k+1} = \max \left( 0, A^k + \alpha_k \nabla_A f(M^k, A^k) \right) \]

\[ M^{k+1} = \max \left( 0, M^k + \beta_k \nabla_M f(M^k, A^{k+1}) \right) \]

in which

\[ \nabla_A f(M, A) = M^T (MA - R) \]

\[ \nabla_M f(M, A) = (MA - R) A^T + \nabla_M J(M) \]

\[ \nabla_M J(M) = \sum_{i=1}^{c} M \left( I_b + D_i + D_i^T + D_i^T D_i \right) \]
3. Experiments

- Experiments with Synthetic hyperspectral pixels
  - Spectral liberty: Five spectra of minerals – USGS
  - LMM
  - The abundance -- generated randomly based on the Sum to one constraint and the Nonnegative constraint
  - In order to simulated highly mixed pixels, all the pixels whose abundance is larger than 80% are regenerated
  - zero-mean white Gaussian noise is added to simulate possible errors and sensor noises.
Obviously, no matter how noisy the data is, the proposed MewD-NMF algorithm outperforms the other two constrained NMF algorithms – MVC-NMF and MiniDisco-NMF.
Experiments with AVIRIS dataset

- acquired on June 19th, 1997 by AVIRIS
- 224 channels covering from 370 nm to 2510 nm with a Ground Instantaneous Field of View of 20 m.
- A 200×200 pixels subset of the eastern hydrothermal alteration zone is selected for evaluation.
- Of the 224 atmospherically corrected channels, only 185 bands are adopted by removing the channels associated with H₂O and OH absorption features near 1400 and 1900 nm.
Abundance maps

pure white denotes that the percentage of the ground objects in the mixture is 100%, while pure black denotes 0.

Visually consistent with the results presented by previous researchers.
Fourteen endmembers are extracted (Hysim algorithm). The right figure shows the extracted spectra (five highly representative minerals) and their corresponding library spectra in USGS spectral library.

Endmembers extracted by the proposed MewDC-NMF algorithm have similar absorption and reflection characteristics with their corresponding ground-truth spectra.
4. Conclusions

- MewDC-NMF (minimum endmember-wise constrained NMF) algorithm is proposed for unsupervised unmixing of highly mixed hyperspectral data, simultaneously in spectral and spatial domain.
- The proposed algorithm utilizes cumulative distance between endmembers to optimize endmember spectra as compact as possible, convex function → convergence guaranteed.
- As a result, the non-uniqueness problems in NMF based unmixing can be alleviated.
- Experiments on both synthetic and real data have demonstrated its effectiveness.
Any Question?

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Thank you!