SAR Image Despeckling Based on Improved Directionlet Domain Gaussian Mixture Model

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The main contents consist of the following parts:

1. Introduction
2. Cartoon-Texture Decomposition
3. Despeckling procedure
4. Results
Speckle reduction is an important issue in SAR image processing.

Typical methods:
- Spatial domain: Lee filter, Gamma MAP filter etc al;
- Transform domain: wavelet, many multiscale geometric analysis-based techniques.

In 2005, V. Velisavljevic et al. proposed Directionlet, which represents an image sparsely while capturing anisotropic structures.

We improve the Directionlet transform to represent SAR images much more sparsely and efficiently.

In this topic, we will address an efficient SAR image despeckling method based on:

- Non-log additive model;
- Cartoon texture decomposition;
- The improved Directionlet transform;
- Gaussian Mixture Model.
Speckle noise model

- Speckle noise is a natural consequence of image formation under coherent radiation, which is generally modeled as multiplicative random noise, i.e.

\[ X = YF \]

- Log-transform is usually employed to make the multiplicative speckle noise additive. Multiplicative speckle noise can also be converted into additive noise without the use of the log-transform:

\[ X = YF = Y + Y (F - 1) \]
The model is derived from the ROF model with total variation minimization combining with the space of oscillating functions.

In the ROF model, a SAR image $f \in L^2(R^2)$ is represented as:

$$f(x, y) = u(x, y) + v(x, y)$$

$u$ is the cartoon part and $v$ is the texture part.

- Piecewise regions should be retained.
- Remaining texture needs to be processed.
- Shape edges.
- Noise.
An example of the cartoon texture decomposition.

The noise is mainly contained in the texture part.
Directionlet

- Directionlet is an extension of wavelet transform to many directions.

- Using Directionlet transform, an image can be partitioned into several independent cosets, where the skewed anisotropic wavelet transform (AWT) is performed in different directions.

- Directionlets trace the discontinuity efficiently with fewer significant coefficients, so it can represent highly anisotropic objects like contours and edges more precisely.
The improved Directionlet

- Performing the skewed-AWT directly to the cosets obtained by the first decomposition and fitting model result in over-sharp a peak at zero point, so that the curve cannot be fitted correctly.

- We improve the coset decomposition, proposing a 2-level coset decomposition, that is, coset decomposition is performed once more on the cosets obtained by the first decomposition.

- The 2-level coset decomposition is reversible.
The improved Directionlet

1. **SAR image**
   - Rotation → **Coset1**
     - Rotation → **Coset1-1** → Remove zeros → AWT
     - Rotation → **Coset1-2** → Remove zeros → AWT

2. Rotation → **Coset2**
   - Rotation → **Coset2-1** → Remove zeros → AWT
   - Rotation → **Coset2-2** → Remove zeros → AWT

3. **Directionlet coefficients**
Assuming that $W_x$ is the noisy Directionlet coefficients, its mixture probability density function is given by

$$p_{w_x}(W_x) = \sum_{k=0,1} p(S = k)p(W_x | S = k)$$

$$\sum_{k=0,1} p(S = k) = 1$$

$$p(W_x | S = k) = \frac{1}{\sqrt{2\pi d_k}} \exp\left(-\frac{x^2}{2d_k}\right)$$

where $p(S = k)$ is a mixture parameter, and $p(W_x | S = k)$ is the probability density function of the $k$-th Gaussian component, with mean 1 and variance $d_k$. 
Fitting results

The solid lines with violent oscillation denote coefficient curves of the high-frequency sub-bands in the cosets, and the smoothing solid lines are the results of fitting.
GMM-based estimate of noise

- For the high-frequency coefficients $W_Y(i, j)$, Bayes estimate of the noise-free Directionlet coefficients $W_Y$ is given by

$$\hat{W}_Y = \eta W_X$$

- Based on a GMM of the PDF, the MMSE estimate of noise-free Directionlet coefficients is

$$\hat{W}_Y = \sum_{k=0,1} p(S = k | W_X) \frac{\sigma_{W_x}^2 - \sigma_{W_y}^2}{\sigma_{W_x}^2} W_X$$
According to the Bayesian rule, we have

\[ p(S=k \mid W_x) = \frac{p(W_x \mid S=k)p(S=k)}{p(W_x)} \].

The noise standard deviation is estimated with a heuristic priori:

\[ \sigma_{W_B} = \frac{\text{median}(|W_x| - \text{median}(W_x))}{0.6745} \]
Flowchart

Input SAR image

Cartoon texture decomposition

Cartoon part
Texture part

Processing

Keep it the same

Improved Directionlet transform

Estimate high-frequency coefficients by GMM

MMSE denoising

Inverse transform

output
Results

Comparison of denoising results of the SAR field image.
The details of the denoising results.
## Performance comparison

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We address a novel speckle reduction method, combining the cartoon-texture model with the GMM in the improved Directionlet domain.

The results demonstrates that the method achieves effective despeckling performance in:

- subjective visual quality
- details preservation
Thank You!