TUTORIAL

July 6, 2008

Boston, Massachusetts, USA

SAR Polarimetry: Basics, Processing Techniques and Applications

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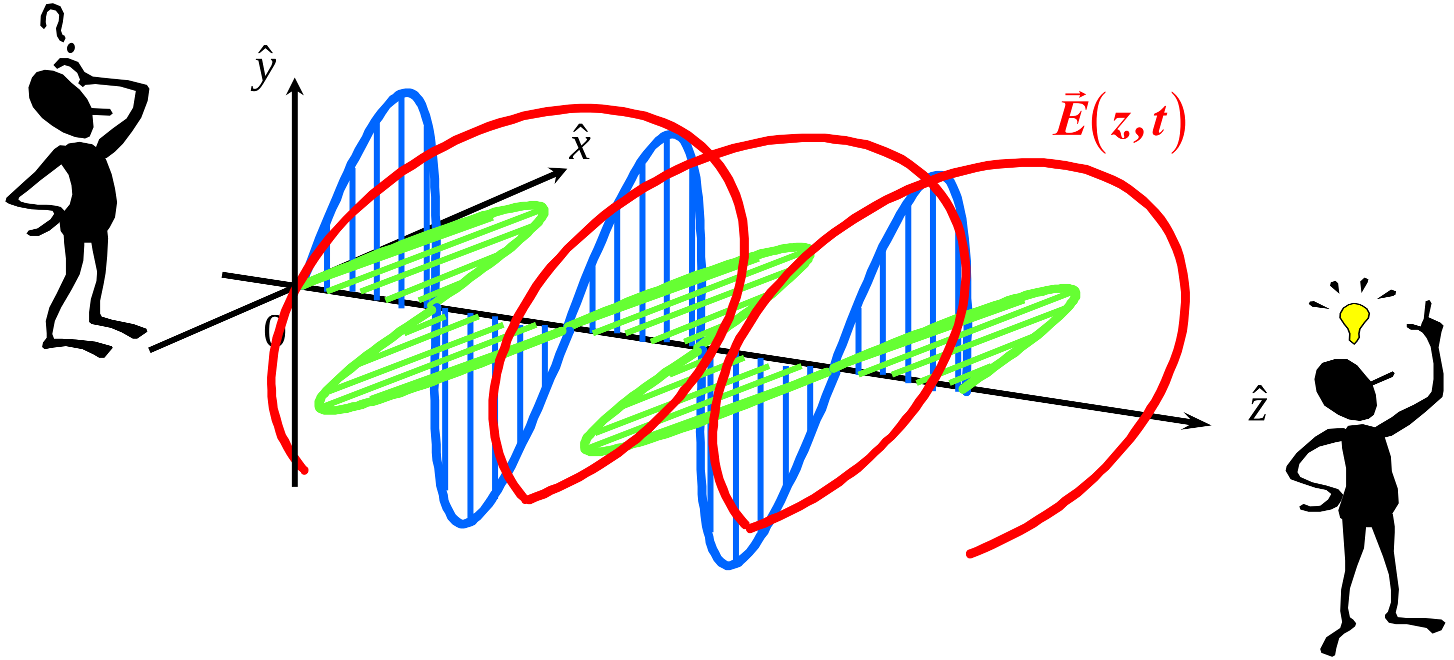
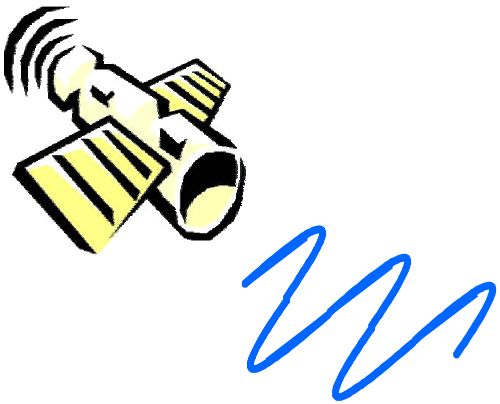
RADAR POLARIMETRY



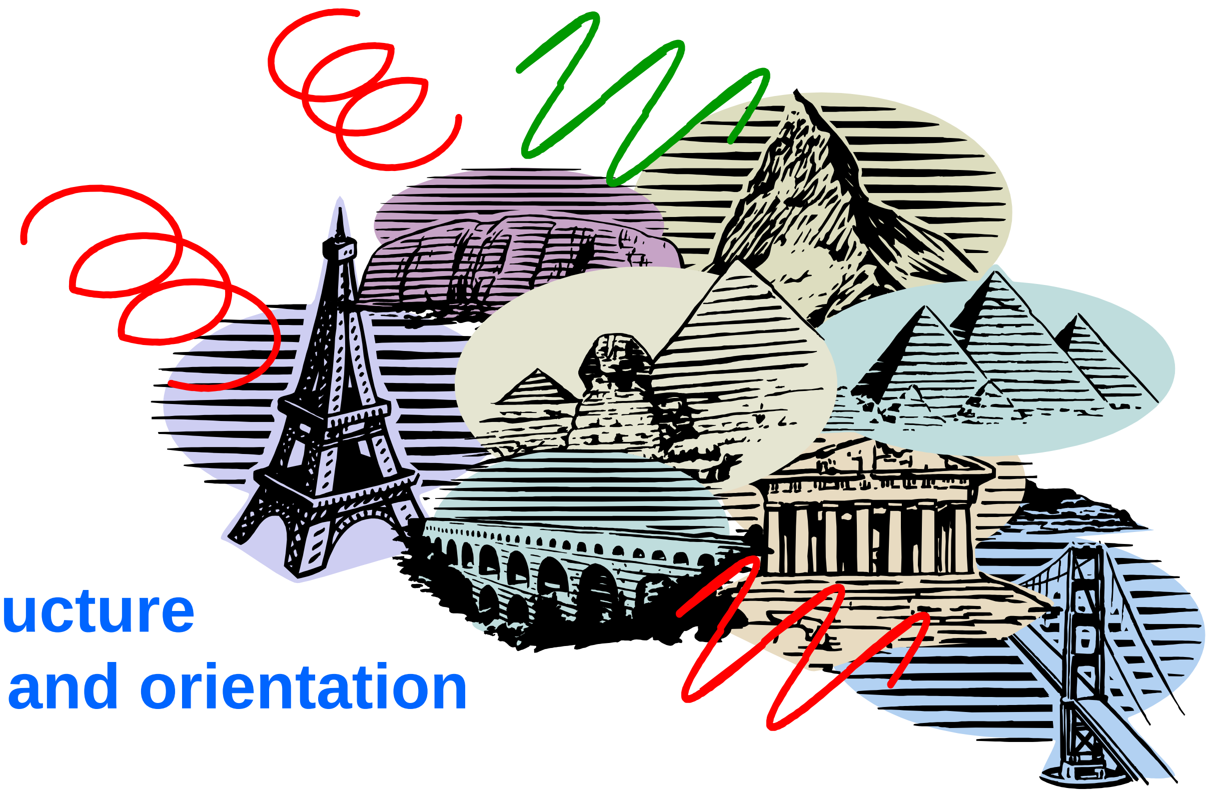
Radar Polarimetry (Polar : polarisation Metry: measure) is the science of acquiring, processing and analysing the polarization state of an electromagnetic field

Radar Polarimetry deals with the full vector nature of polarized electromagnetic waves

RADAR POLARIMETRY



The POLARISATION information
Contained in the waves backscattered
from a given medium is highly related to:



its geometrical structure
reflectivity, shape and orientation

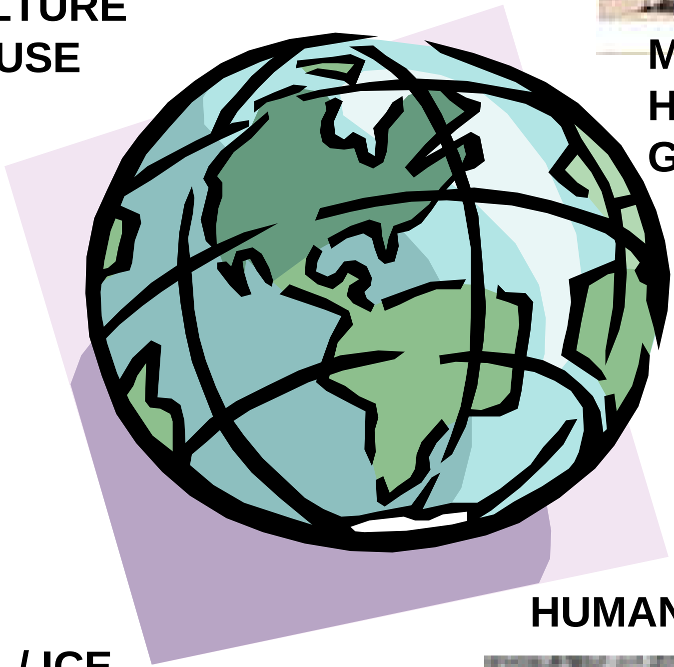
its geophysical properties such as humidity, roughness, …

RADAR POLARIMETRY

APPLICATIONS OF RADAR POLARIMETRY IN REMOTE SENSING (EARTH MONITORING)

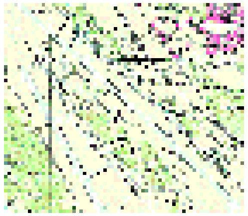
AGRICULTURE
LAND USE

METEOROLOGY
HYDROLOGY
GEOLOGY



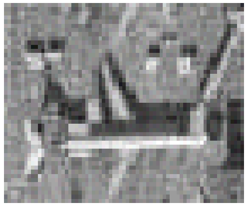
FORESTRY

TOPOGRAPHY
CARTOGRAPHY



SECURITY
HUMANITARIAN DEMINING

SEA / ICE
OCEANOGRAPHY



RADAR POLARIMETRY

Forest Vegetation

- Forest Height
- Forest Biomass
- Forest Structure
- Canopy Extinction
- Underlying Topography

- Forest Ecology
- Forest Management
- Ecosystem Change
- Carbon Cycle

Agriculture

- Soil Moisture Content
- Soil roughness
- Height of Vegetation Layer
- Extinction of Vegetation Layer
- Moisture of Vegetation Layer

- Farming Management
- Water Cycle
- Desretification

Snow and Ice

- Topography
- Penetration Depth / Density
- Snow Ice Layer
- Snow Ice Extinction
- Water Equivalent

- Ecosystem Change
- Water Cycle
- Water Management

Urban Areas

- Geometric Properties
- Dielectric Properties

- Urban Monitoring

Courtesy of Dr I. Hajnsek (DLR-HR)

RADAR POLARIMETRY

POLARIMETRIC SAR SENSORS

AIRBORNE SENSORS

AES1
AeroSensing (D)

AIRSAR
NASA / JPL (USA)

DOSAR
EADS / Dornier GmbH (D)

ESAR
DLR (D)

EMISAR
DCRS (DK)

MEMPHIS / AER II-PAMIR
FGAN (D)

PHARUS
TNO - FEL (NL)

PISAR
NASDA / CRL (J)

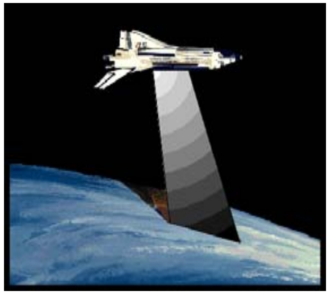
RAMSES
ONERA (F)

RENE
UVSQ / CETP (F)

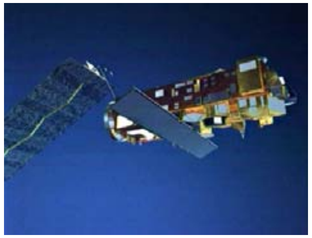
STORM
UVSQ / CETP (F)

SAR580
Environnement Canada (CA)

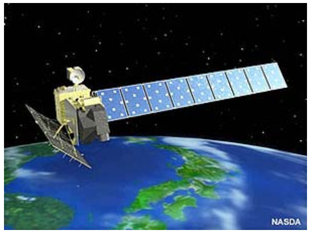
SHUTTLE / SPACEBORNE SENSORS



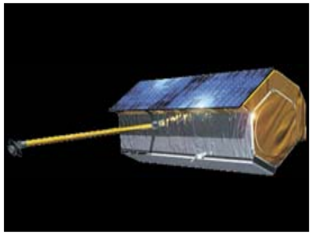
SIR-C
NASA / JPL (USA)



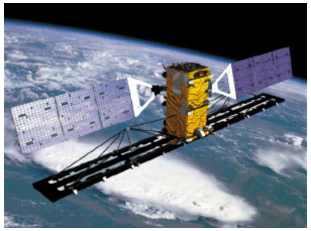
ENVISAT / ASAR
ESA (EU)



ALOS / PALSAR
NASDA / JAROS (J)



TerraSAR - X
BMBF / DLR / ASTRIUM



RADARSAT 2
CSA - MDA (CA)

RADAR POLARIMETRY

Innovation

- Specifications needed for future satellite sensors
- Test advanced imaging modes

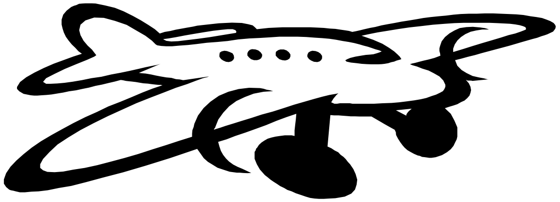
Development

- Development of algorithm for quantitative parameter inversion
- Development of new application products

Data Availability

- Detailed information in critical areas
- Key information that cannot currently be measured from space
- Young researcher education
- Preparation to satellite SAR sensors

Requirements to an Airborne system



- Flexible and modular SAR system
- System avalaibility
- Complete processing chain
- Fast data delivery
- High data quality

Courtesy of Dr I. Hajnsek (DLR-HR)

POLARIMETRIC AIRBORNE SAR SENSORS

AES1
InterMap Technologies (D)
GulfStream Commander
X-Band (HH), P-Band (Quad)

AIRSAR
NASA / JPL (USA)
DC8
P, L, C-Band (Quad)

AuSAR - INGARA
D.S.T.O (Aus)
DC3 (97) KingAir 350 (00) Beach 1900C
X-Band (Quad)

DOSAR
EADS / Dornier GmbH (D)
DO 228 (89), C160 (98), G222 (00)
S, C, X-Band (Quad), Ka-Band (VV)

ESAR
DLR (D)
DO 228
P, L, S-Band (Quad)
C, X-Band (Sngl)

EMISAR
DCRS (DK)
G3 Aircraft
L, C-Band (Quad)

MEMPHIS / AER II-PAMIR
FGAN (D)
Transal C160
Ka, W-Band (Quad) / X-Band (Quad)

STORM
UVSQ / CETP (F)
Merlin IV
C-Band (Quad)

PHARUS
TNO - FEL (NL)
CESSNA – Citation II
C-Band (Quad)

PISAR
NASDA / CRL (J)
GulfStream
L, X-Band (Quad)

RAMSES
ONERA (F)
Transal C160
P, L, S, C, X, Ku, Ka, W-Band (Quad)

SAR580
Environnement Canada (CA)
Convair CV-580
C, X-Band (Quad)

+ CASSAR (China), MIT/Lincoln Lab (USA), P3-SAR (NADC / ERIM -USA), Military Systems …

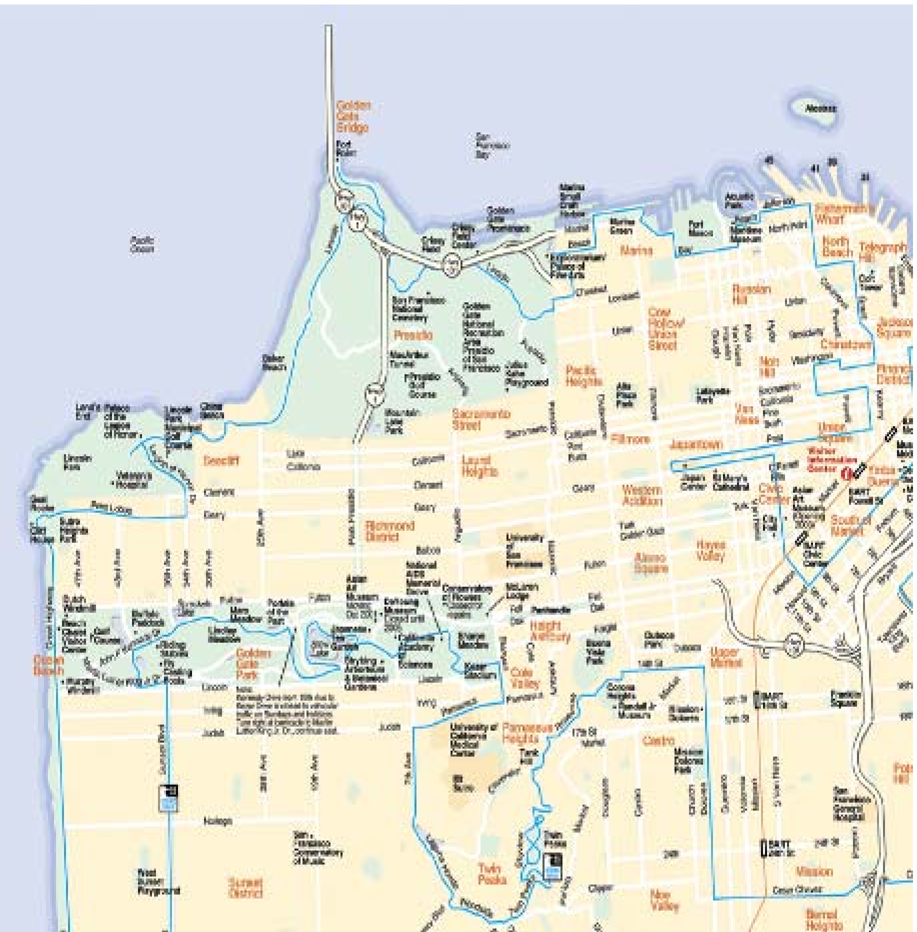
AIRSAR

DC8

P, L, C-Band (Quad)

RADAR POLARIMETRY

AIRSAR JPL



|HH+VV| |HV | |HH-VV|

San Francisco Bay (1988) – (L-Band)

RADAR POLARIMETRY

ESAR

DO 228

P, L, S-Band (Quad)
C, X-Band (Sngl)

P-Band

X-Band

L-Band

C-Band

RADAR POLARIMETRY

|HH+VV| |HV | |HH-VV|

Oberpfaffenhofen Airfield – (L-Band)

RADAR POLARIMETRY

F – SAR DLR

X-Band – Quad Pol

Courtesy of Prof. W.M. Boerner

RADAR POLARIMETRY

EMISAR

DCRS (DK)

G3 Aircraft

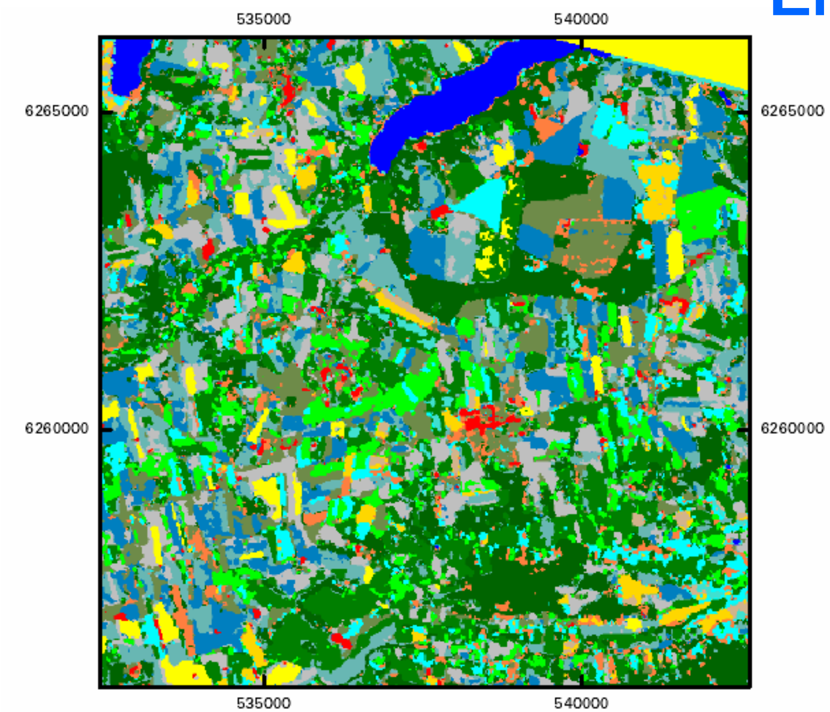
L, C-Band (Quad)

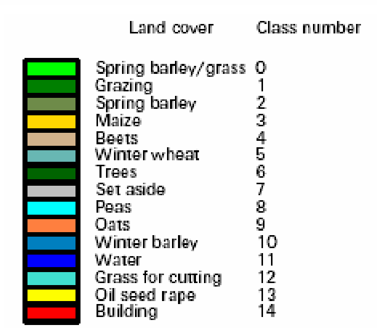
RADAR POLARIMETRY

Copenhagen (1999)
C-Band

RADAR POLARIMETRY

EMISAR





LandSat TM Classification

Foulum (C-Band) Land Cover Monitoring

RADAR POLARIMETRY

PISAR

NASDA / CRL (J)

GulfStream

L, X-Band (Quad)

Tsukuba Science City (1997)

L-Band

IMAGE SIZE : 700 (Range) x 700 (Azimut)
PIXEL SIZE : 2.5m (Range) x 2.5m (Azimut)

X-Band

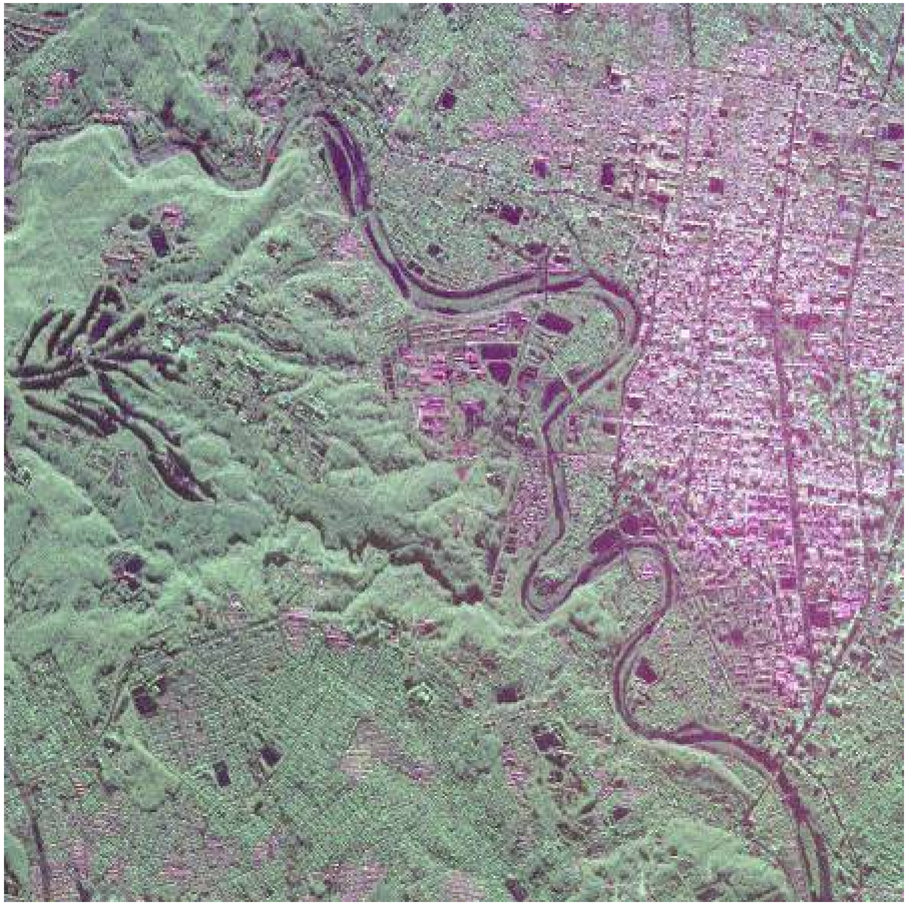
IMAGE SIZE : 700 (Range) x 700 (Azimut)
PIXEL SIZE : 2.5m (Range) x 2.5m (Azimut)

|HH-VV|, |HV|, |HH+VV|

RADAR POLARIMETRY

Tohoku University (2001)

L-Band **X-Band**



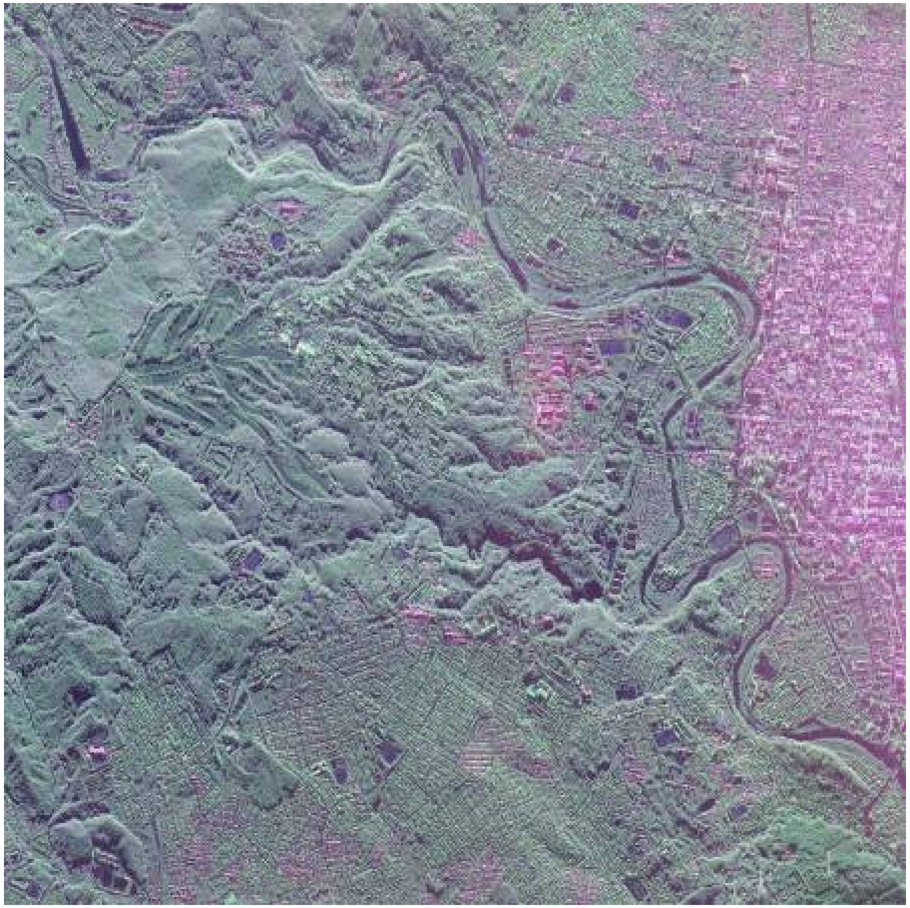


IMAGE SIZE : 5000m (Range) x 5000m (Azimut)
PIXEL SIZE : 2.5m (Range) x 2.5m (Azimut)

IMAGE SIZE : 5000m (Range) x 5000m (Azimut)
PIXEL SIZE : 2.5m (Range) x 2.5m (Azimut)

|HH-VV|, |HV|, |HH+VV|

RADAR POLARIMETRY

RAMSES

ONERA (F)

Transal C160

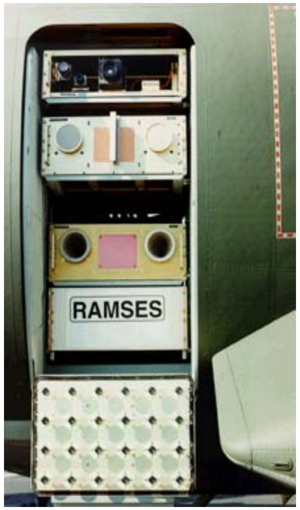
P, L, S, C, X, Ku, Ka, W-Band (Quad)

RADAR POLARIMETRY

MAIN CHARACTERISTICS OF THE RAMSES SYSTEM

Band	Ctr Freq [GHz]	λ [cm]	Bwidth [MHz]	Rés [m]	Anten.	Elev x azimut	Peak Power	Power Stage	Polar	Mode
P	0.43	69,7	75	2	Array	40° x 30°	300	SSA	Full	
L*	1.3	23	200	0,75	Array	23° x 16°	100	SSA	Full	
S	3.2	9,4	300	0,5	Array	30° x 10°	150	SSA	Full	
C	5.3	5,6	300	0,5	Array	33° x 8°	500	TWT	Full	
X	9.6	3,1	1200	0,13	Both	16° x 16°	200	TWT	Full	IFPOL, IF MB
Ku	14.3	2,1	1200	0,13	Horn	14° x 14°	200	TWT	Full	IFPOL, IF MB
Ka	35	0,8	1200	0,13	Horn	20° or 5°	100	TWT	VV	
W	95	0,3	500	0,3	Horn	3° 5° 10° or 20°	50	EIA	LR, LL	

- Boresight incidence angle can be adjusted from 30° to 75°(except at P-band)
- Flexible waveform: Bandwidth, number of recorded channels, swath width
- Two frequencies can be operated simultaneously



RADAR POLARIMETRY

RAMSES
X -Band (Quad - PolInSar)
Resolution: 0.9m (range) x 0.9m (azimut)
Swath: 800m

|HH-VV| **|HV |** **|HH+VV|**

Campaign **RITAS** (Radar Imagerie Thématique Agricole et Sols)
INRA, CETP, BRGM, CEMAGREF et ONERA – March 2002

RADAR POLARIMETRY

SETHI

ONERA (F)

Mystere 20

P, L, S, C, X (Quad - Pol)

ONERA

P-Band Quad-Pol
B=70MHz

(P-Vh, X-Hh, L-Vh)

SAR580

Environnement Canada (CA)

Convair CV-580

C, X-Band (Quad)

Developed by Canada Centre for Remote Sensing CCRS – 1974

Fully Polarimetric SAR at C-Band

Now owned and operated by Environment Canada

Viewed as a primary research tool to support CCRS work for RADARSAT 2 and ENVISAT

RADAR POLARIMETRY

SAR580

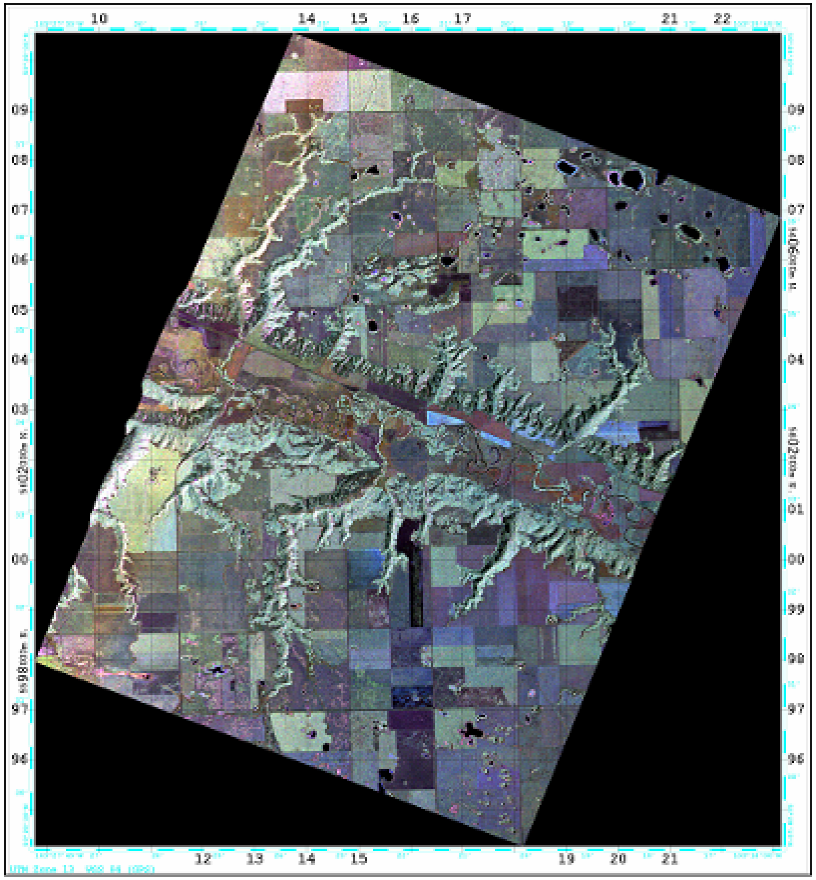
Environnement Canada (CA)

Convair CV-580

C, X-Band (Quad)

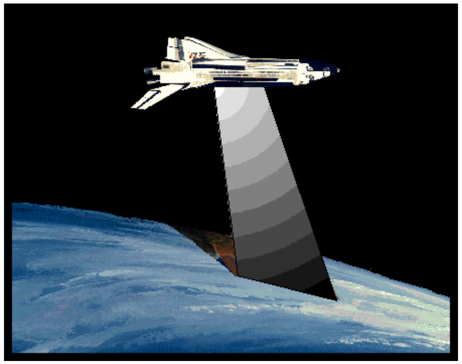
Qu'Appelle River (June 2000) Geocoded Product

Parameter	*Units*	*Value*
Antenna		
Polarization		**H and V**
Peak Gain	dB	**27**
Elevation Width	o	**16**
Azimuth Width	o	**3**
Transmitter		**1**
		TWT
Chirp Generation		**SAW**
Power	kW	**16**
Frequency	GHz	**5.3**
PRF/V	1/m	**3.32 or 2.57**
PRF_max	Hz	**383x2**
Receivers		**2**
Compression		**SAW^{-1}**
Digitization		**I +Q**
		6-bit

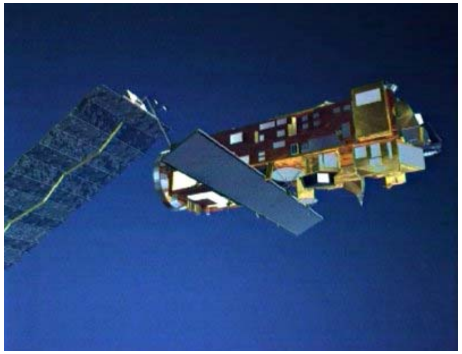


RADAR POLARIMETRY

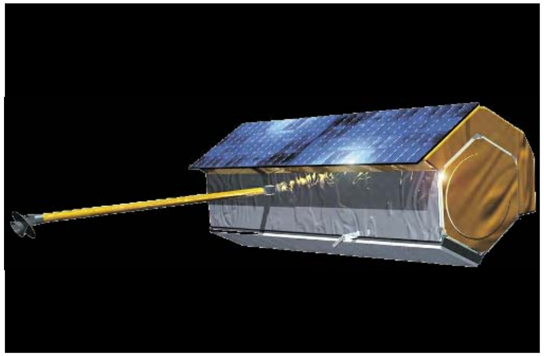
POLARIMETRIC SPACEBORNE SAR SENSORS



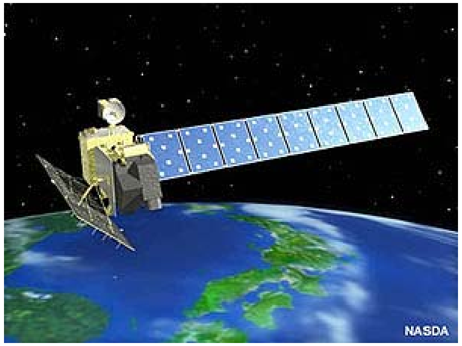
SIR-C
NASA / JPL (USA)
April 1994 (10 days)
October 1994 (10 days)
L, C-Band (Quad)



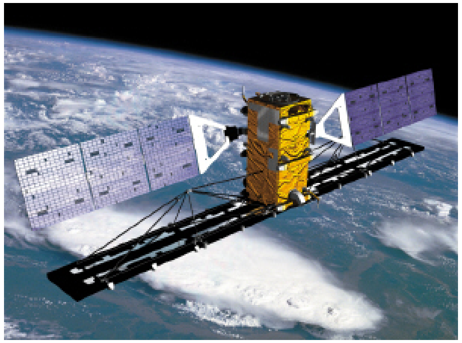
ENVISAT / ASAR
ESA (EU)
2002
C-Band (Sngl / Twin)



TerraSAR-X
DLR / EADS – ASTRIUM / Infoterra GmbH
June 2007
X-Band (Sngl / Twin / Quad ?)

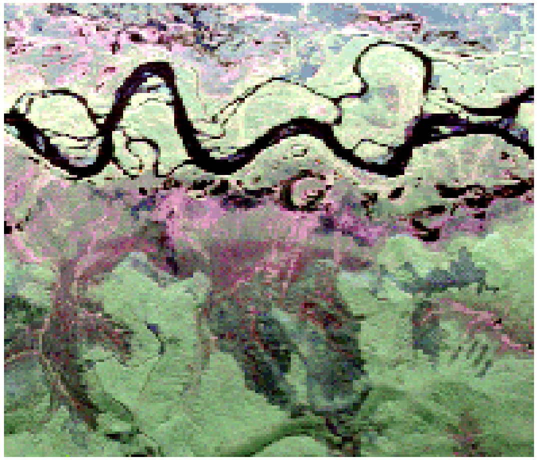


ALOS / PALSAR
JAXA (J)
January 2006
L-Band (Sngl / Twin / Quad)



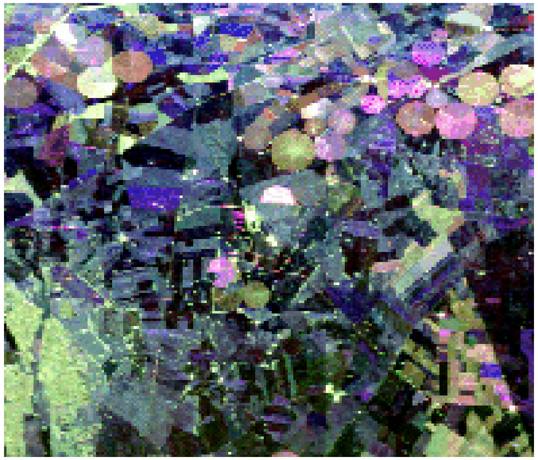
RADARSAT 2
CSA / MDA (CA)
December 2007
C-Band (Quad)

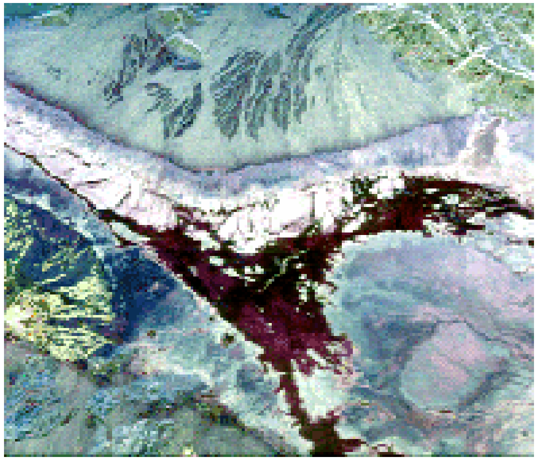
RADAR POLARIMETRY

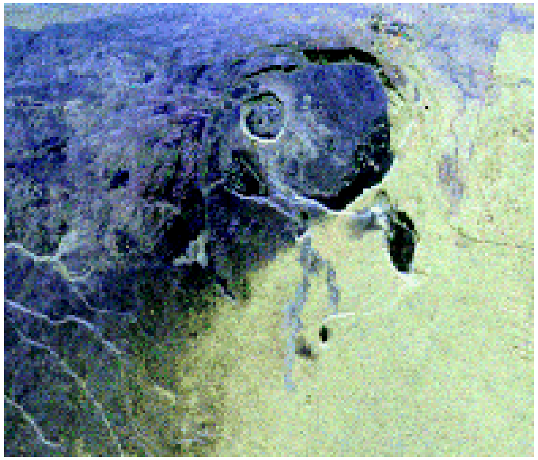

BONANZA CREEK (USA)

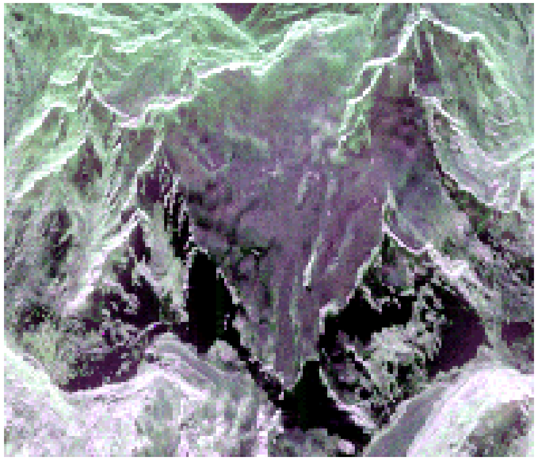
SIR-C

1994 (2 MISSIONS)


LAMANCHA (S)


DEATH VALLEY (USA)


KILAUEA CRATER (USA)


OETZTAL (AUT)

RADAR POLARIMETRY

Pol-InSAR Optimisation

Selenge-River, Kudara/Buryatia,Russia

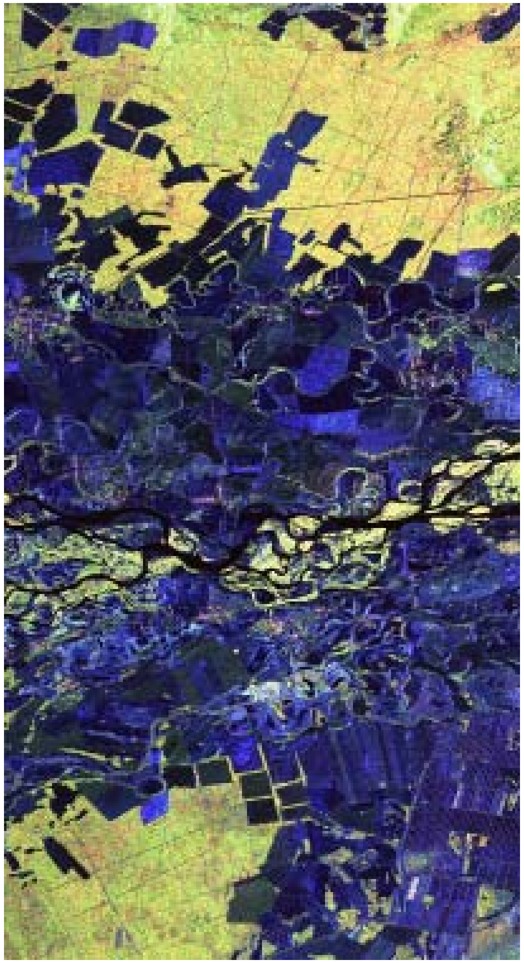
C-band

< Selenge River >

HH-VV

2HV

HH+VV



L-band

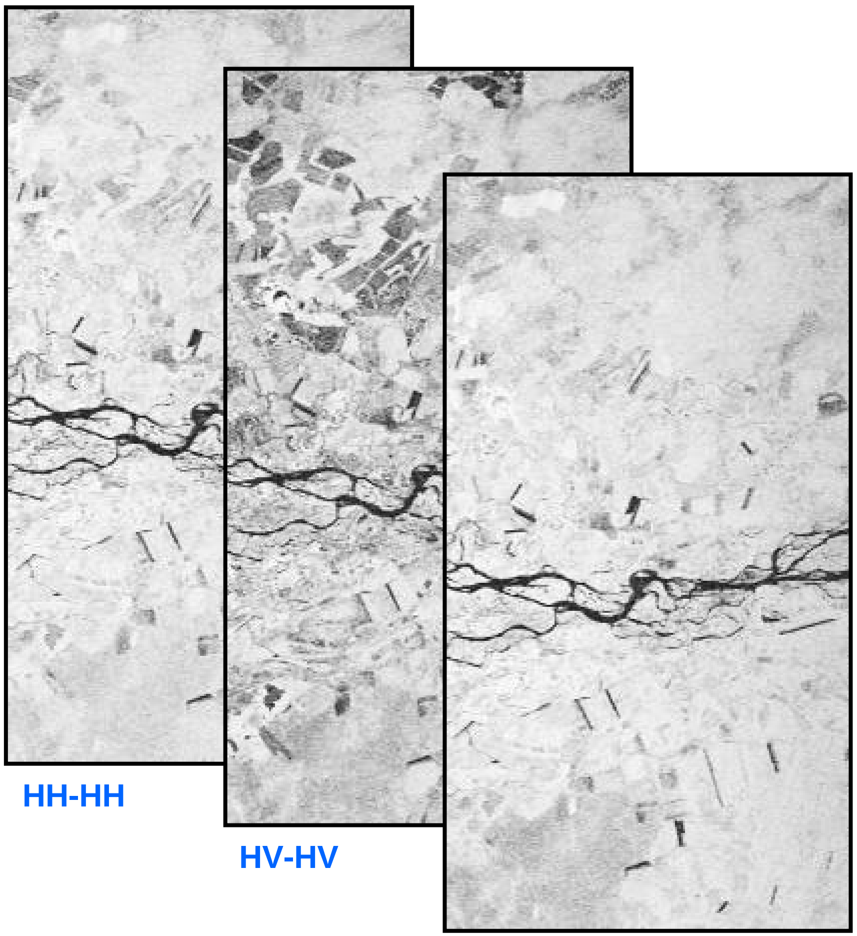
K. Papathanassiou (PhD Dissertation)

Courtesy of Prof. W.M. Boerner

Pol-InSAR Optimisation

Selenge-River, Kudara/Buryatia,Russia

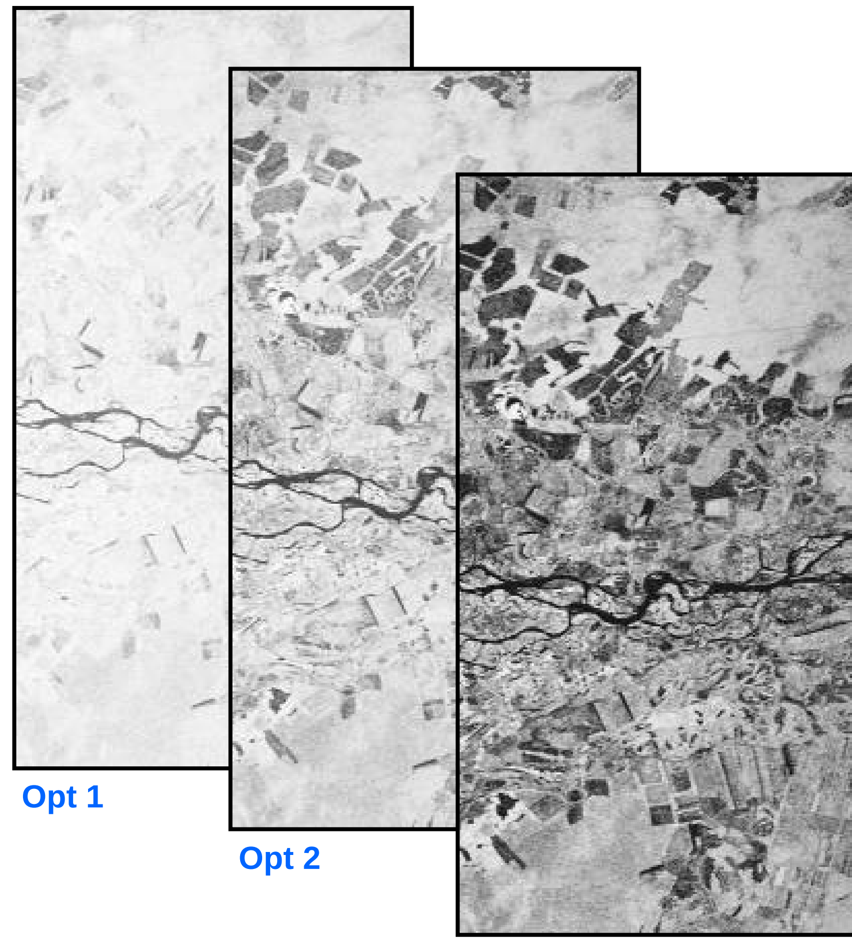
Interferometric Coherence Images



HH-HH

HV-HV

VV-VV



Opt 1

Opt 2

Opt 3

K. Papathanassiou (PhD Dissertation)

Courtesy of Prof. W.M. Boerner

Pol-InSAR Optimisation

Selenge-River, Kudara/Buryatia,Russia

Phase Difference between Scattering Mechanisms

K. Papathanassiou (PhD Dissertation)

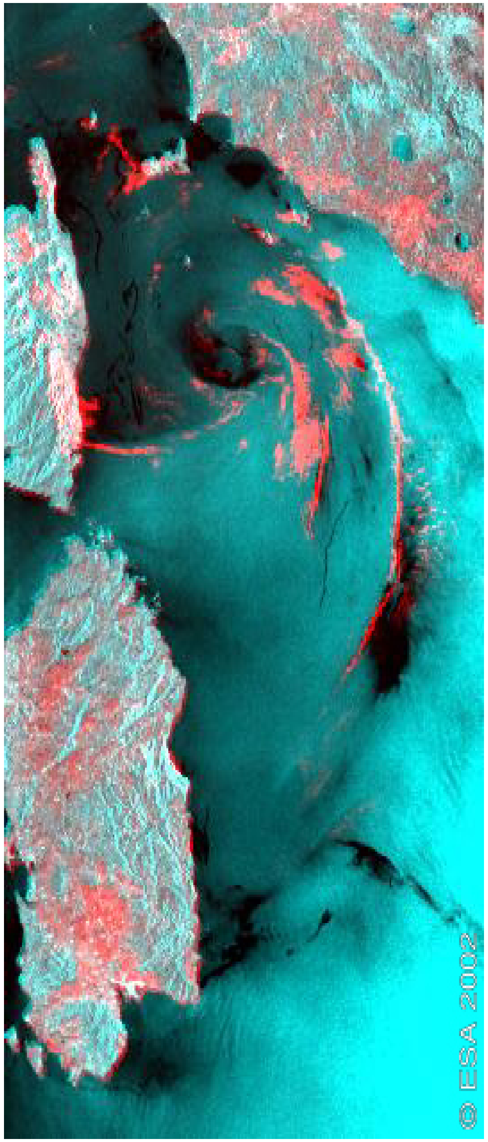
Courtesy of Prof. W.M. Boerner

ENVISAT - ASAR

2002
C-Band
(Sngl / Twin)

RADAR POLARIMETRY

ENVISAT – ASAR – MERIS



RADAR POLARIMETRY

ENVISAT - ASAR

BRETAGNE

Instrument: (ASAR)

Date of Acquisitions:
5 October 2003
20 December 2003
22 February 2004

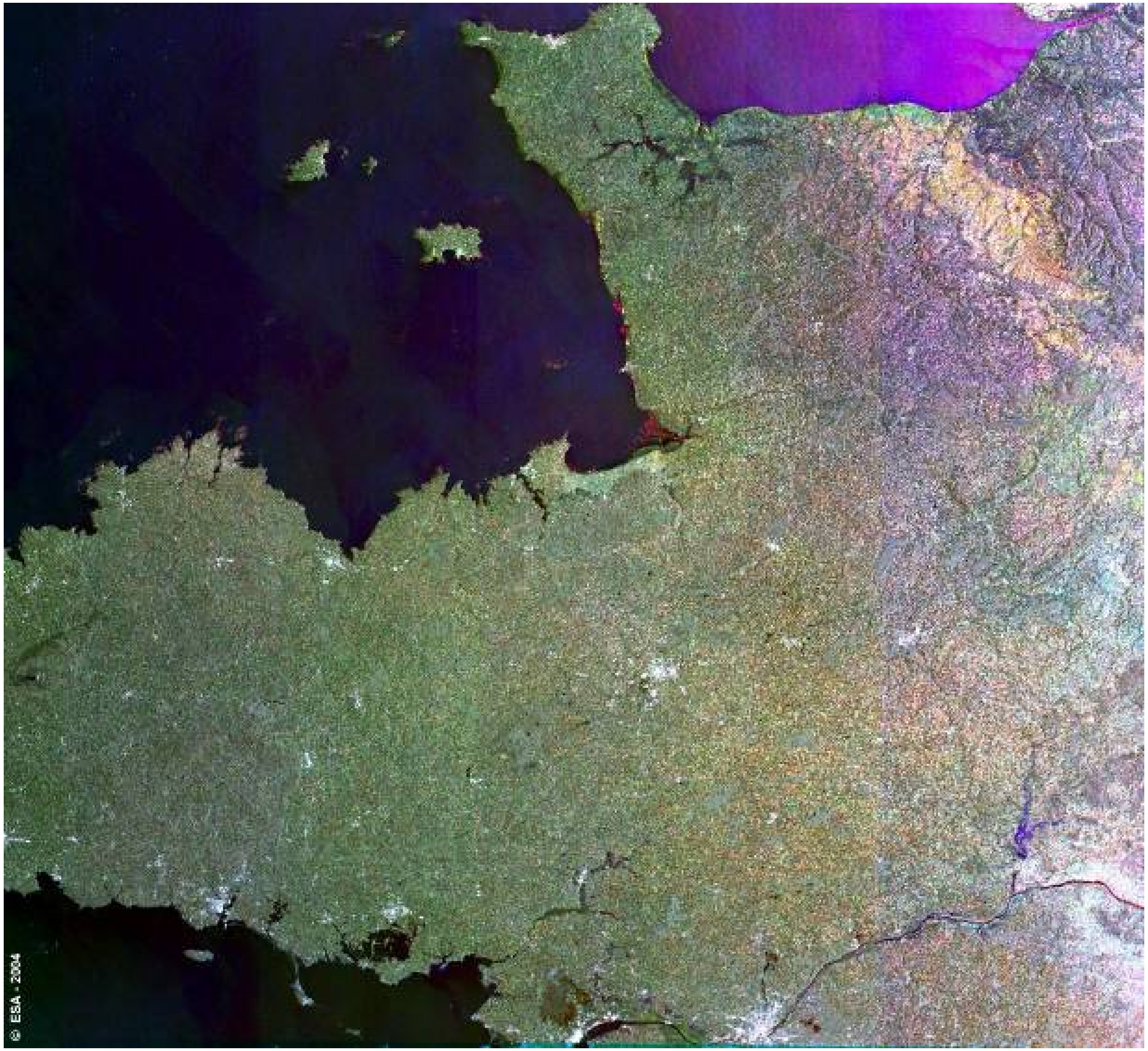
Instrument features:
Image Mode Precision (150 m resolution)

ASAR Mode: Image Swath 2

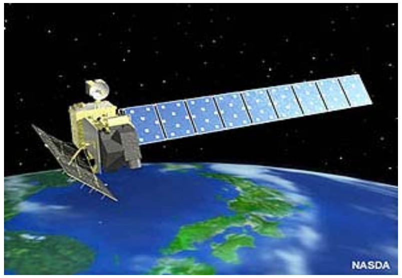
Orbit Direction: Descending
Orbit number: 04249

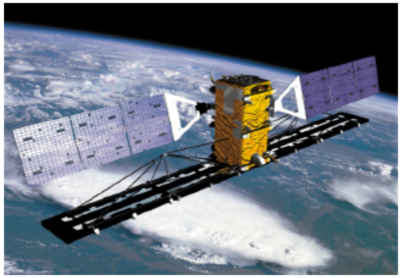
ASAR Polarization: V/V

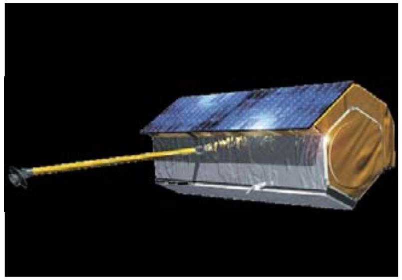
Coordinates:
NW Lat/Long: N 50.42 / W 3.37
NE Lat/Long: N 49.56 / E 2.23
SE Lat/Long: N 46.22 / E 1.07
SW Lat/Long: N 47.06 / W 4.19



RADAR POLARIMETRY

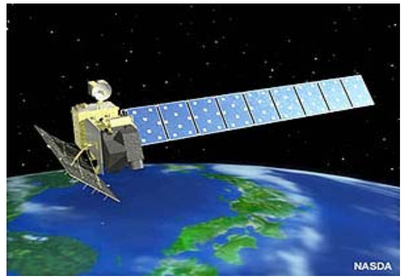
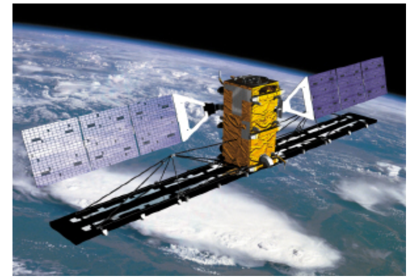
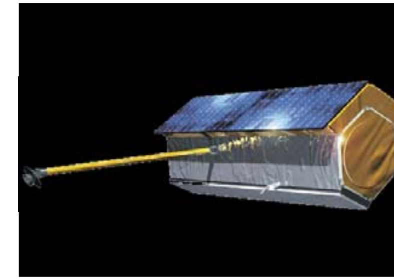






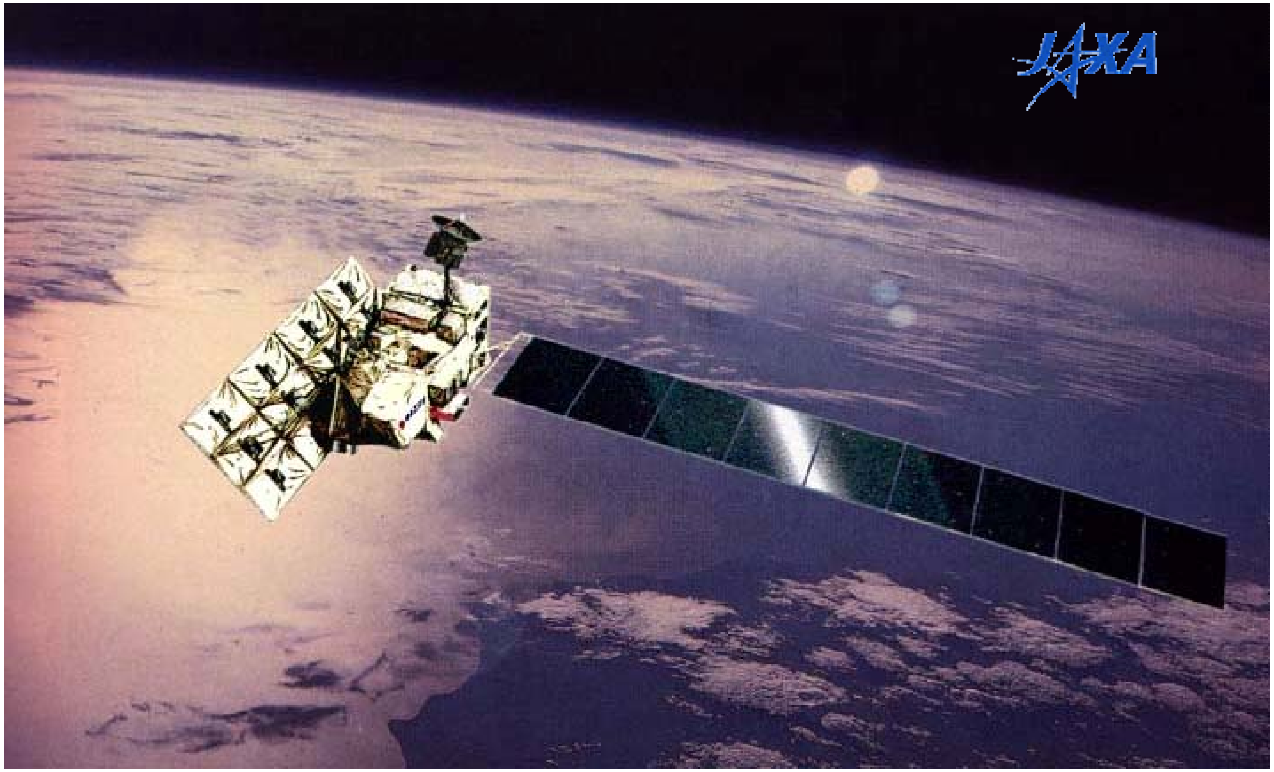
Orbit: LEO, Circular	Sun-synchronous	Sun-synchronous	Sun-synchronous
Repeat Period	46 days	24 days	11 days
Equatorial Crossing Time (*hrs*)	22:30 (ascending)	18:00 (ascending)	18:00 (ascending)
Inclinaison (*deg*)	98.16	98.60	97.44
Equatorial Altitude (*km*)	692	798	515
Wavelegth - Band	23cm (L)	5.6 cm (C)	3 cm (X)

RADAR POLARIMETRY

MODE (Resolution / Swath Looks / Polar)			
Standard Stripmap	20 x 10 m / 70 km 2 / HH or VV	25 m / 100 km 4 / HH or VV	3 m / 30 km 1 / HH or VV
Fine	10 m / 70 km 1 / HH or VV	8 m / 50 km 8 / HH or VV	1 m (spolight) / 10 km 1 / HH or VV
ScanSAR	100 m / 350 km 8 / HH or VV	100 m / 500 km 8 / HH or VV	16 m / 100 km 1 / HH or VV
Quad Polarisation	30 x 10 m / 30 km 2 / Quad Pol	*Standard* 25 x 8 m / 25 km 4 / Quad Pol *Fine* 8 x 8 m / 25 km 1 / Quad Pol	3 m / 15 km 1 / Quad Pol

ALOS - PALSAR

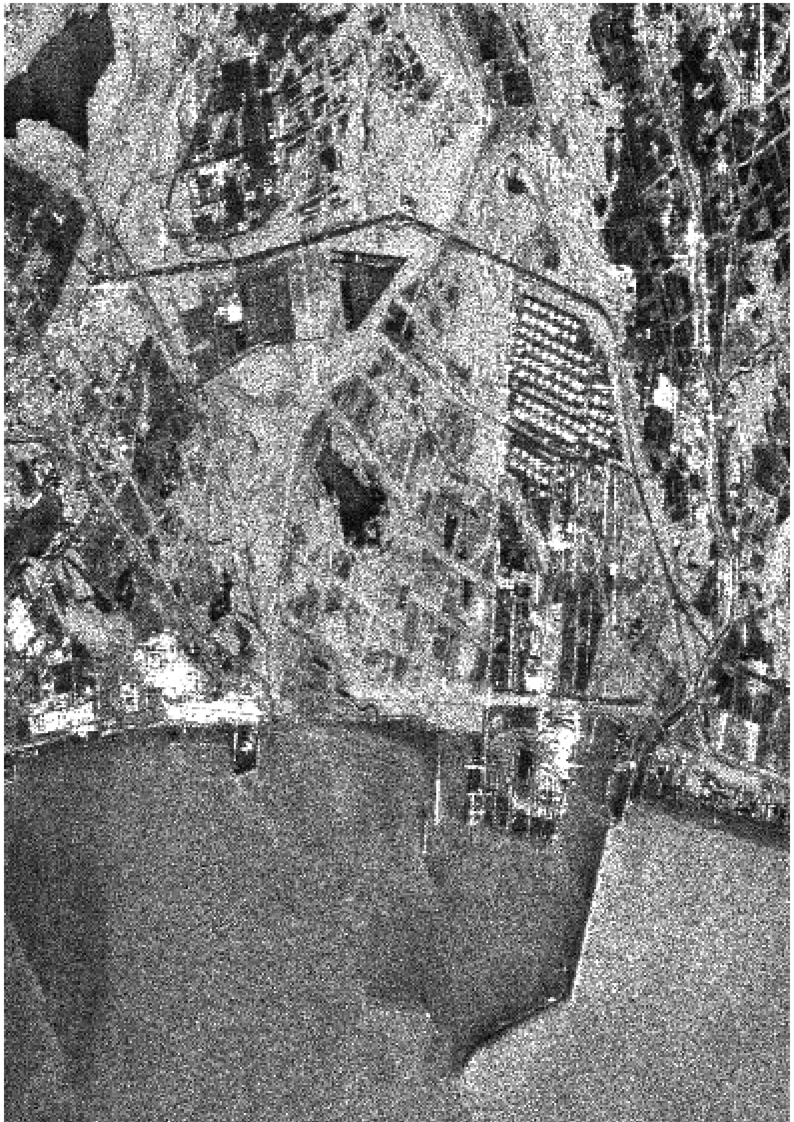
January 2006
L-Band (Sngl / Twin / Quad)



ALOS : Advanced Land Observing Satellite
PALSAR : Phase Array L-Band SAR

RADAR POLARIMETRY

Tamakomai – Sapporo (Japan)

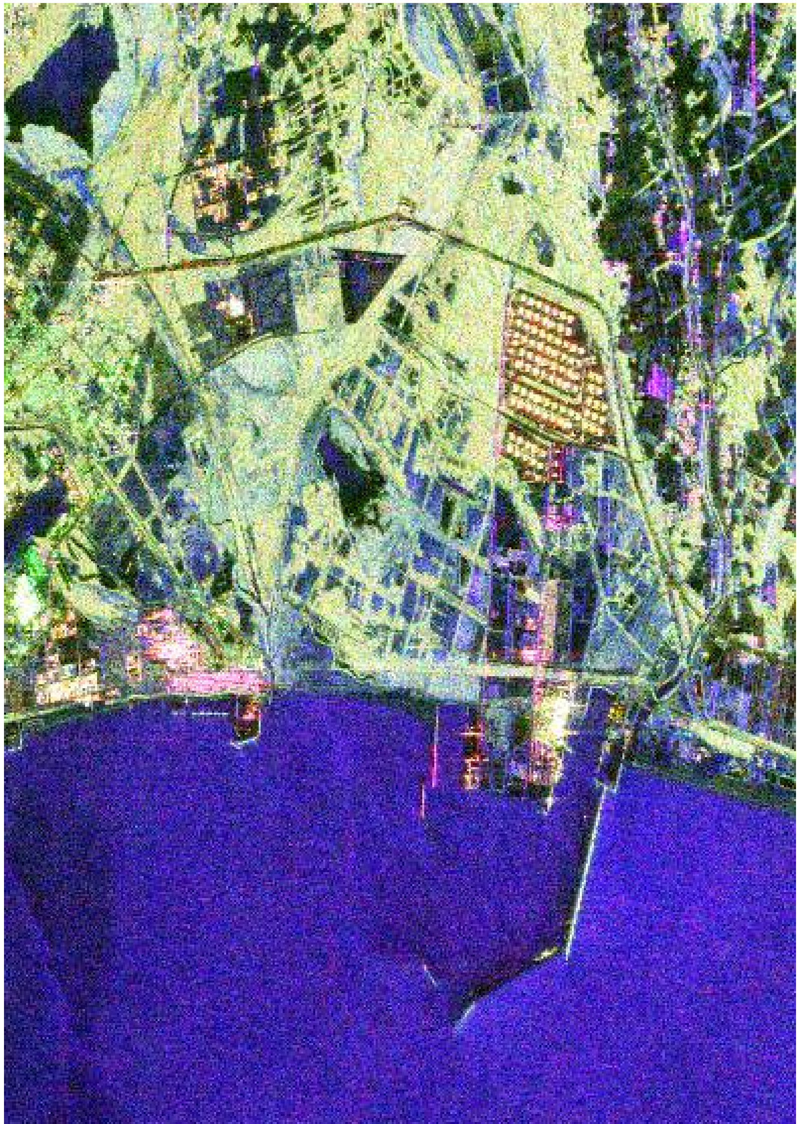


|HH|

GOOGLE EARTH

RADAR POLARIMETRY

Tamakomai – Sapporo (Japan)

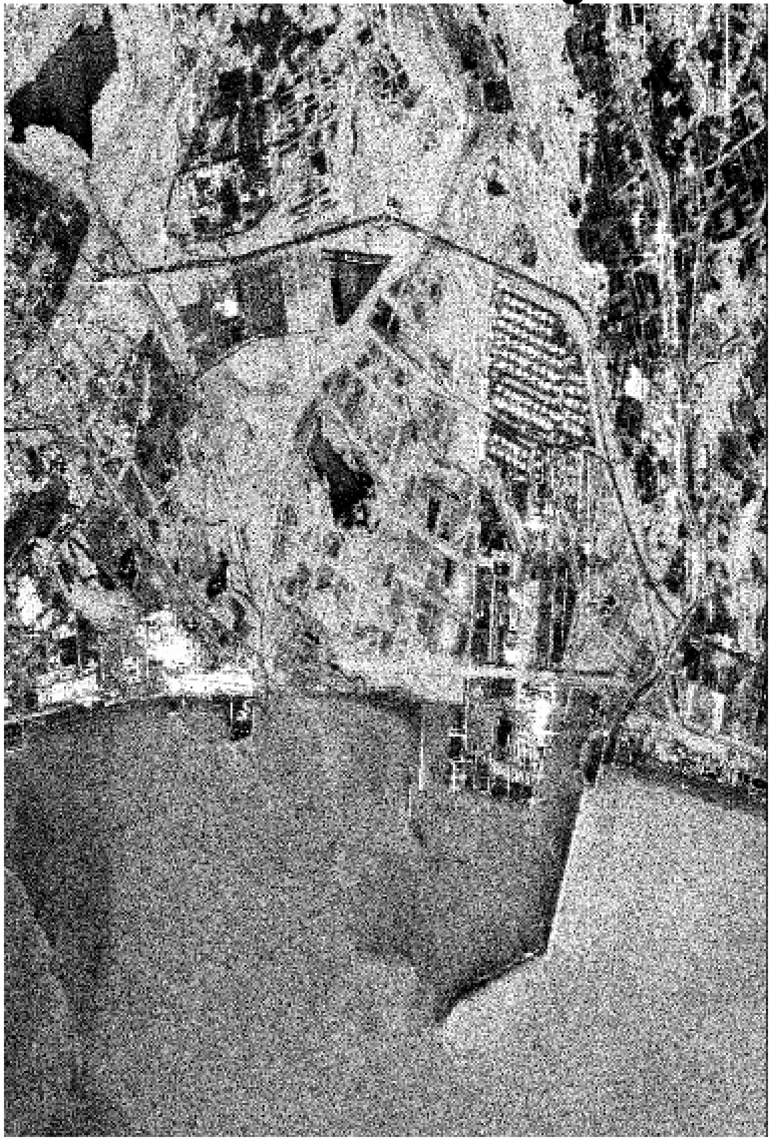


|HH-VV|, |HV|, |HH+VV|

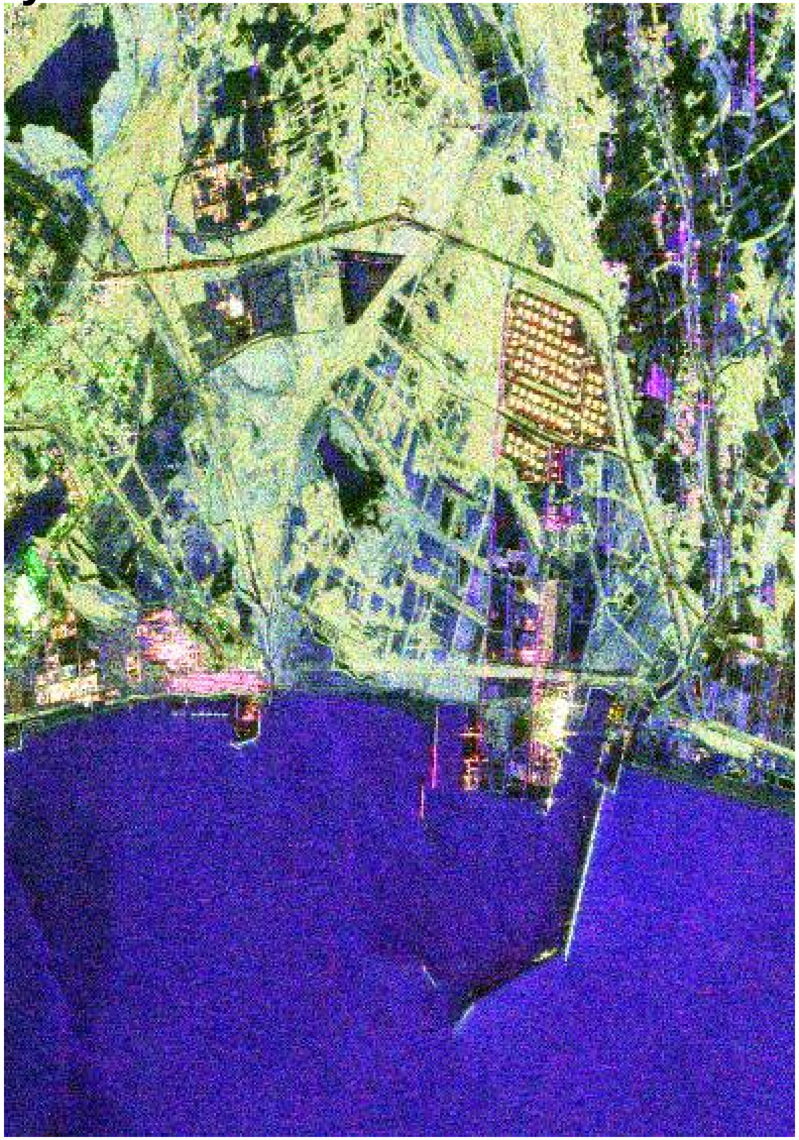
GOOGLE EARTH

RADAR POLARIMETRY

Single vs. Fully Polarized

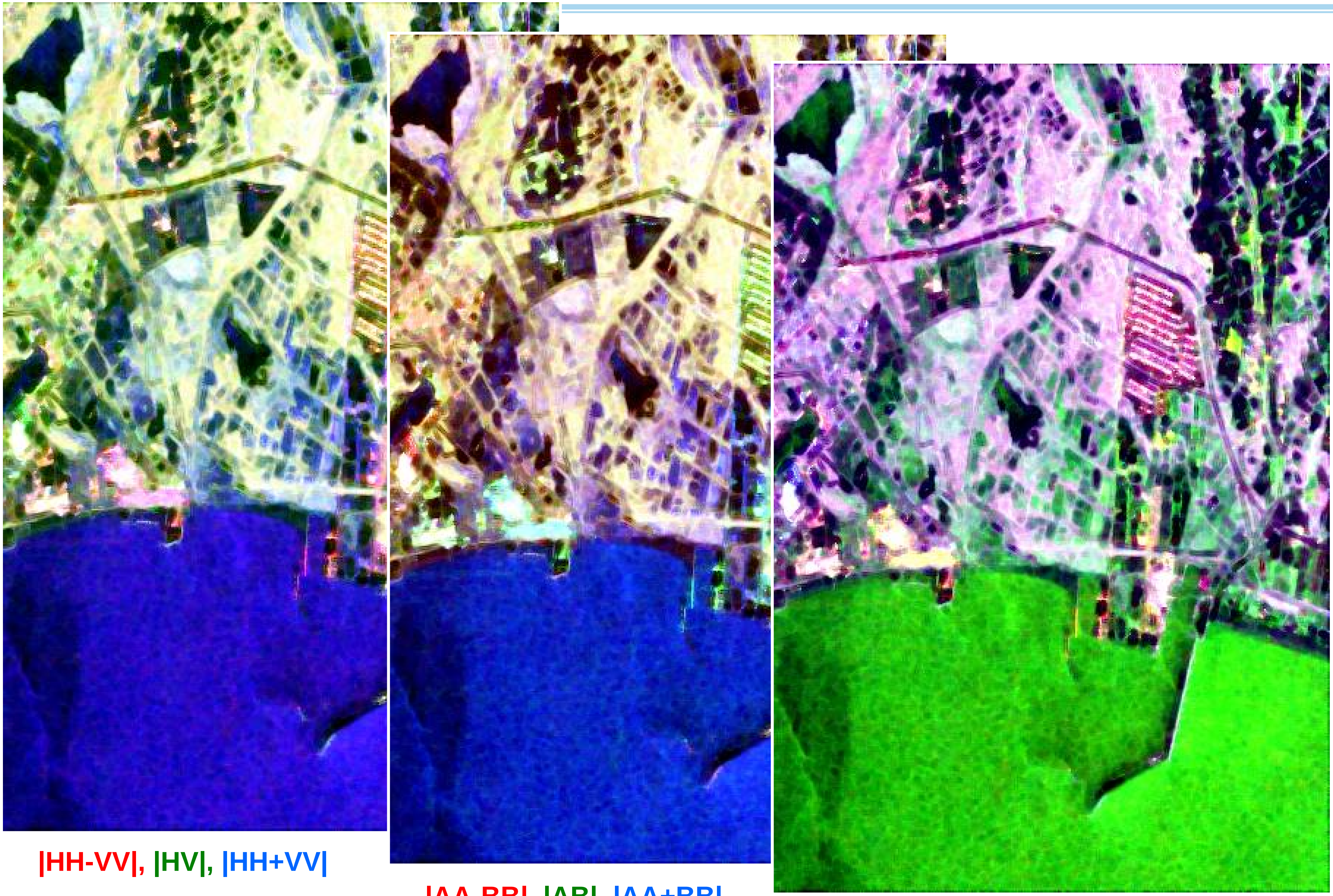


|HH|



|HH-VV|, |HV|, |HH+VV|

RADAR POLARIMETRY

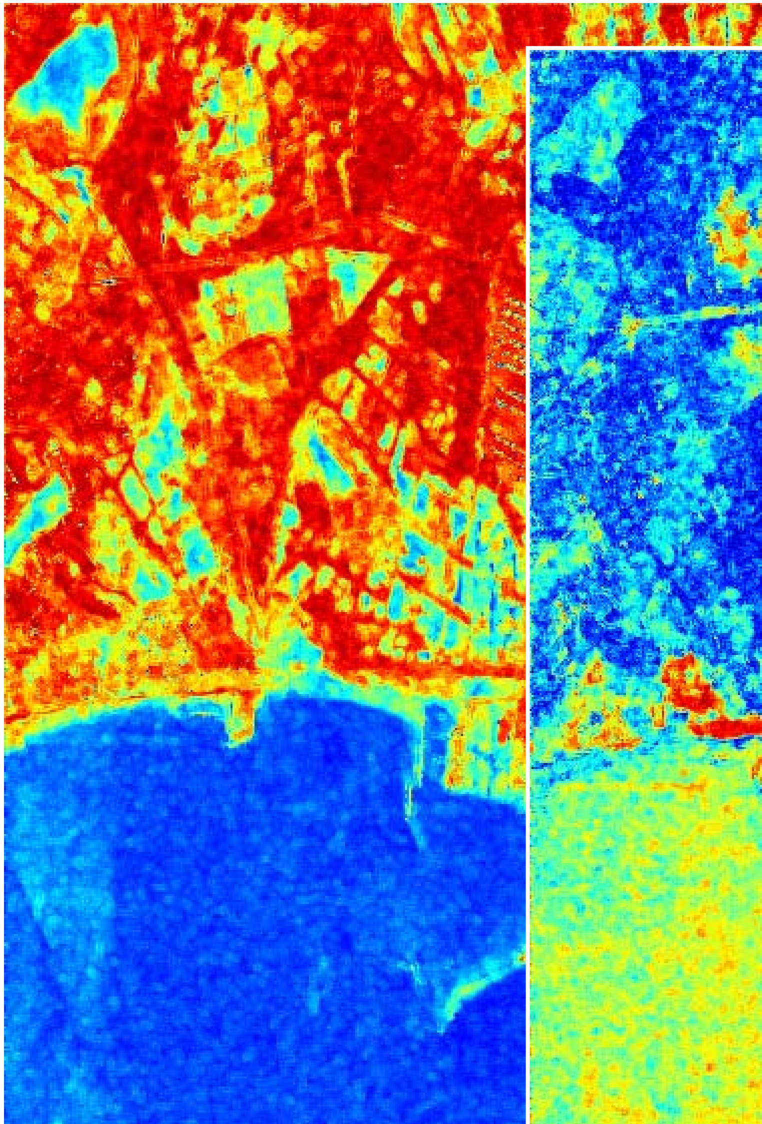


|HH-VV|, |HV|, |HH+VV|

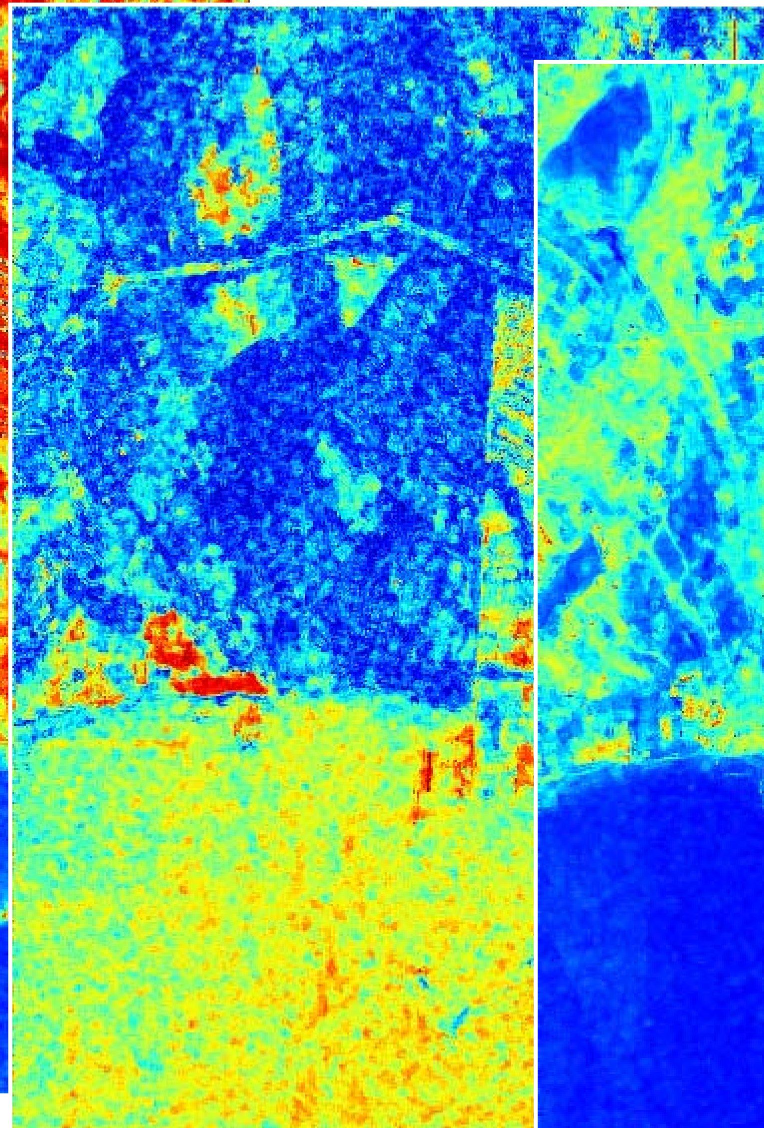
|AA-BB|, |AB|, |AA+BB|

|LL-RR|, |LR|, |LL+RR|

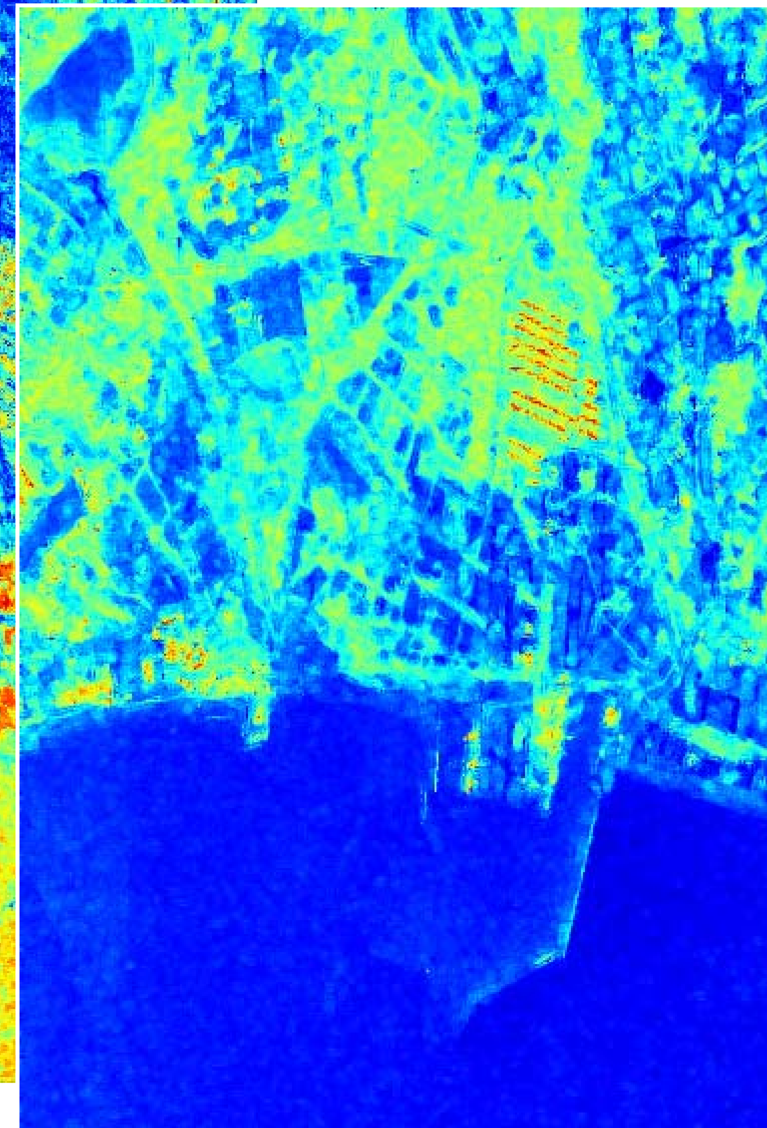
RADAR POLARIMETRY



Entropy

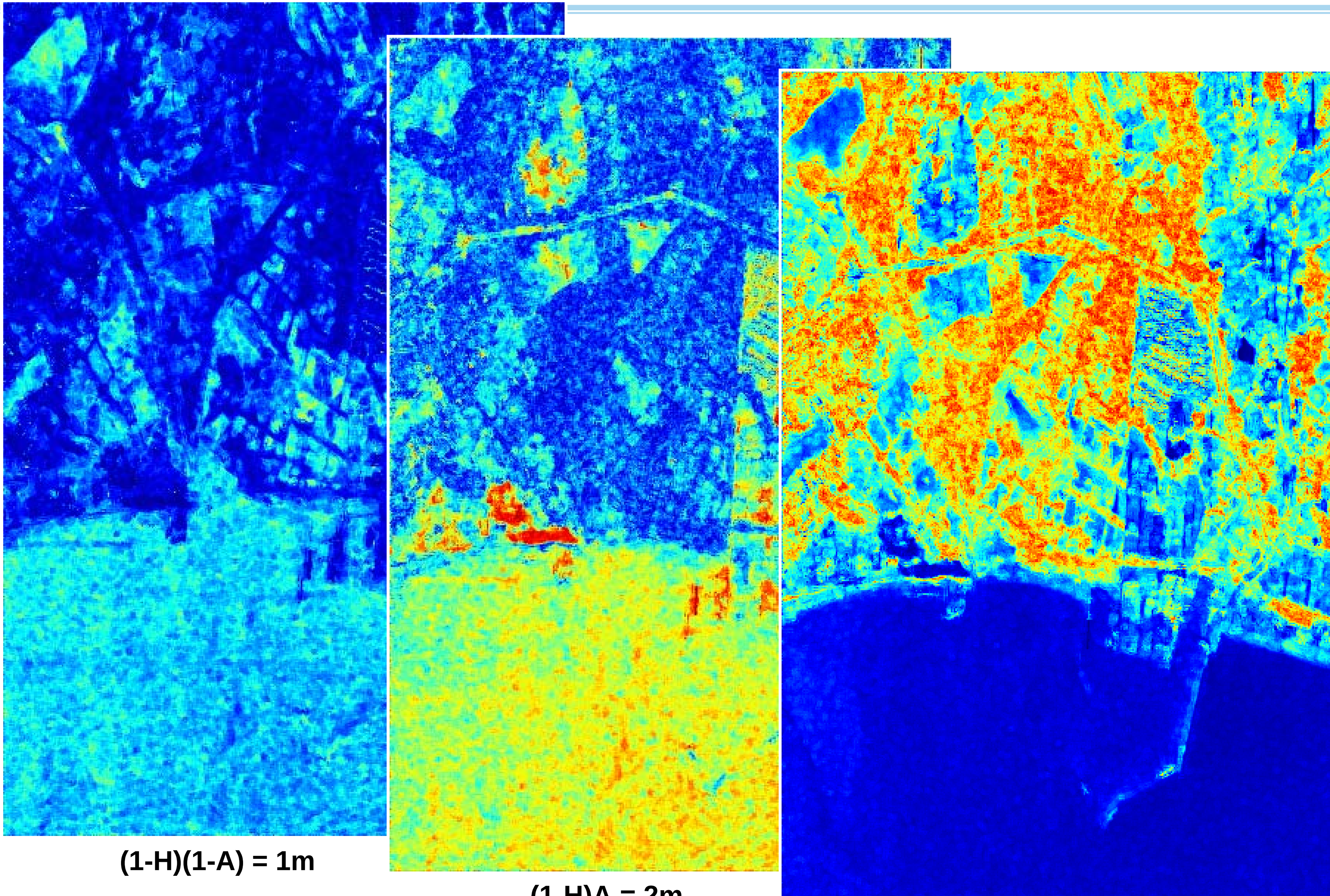


Anisotropy



Alpha

RADAR POLARIMETRY

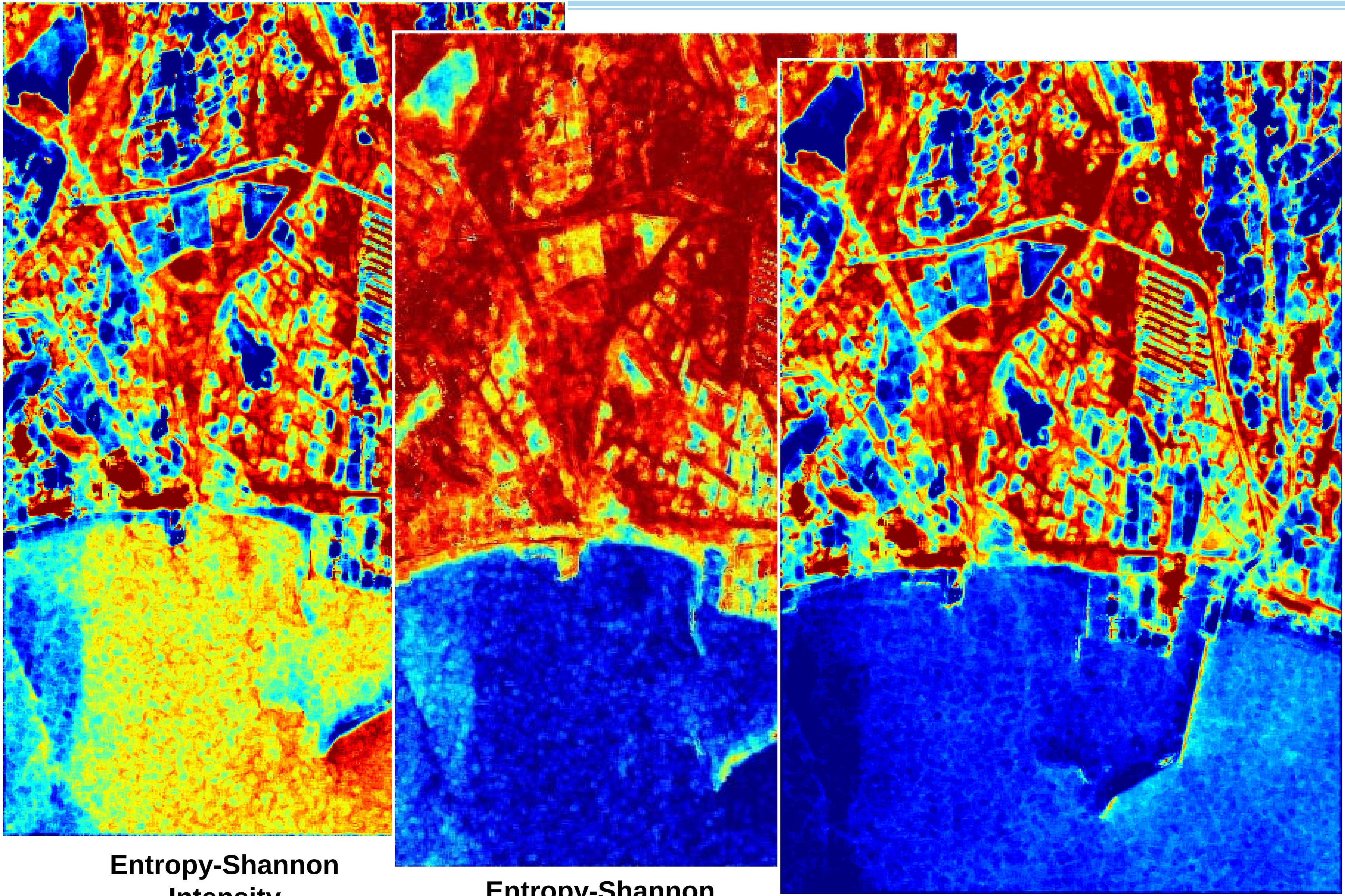


(1-H)(1-A) = 1m

(1-H)A = 2m

H(1-A) = 3m

RADAR POLARIMETRY



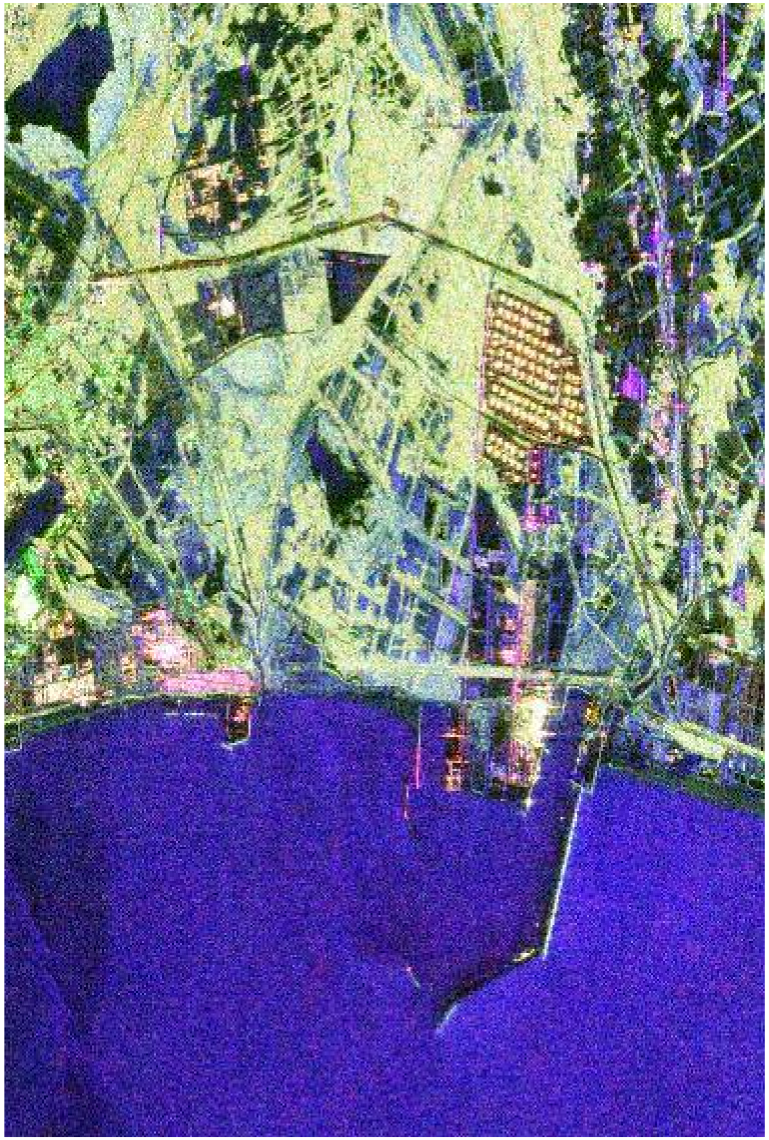
Entropy-Shannon Intensity

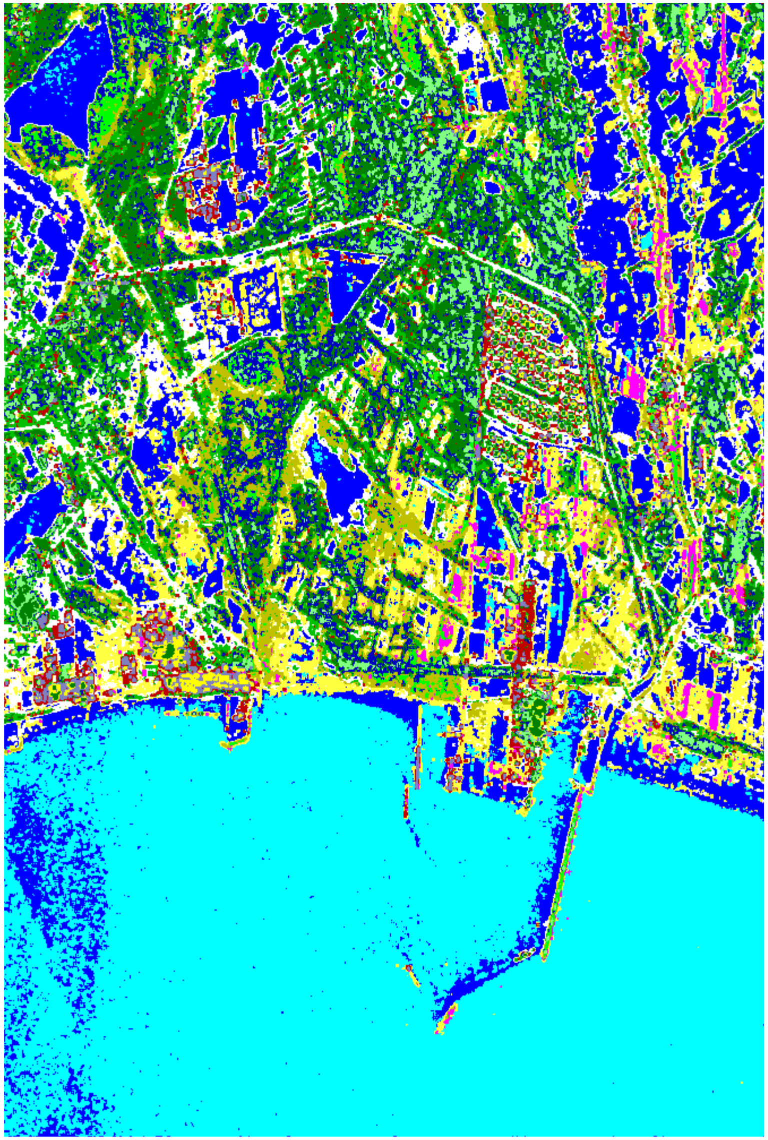
Entropy-Shannon Polarimetry

Entropy-Shannon

RADAR POLARIMETRY

Tamakomai – Sapporo (Japan)



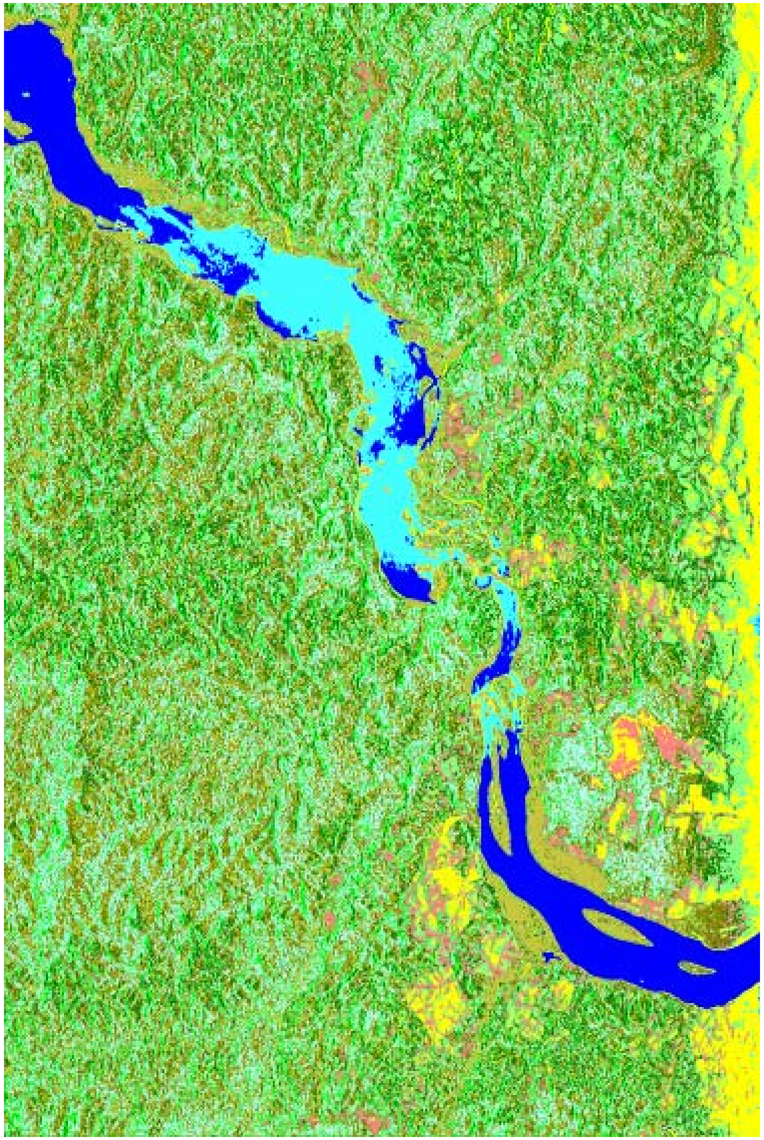


Wishart – H/A/alpha segmentation

E. Pottier **|HH-VV|, |HV|, |HH+VV|**

RADAR POLARIMETRY

Belize (Brazil)

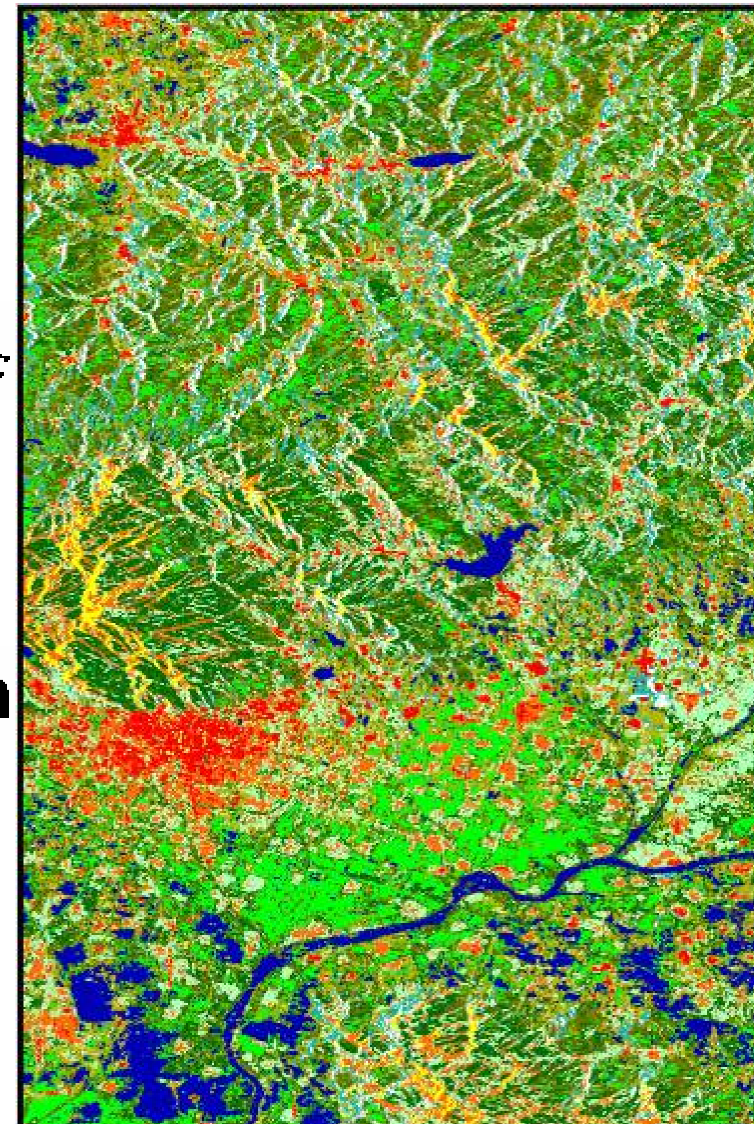


E. P.. |HH-VV|, |HV|, |HH+VV|

Wishart – H/A/alpha segmentation

RADAR POLARIMETRY

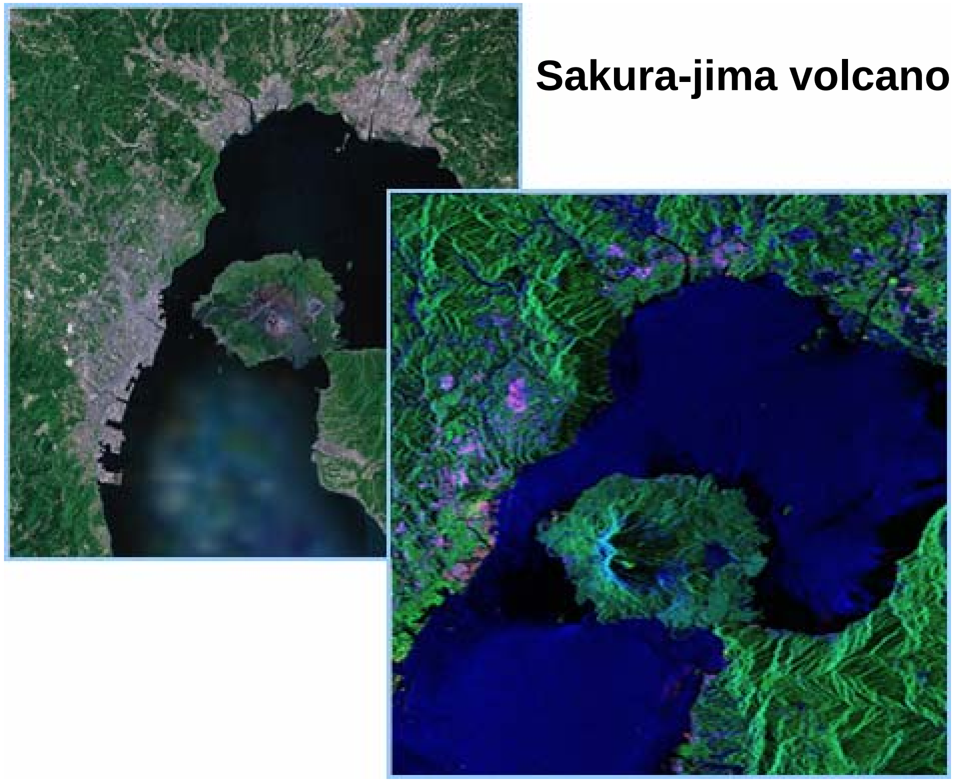
Tai An (China)



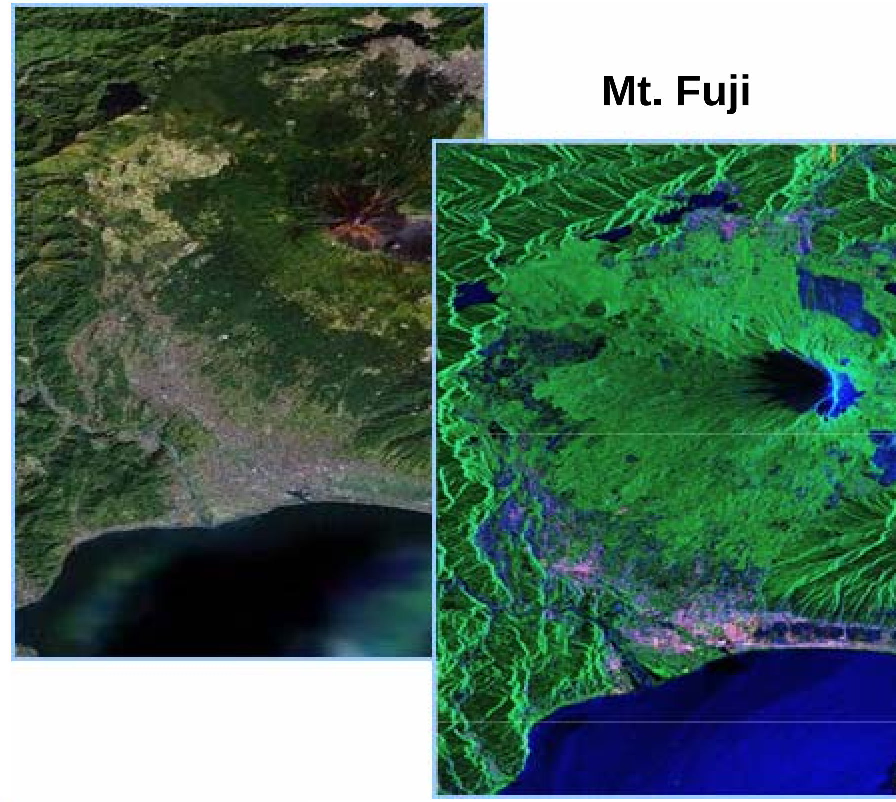
Wishart – H/A/alpha segmentation

|HH-VV|, |HV|, |HH+VV|

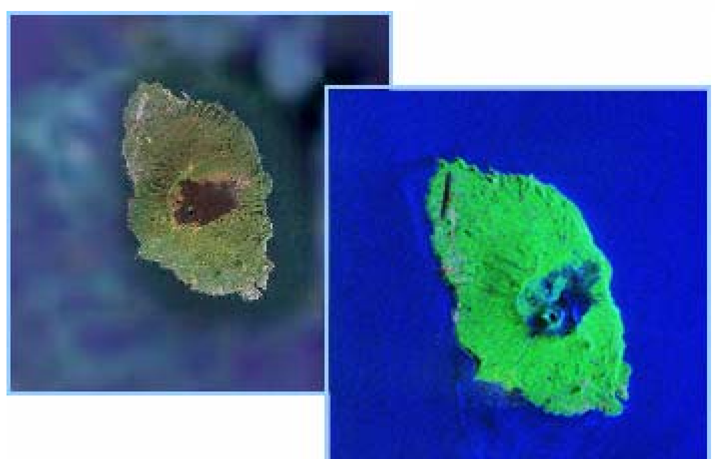
RADAR POLARIMETRY



Sakura-jima volcano



Mt. Fuji



Izu-Oshima island

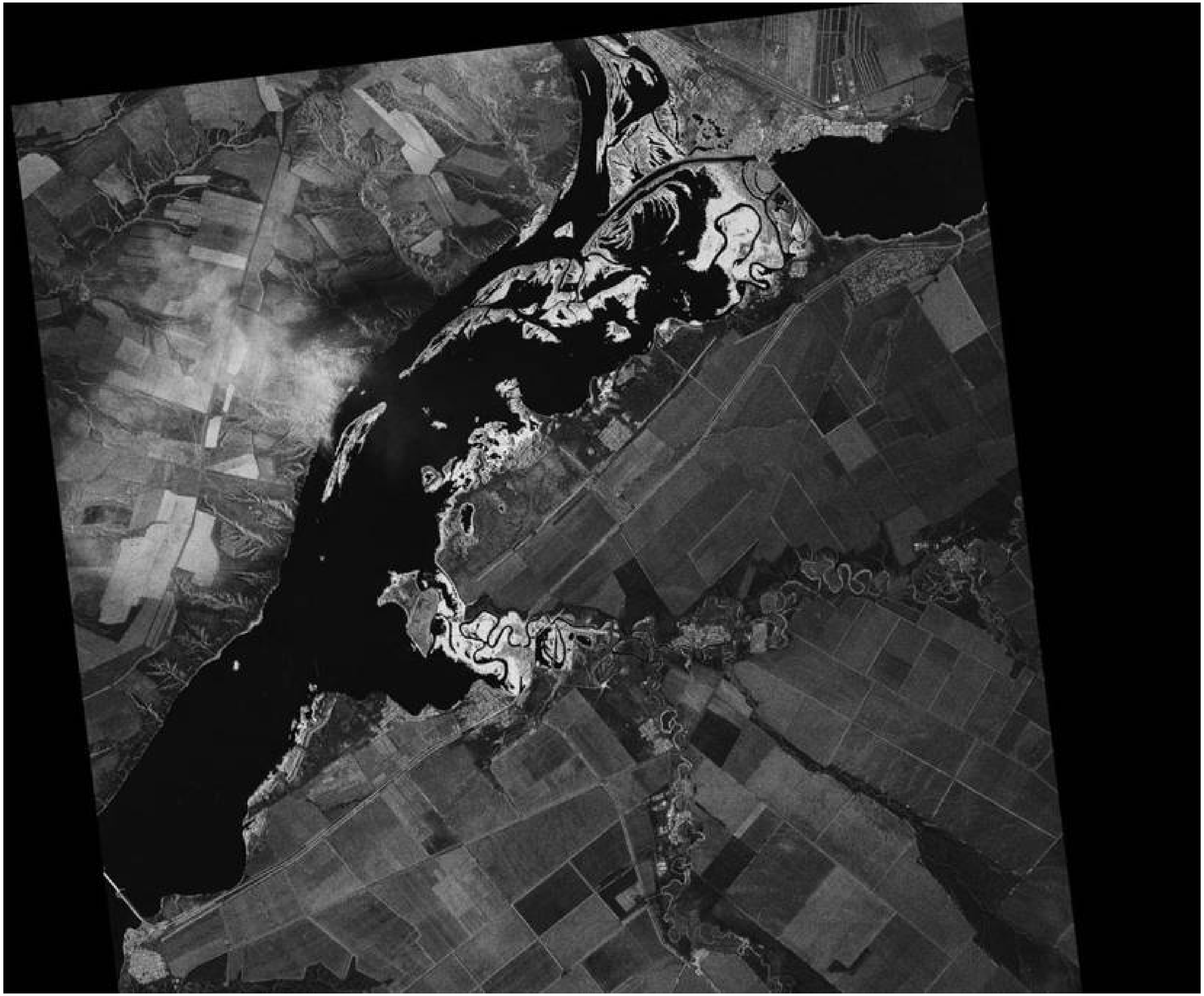
Courtesy of Prof. Y. Yamaguchi

TerraSAR - X

June 2007
X-Band (Sngl / Twin / Quad ?)

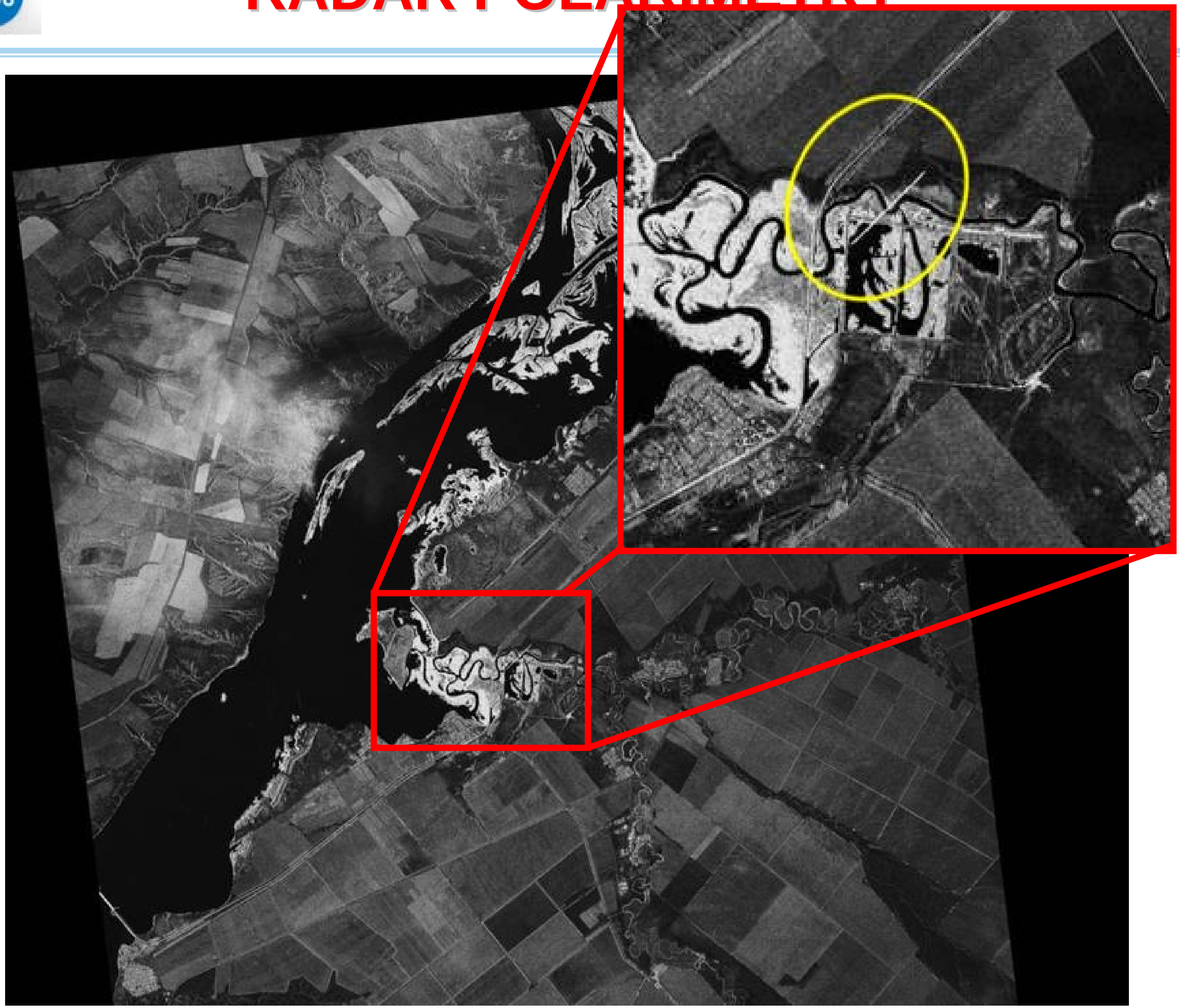
DLR EADS ASTRIUM

RADAR POLARIMETRY



Volgograd (21 / 06 / 07)

RADAR POLARIMETRY



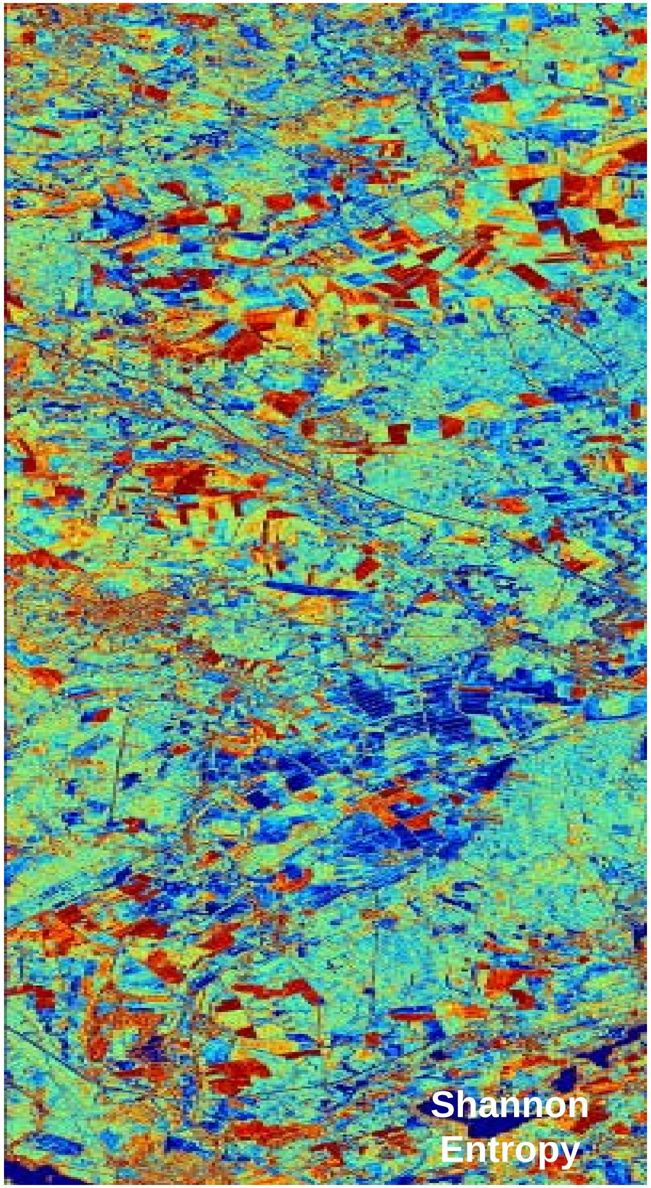
Volgograd (21 / 06 / 07)

RADAR POLARIMETRY

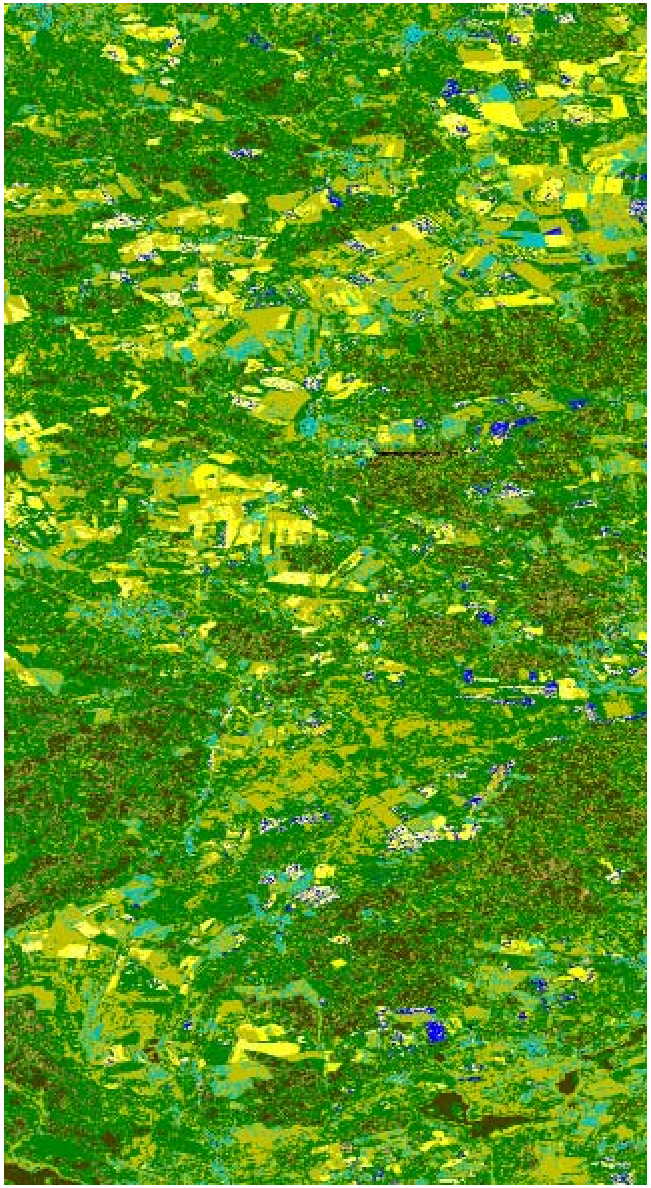
Belzig – Berlin (G)

Dual-Pol (s_{HH}, s_{VV})

RADAR POLARIMETRY



RADAR POLARIMETRY

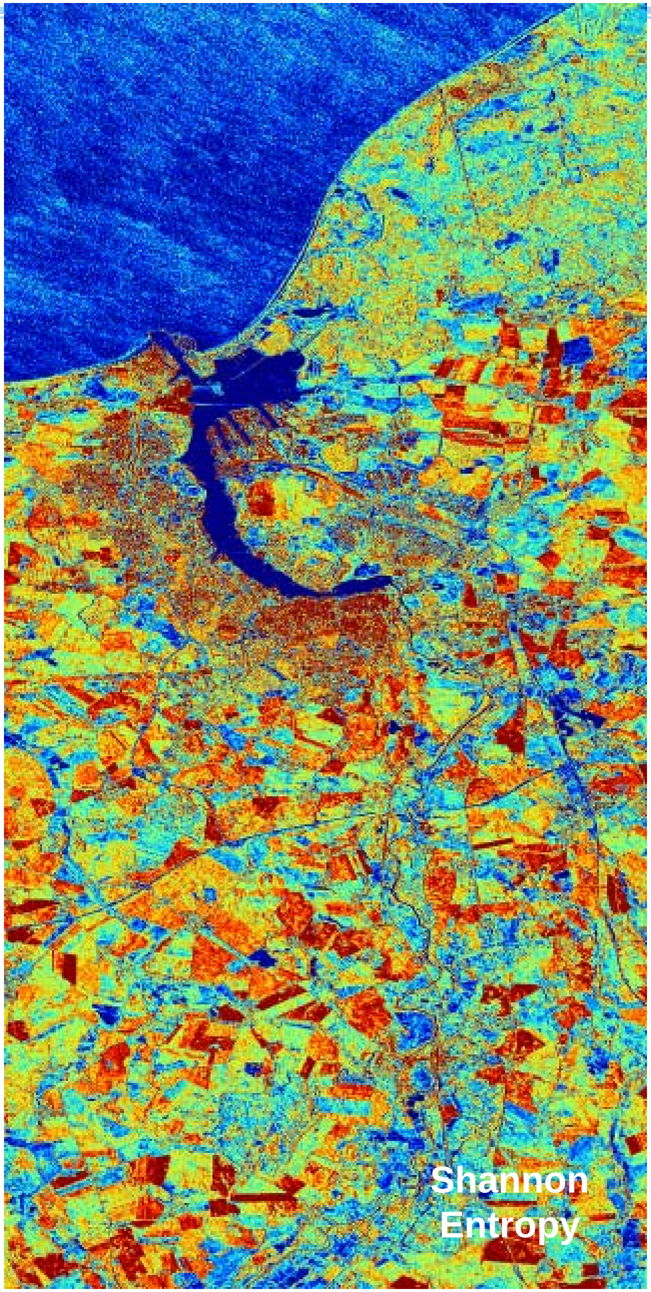


RADAR POLARIMETRY

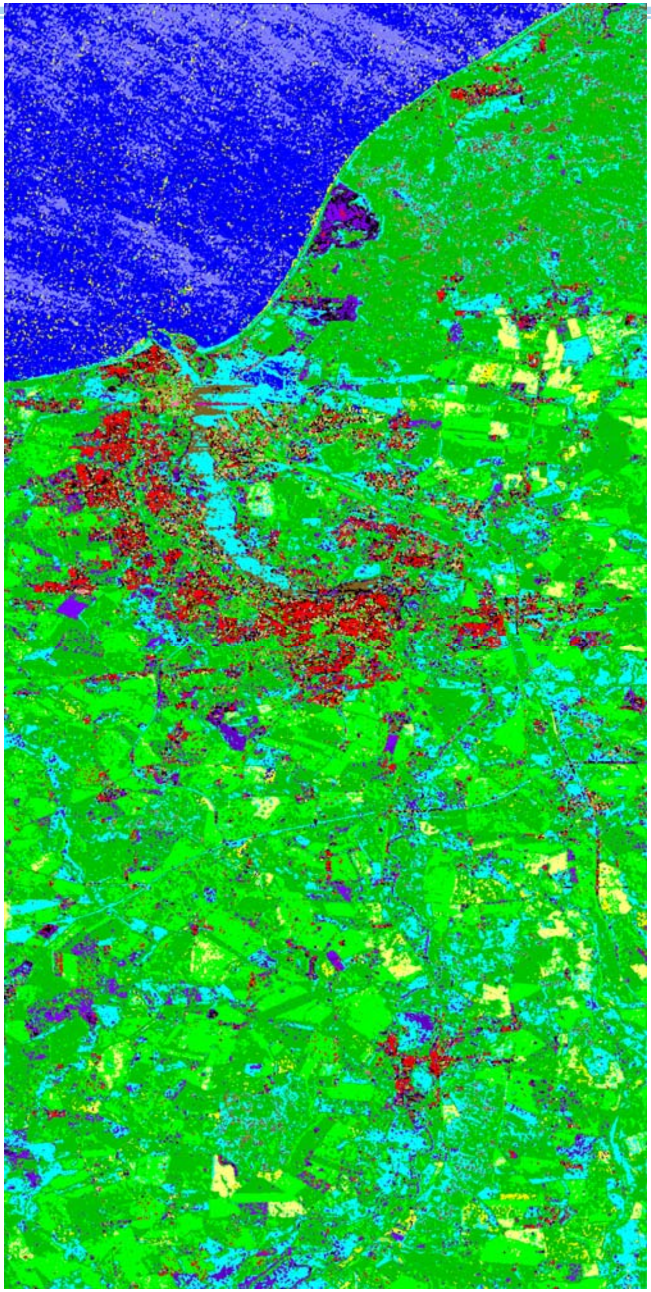
Rostock (G)

Dual-Pol (s_{HH}, s_{VV})

RADAR POLARIMETRY

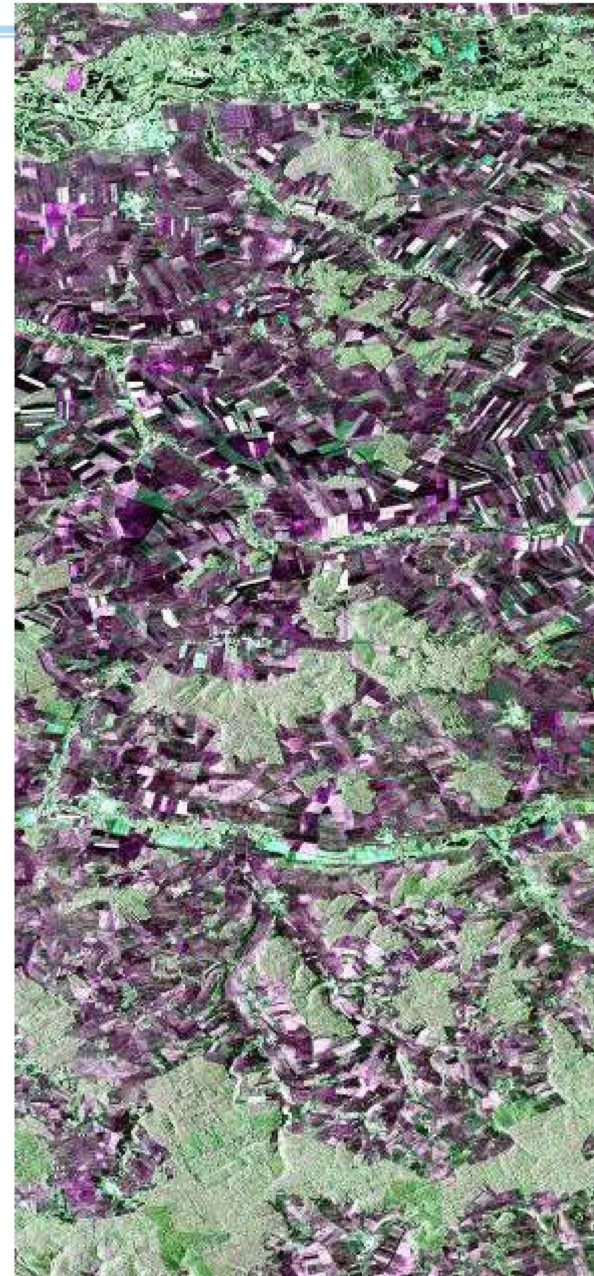


RADAR POLARIMETRY



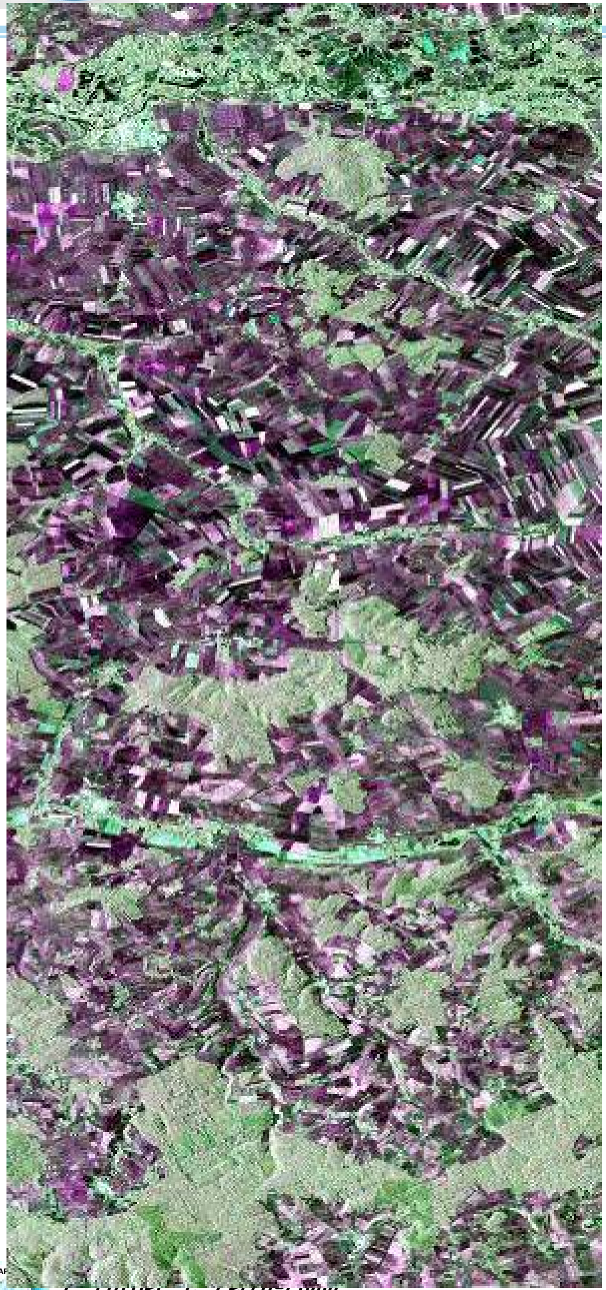
RADAR POLARIMETRY

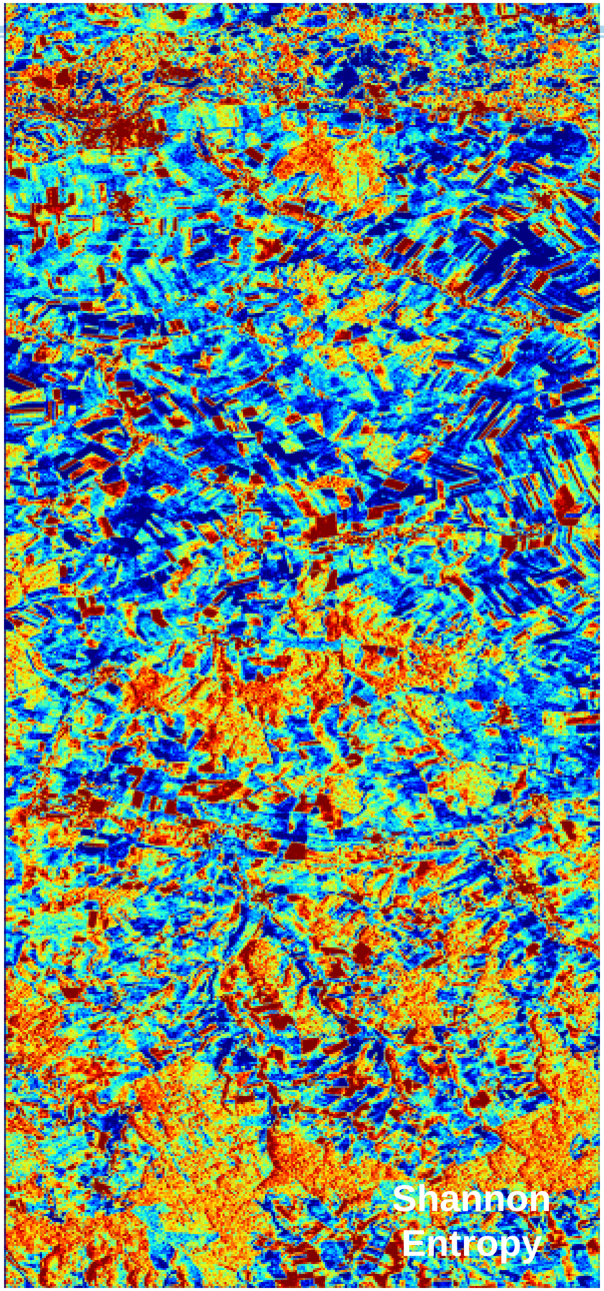
Troyes (F)



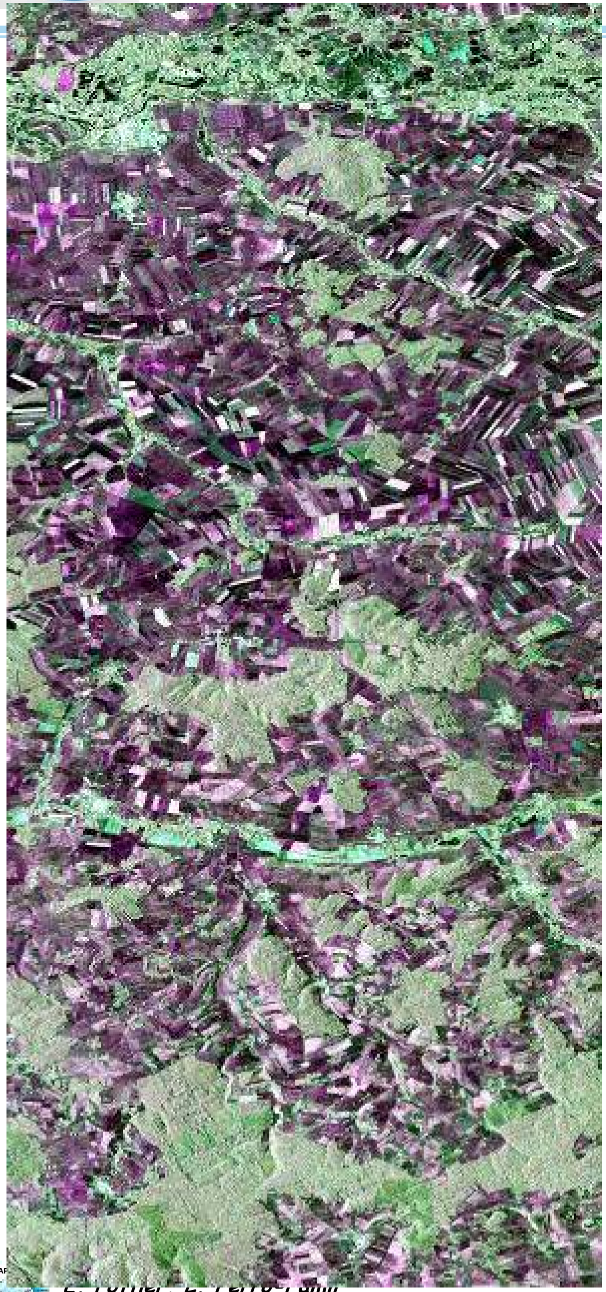
Dual-Pol (s_{HH}, s_{VV})

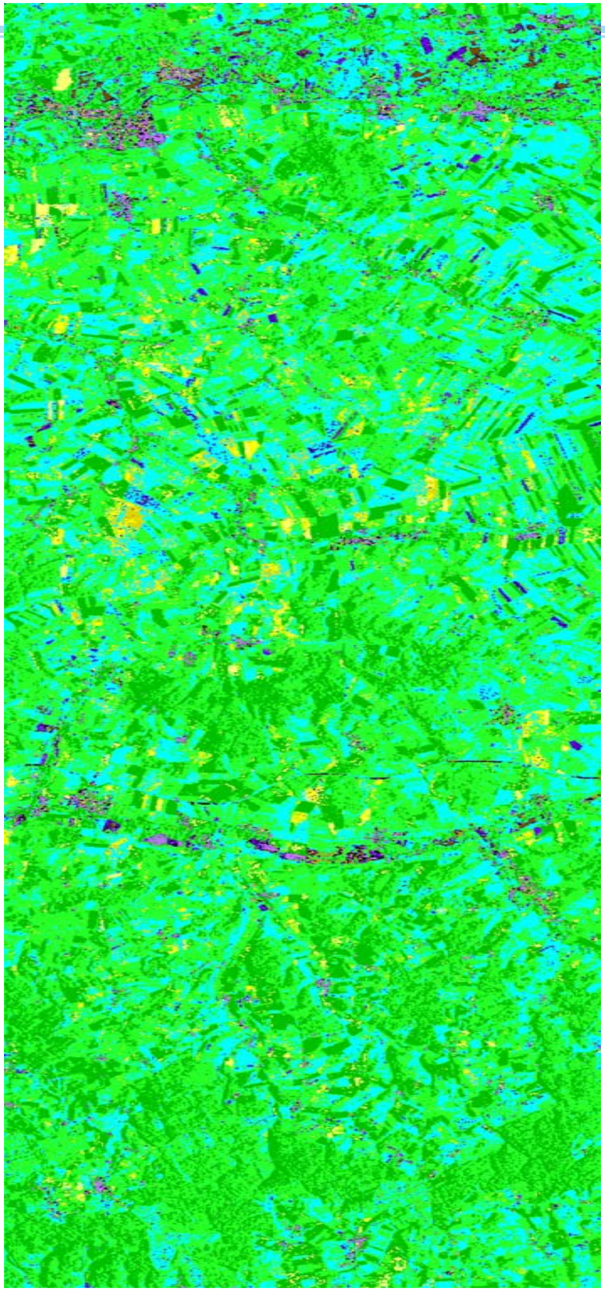
RADAR POLARIMETRY



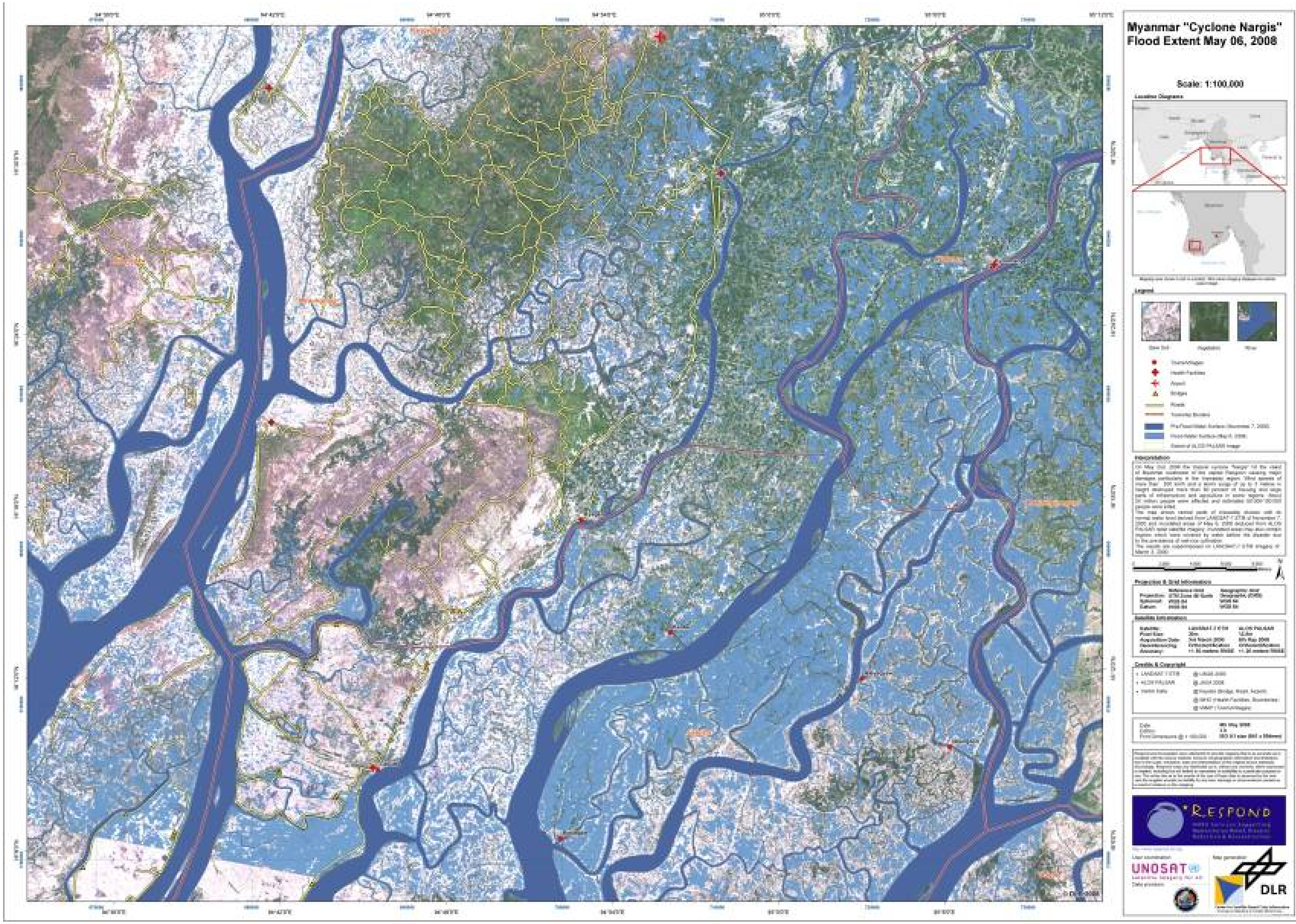


RADAR POLARIMETRY



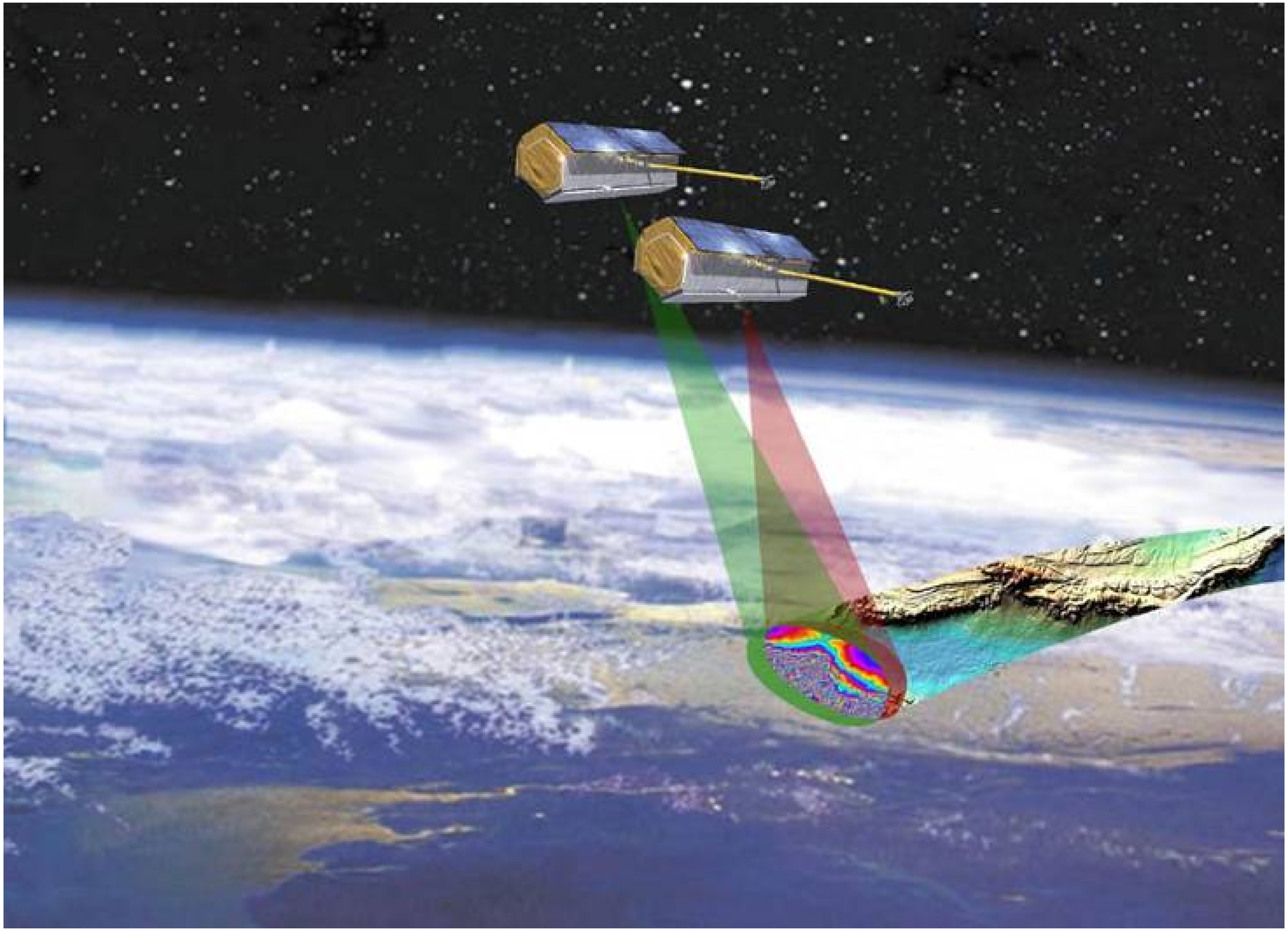


RADAR POLARIMETRY



Courtesy of Prof. W.M. Boerner

TerraSAR–X – TANDEM–X (2009)

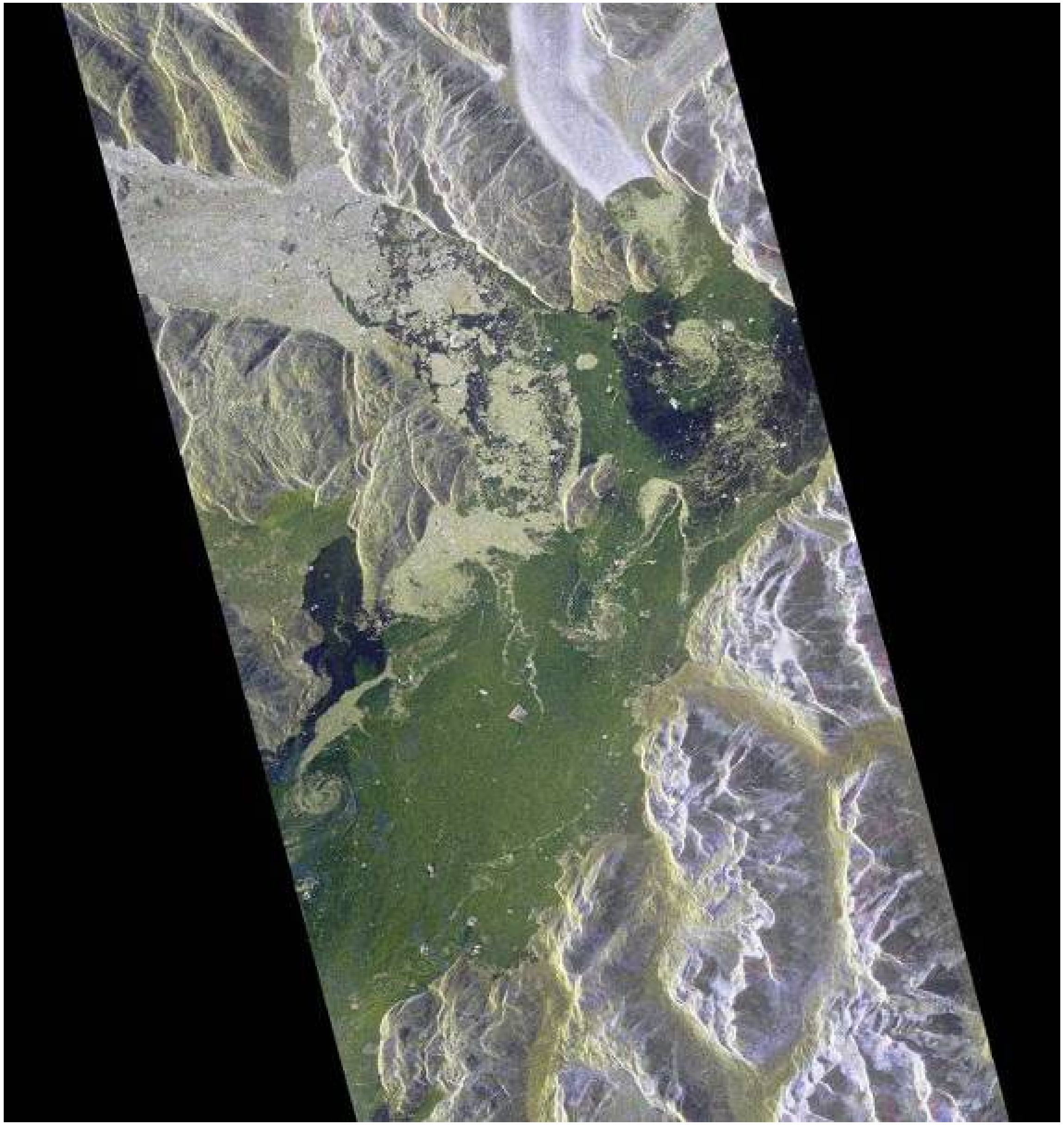


RADAR POLARIMETRY

RADARSAT - 2

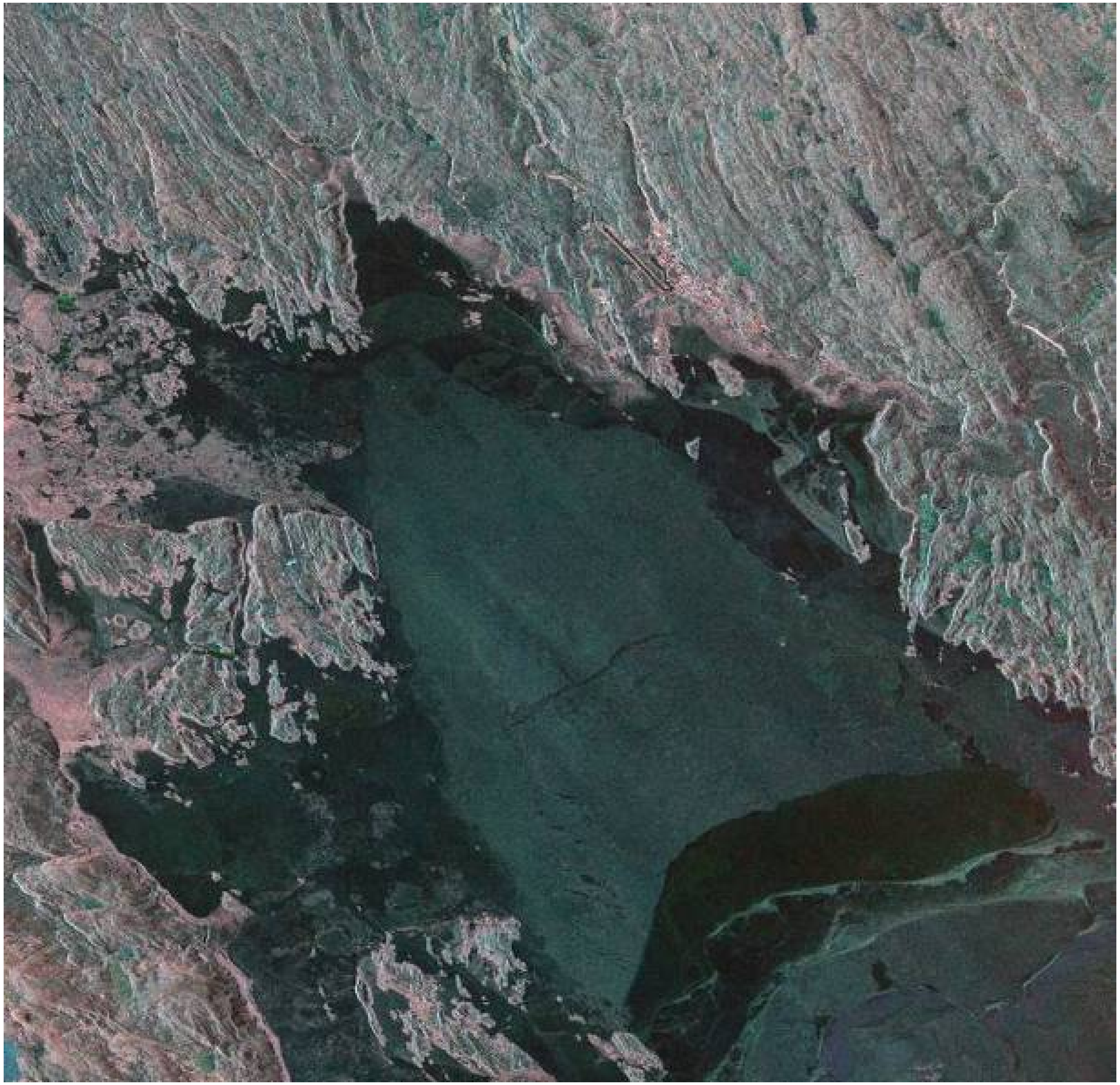
Agence spatiale canadienne   Canadian Space Agency

RADAR POLARIMETRY



 Fenrisgletscher glacier – Sermilik fjord (18/12/2007)

RADAR POLARIMETRY



 Koojesse inlet - Frobisher Bay, southeast Baffin Island (07/01/2008)

RADAR POLARIMETRY

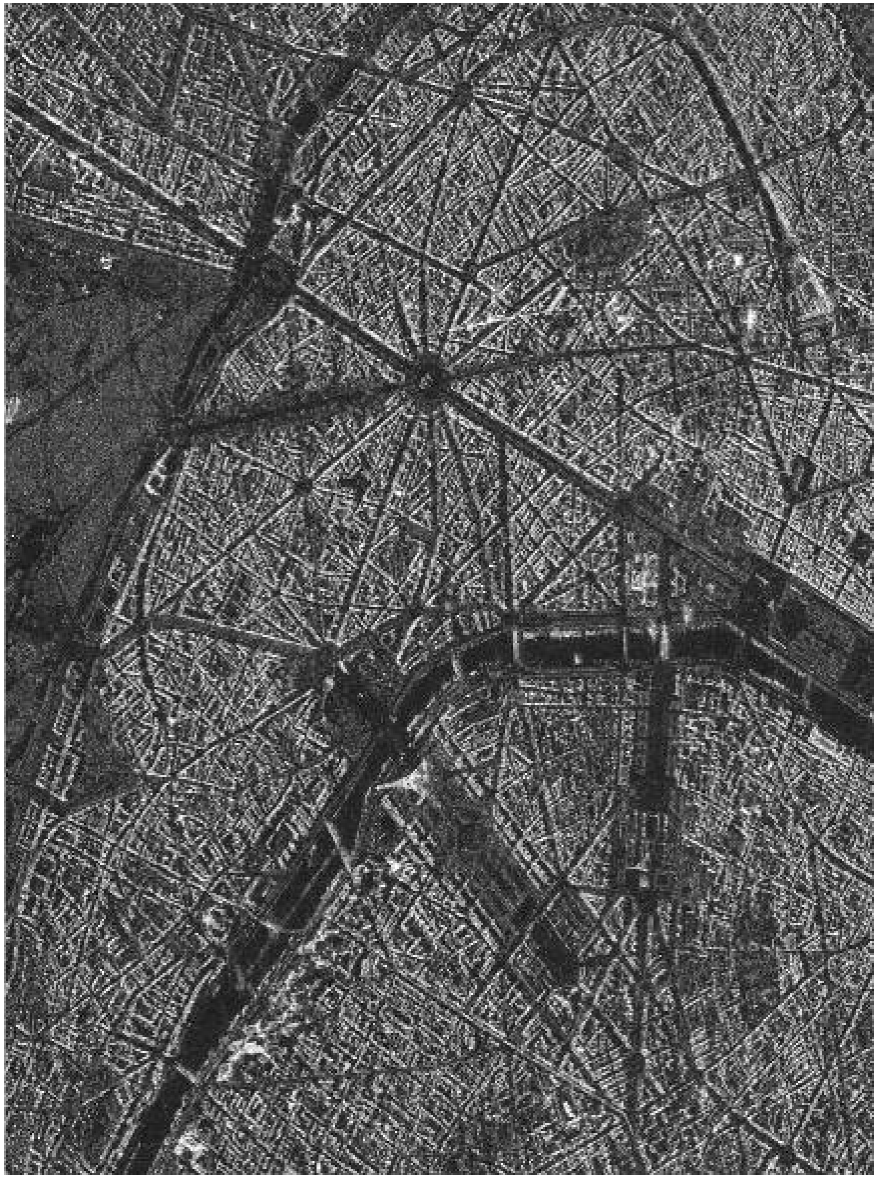
San Francisco (USA)

E. P |HH-VV|, |HV|, |HH+VV|

GOOGLE EARTH

RADAR POLARIMETRY

Paris (F)



|HV|

GOOGLE EARTH

COSMO – SkyMED (07 / 06 / 07)

4 satellite SAR constellation, right & left looking acquisition

- Short revisit time
- Rapid response
- Very high resolution from 1 metre
- High geo-location accuracy
- All-weather day/night global coverage
- High imaging capacity
- Interferometric & polarmetric capability

Interferometric capability

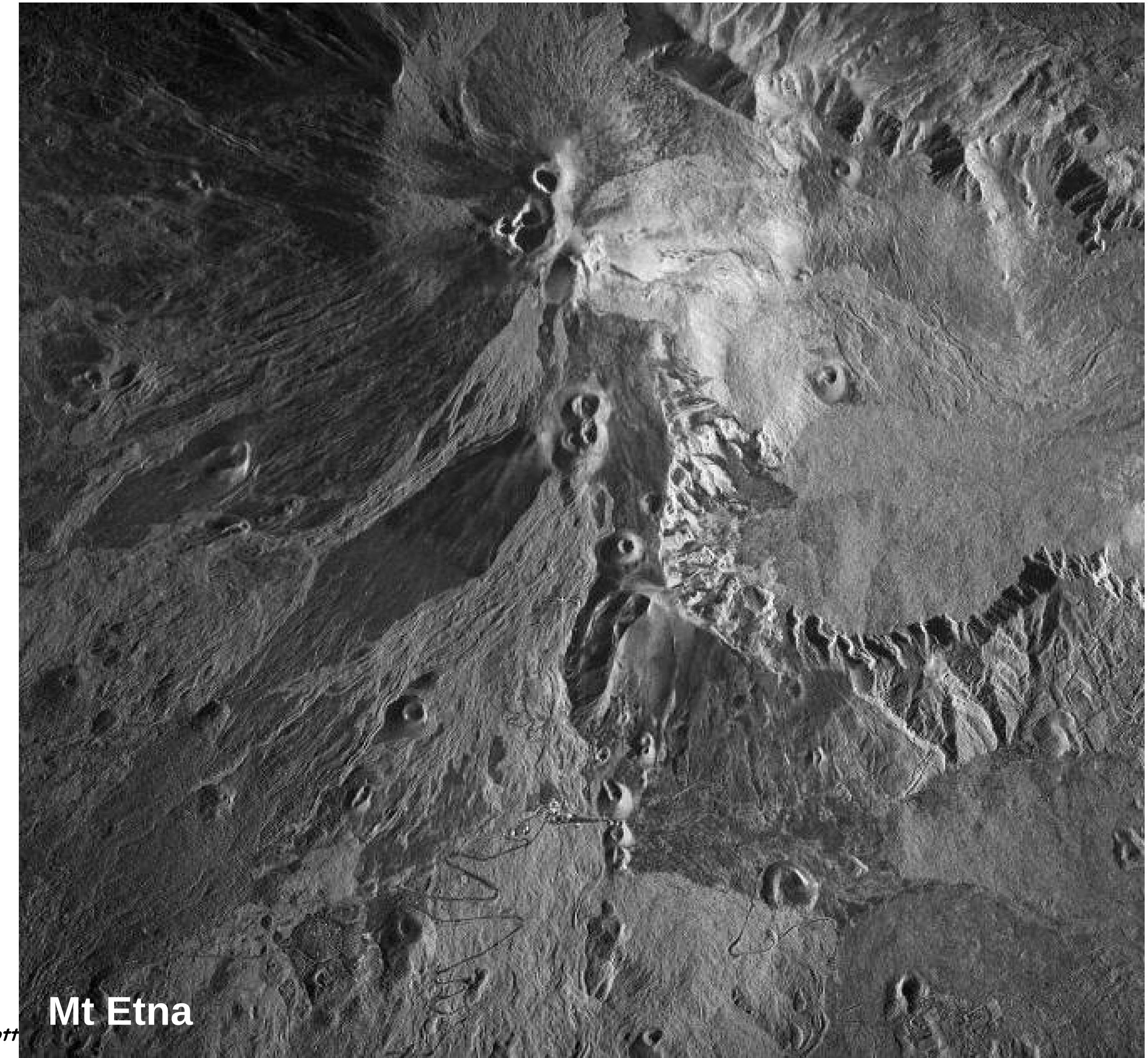
- One-day interferometry
- Tandem-like interferometry mission
- Optimum mean revisit
- Flexible acquisition; time delay of:
    - one day (pairs 1-2, 2-3)
    - two days (1-3, 3-4)
    - three days (2-4)
    - four days (1-4)

asi

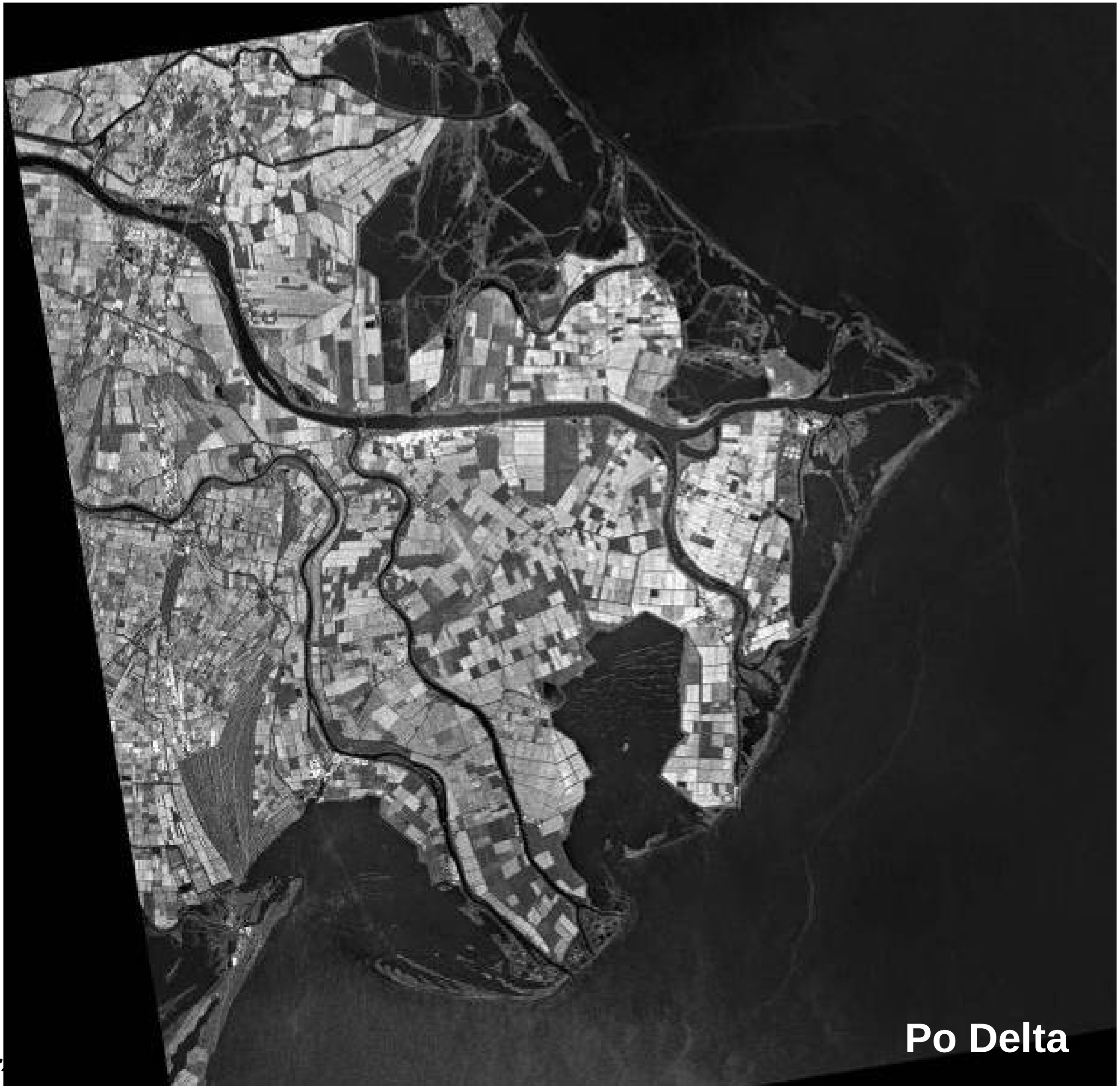
TELESPAZIO
A Finmeccanica / Thales Company

THALES

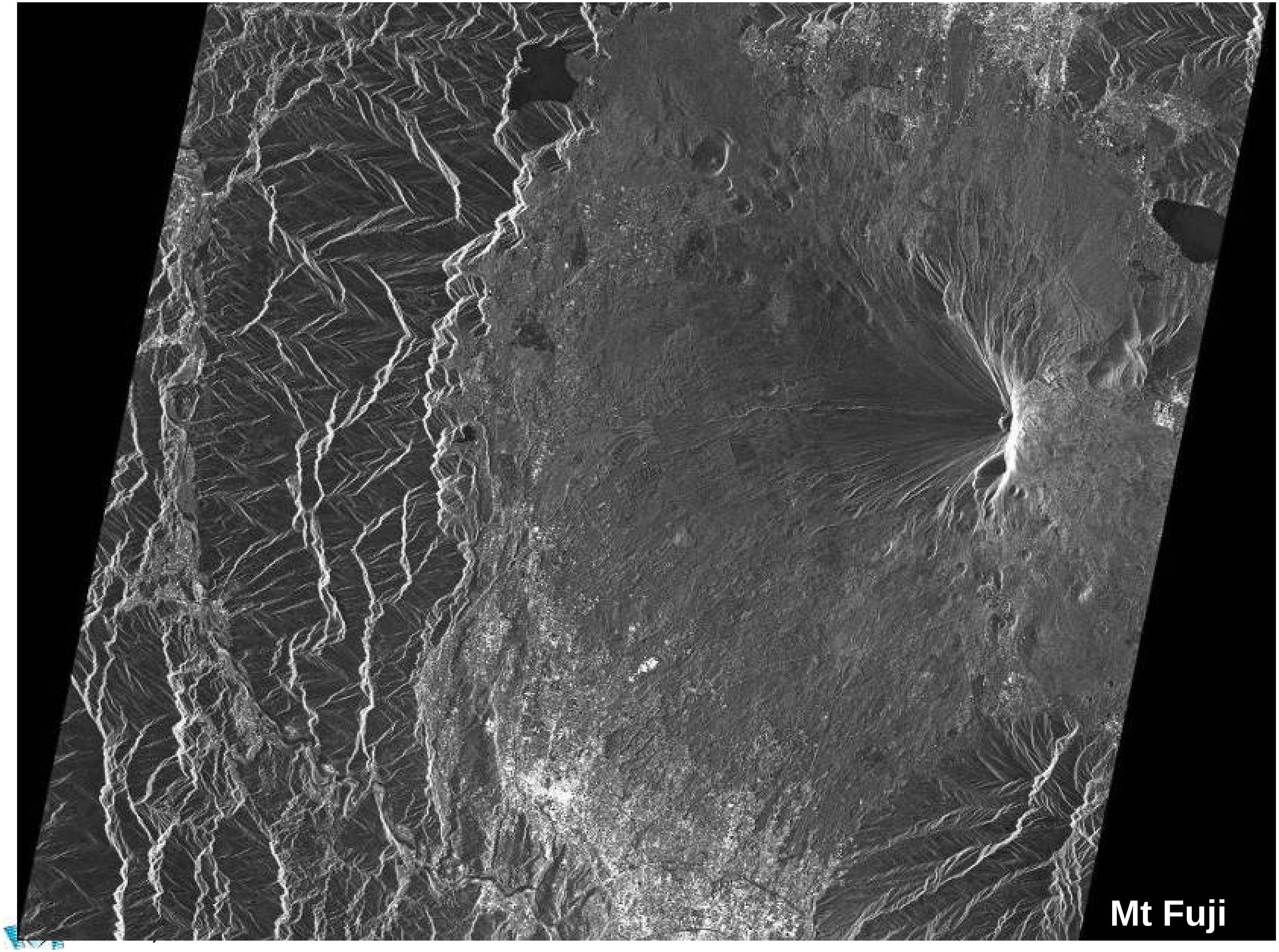
RADAR POLARIMETRY



RADAR POLARIMETRY

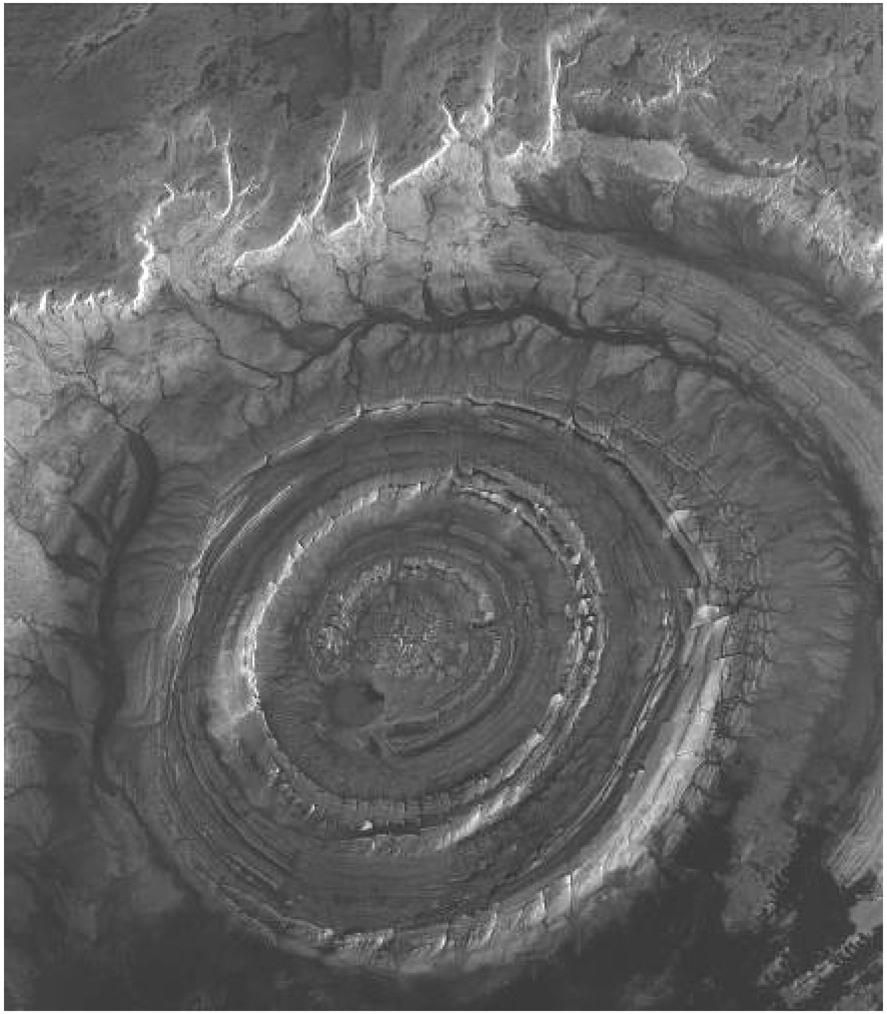


RADAR POLARIMETRY



RADAR POLARIMETRY

Ring structure located in Ouadane-Richat in Mauritania

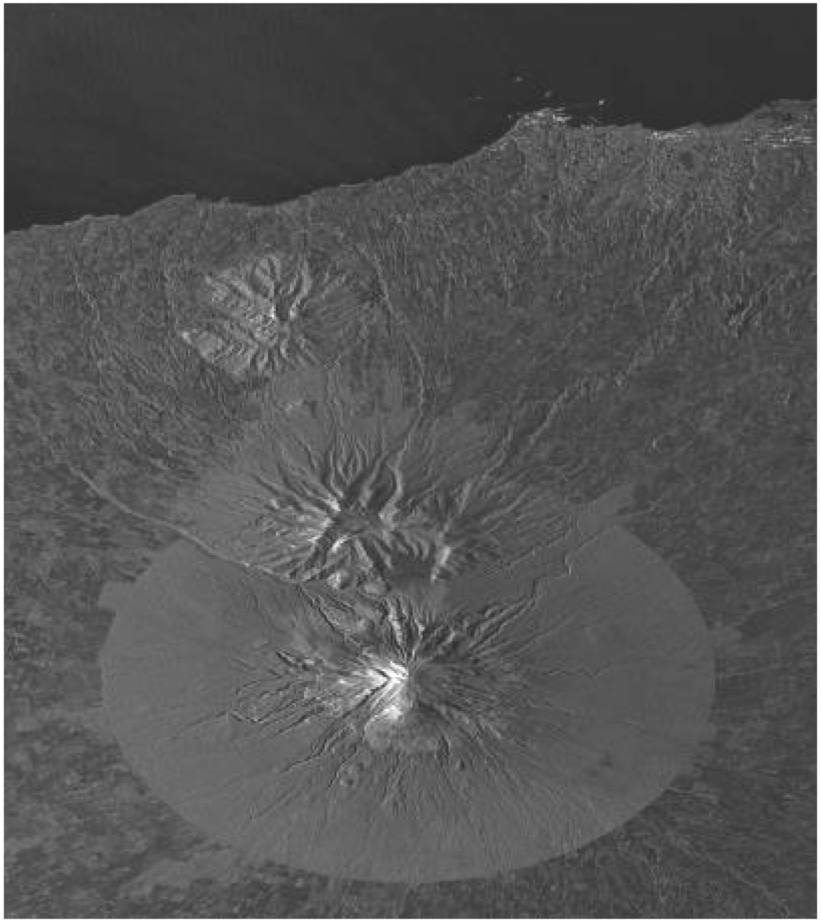


TerraSAR-X
ScanSAR (16m) – VV

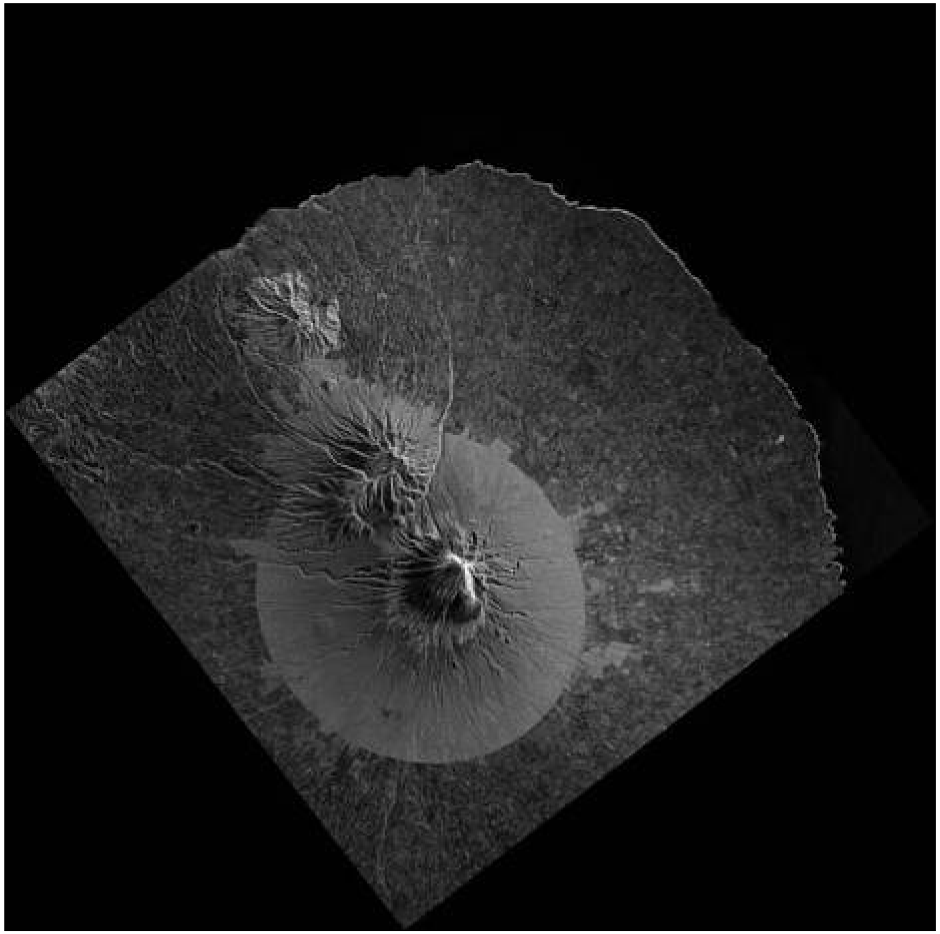
Cosmo – Skymed

RADAR POLARIMETRY

The near-circular, conical peak of Mount Egmont on New Zealand's North Island



TerraSAR-X
StripMap (3m) – VV



Cosmo – Skymed

RADAR POLARIMETRY

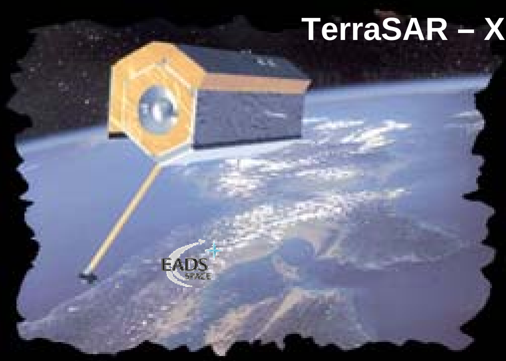
ENVISAT – ASAR

ALOS – PALSAR

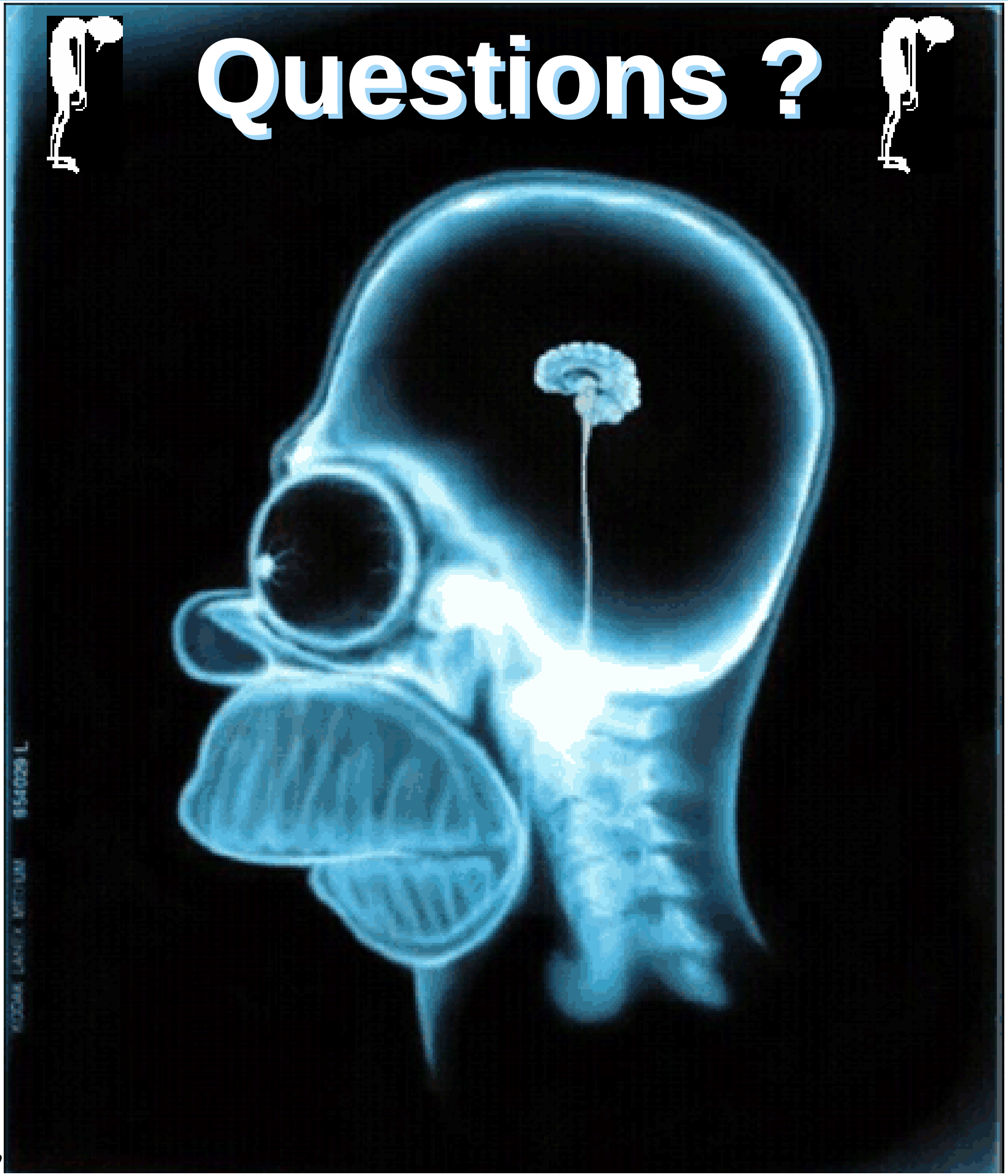
GOLDEN AGE

PolSAR
Pol-InSAR

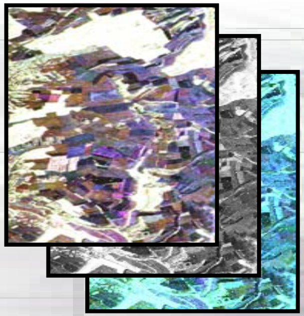
TerraSAR – X



RADARSAT 2

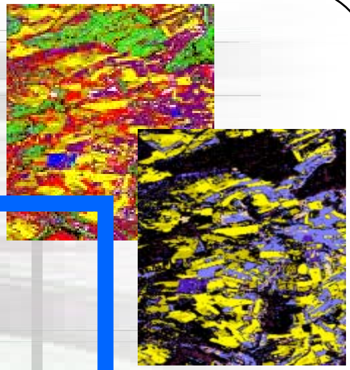

Questions ?

POLARIMETRIC REMOTE SENSING



POLARIMETRIC
SAR DATA SET(S)

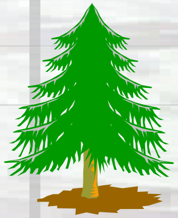
POLSAR - POLINSAR
DATA
PROCESSING



QUALITATIVE ANALYSIS

POLARIMETRIC
DESCRIPTORS
EXTRACTION

TWO-STEP INVERSION PROCEDURE

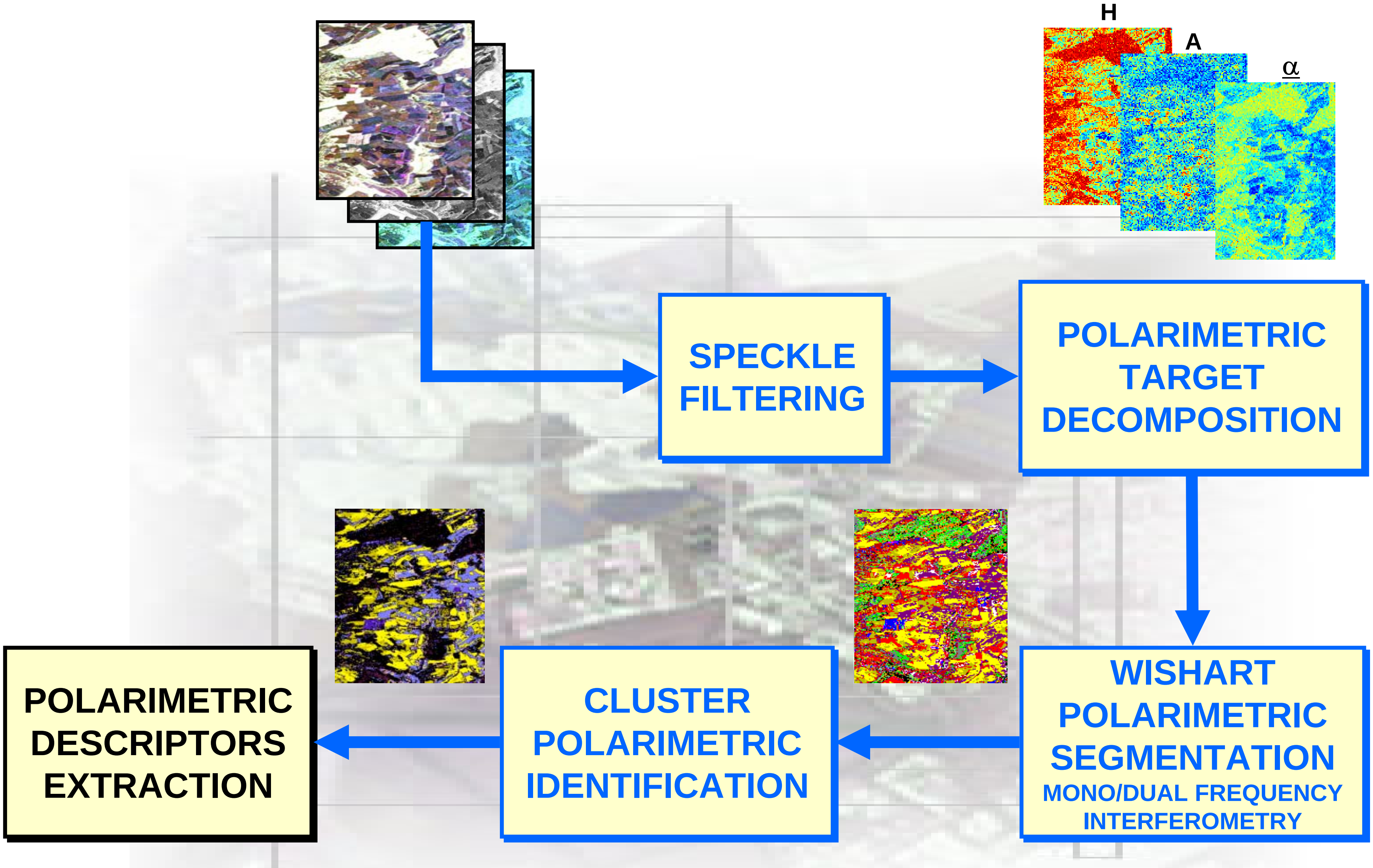


MODEL-BASED
PHYSICAL PARAMETER
INVERSION

BIO and GEO PHYSICAL
PARAMETERS

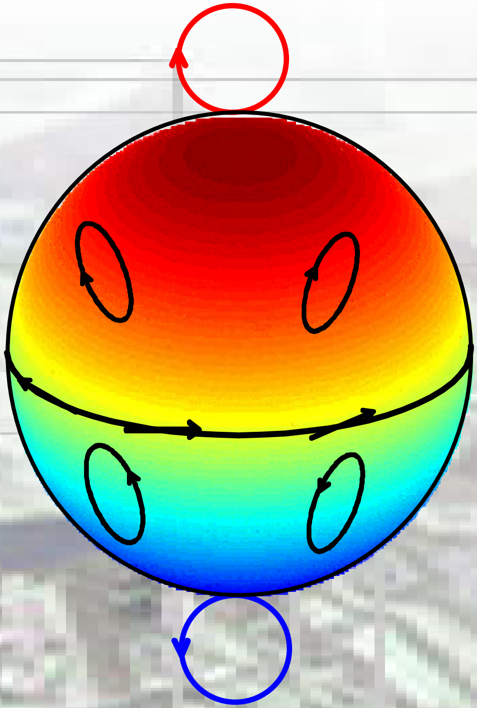
QUANTITATIVE ANALYSIS

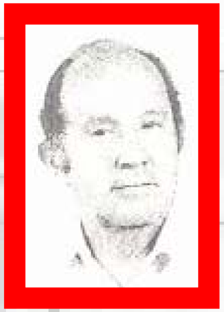
QUALITATIVE ANALYSIS



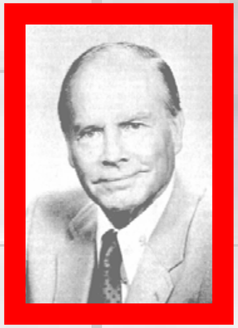
RADAR POLARIMETRY

G.A. DESCHAMPS
(1911 – 1998)

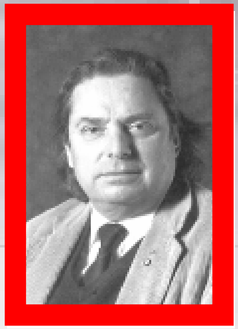




E. M. KENNAUGH
(1922 – 1983)

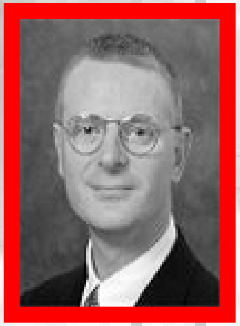


J.R. HUYNEN
(1920 – 2007)



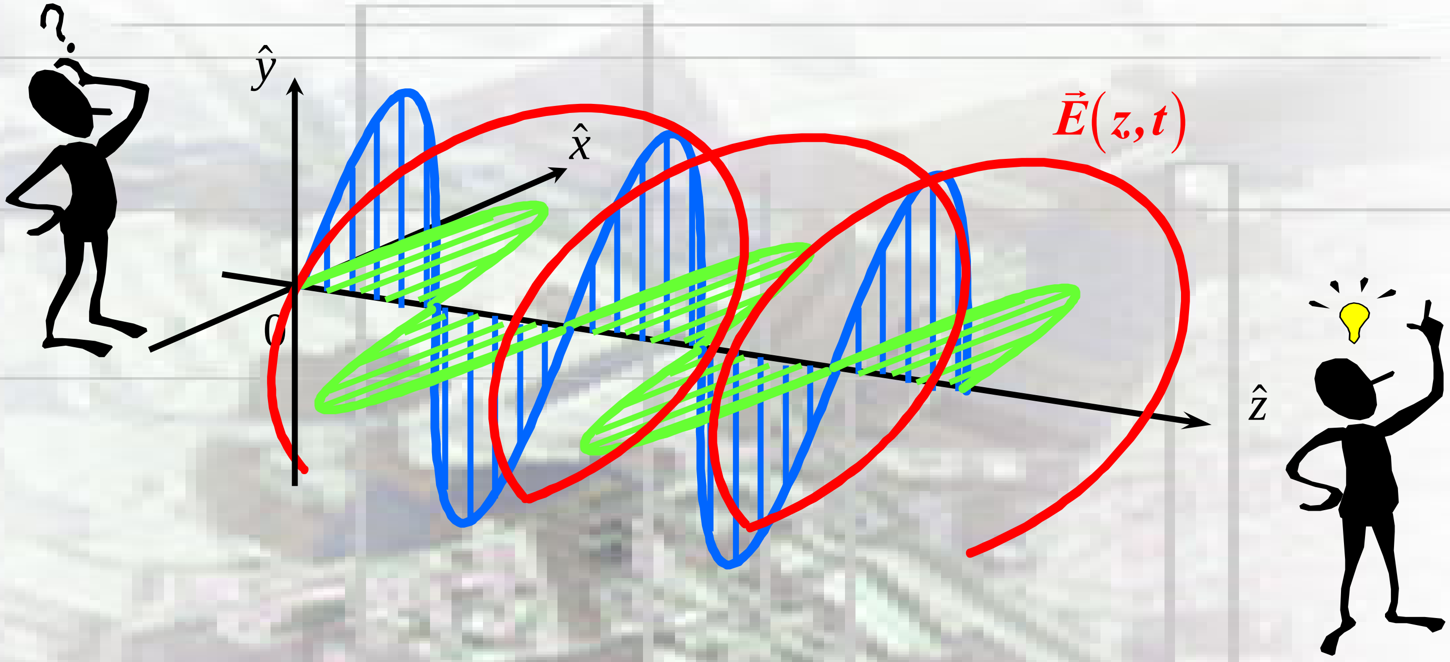
W.M. BOERNER

J. S. LEE



S.R. CLOUDE

E. POTTIER



WAVE POLARIMETRY

PROPAGATION EQUATION

REAL ELECTRIC FIELD VECTOR $\vec{E}(z,t)$

MAXWELL EQUATIONS

MAXWELL – FARADAY EQUATION $\nabla \wedge \vec{E}(z,t) = -\frac{\partial \vec{B}(z,t)}{\partial t}$

MAXWELL – AMPERE EQUATION $\nabla \wedge \vec{H}(z,t) = \vec{J}_T(z,t)$

GAUSS THEOREM $\nabla \cdot \vec{D}(z,t) = \rho(z,t)$

$\nabla \cdot \vec{B}(z,t) = 0$

$$\vec{J}_T(z,t) = \vec{J}_C(z,t) + \frac{\partial \vec{D}(z,t)}{\partial t}$$

$$\vec{J}_C(z,t) = \sigma \vec{E}(z,t)$$

$$\vec{B}(z,t) = \mu \vec{H}(z,t)$$

$$\vec{D}(z,t) = \varepsilon \vec{E}(z,t)$$

σ (Conductivity)

μ (Permeability)

ε (Permittivity)

PROPAGATION EQUATION

$$\nabla \wedge \left(\nabla \wedge \vec{A}\right) = \nabla\left(\nabla \cdot \vec{A}\right) - \nabla \cdot \left(\nabla \vec{A}\right)$$

PROPAGATION EQUATION

$$\nabla^2 \vec{E}(z,t) - \mu\varepsilon \frac{\partial^2 \vec{E}(z,t)}{\partial t^2} - \mu\sigma \frac{\partial \vec{E}(z,t)}{\partial t} = -\frac{1}{\varepsilon}\frac{\partial \rho(z,t)}{\partial t}$$

HELMHOLTZ PROPAGATION EQUATION

$$\nabla^2 \vec{E}(z,t) - \mu\varepsilon \frac{\partial^2 \vec{E}(z,t)}{\partial t^2} = 0$$

Source Free, Linear, Homogeneous, Isotropic, Dielectric and lossless Medium

PROPAGATION EQUATION

COMPLEX ELECTRIC FIELD VECTOR $\underline{E}(z)$ **With:** $\vec{E}(z,t) = \Re\left(\underline{E}(z)e^{j\omega t}\right)$

HELMHOLTZ PROPAGATION EQUATION

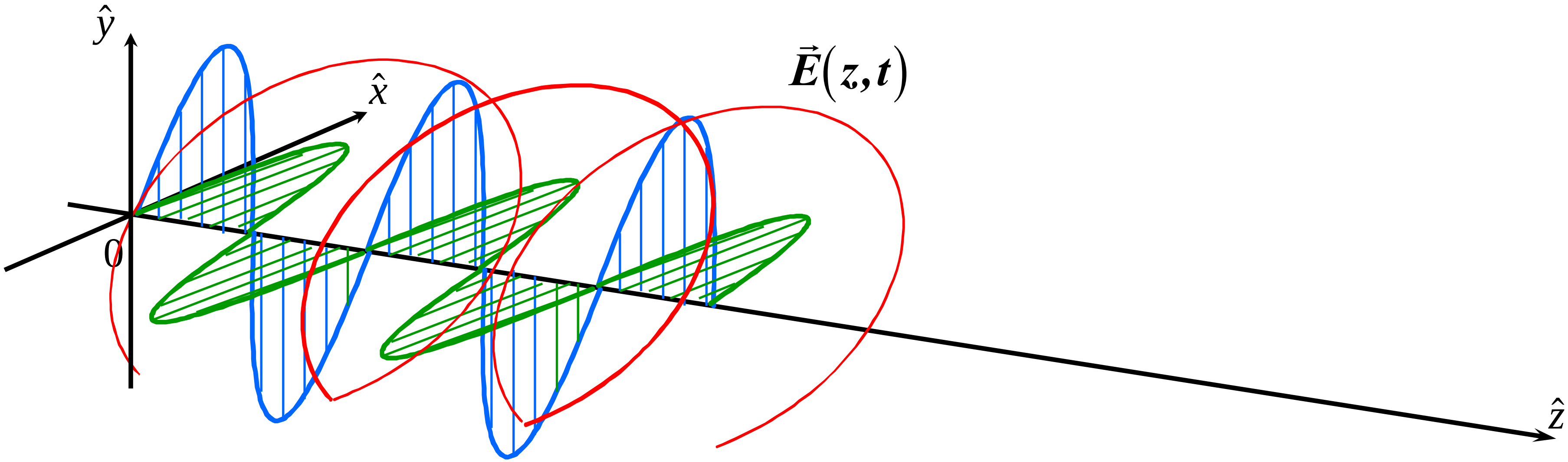
$$\nabla^2 \underline{E}(z) + \underline{k}^2 \underline{E}(z) = 0$$

SOLUTION: $\underline{E}(z) = \underline{E}e^{-jkz}$ **With:** $\underline{E} = \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} = \begin{bmatrix} E_{ox}e^{j\delta_x} \\ E_{oy}e^{j\delta_y} \\ E_{oz}e^{j\delta_z} \end{bmatrix}$

SINUSOIDAL PLANE WAVE

$$\nabla \cdot \vec{E}(z,t) = 0 \quad \Rightarrow \quad \frac{\partial E_z}{\partial z} = 0$$

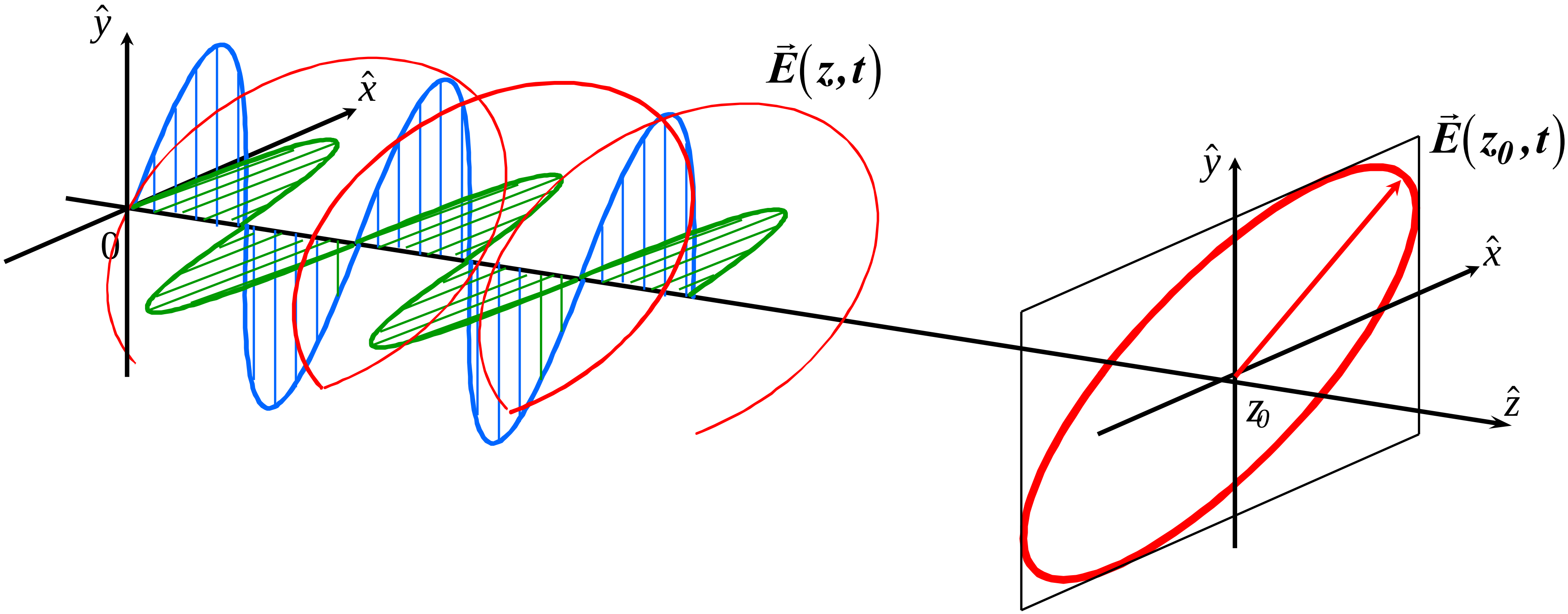
POLARISATION ELLIPSE



REAL ELECTRIC FIELD VECTOR

$$\vec{E}(z,t) = \begin{cases} E_x = E_{0x}\cos(\omega t - kz - \delta_x) \\ E_y = E_{0y}\cos(\omega t - kz - \delta_y) \\ E_z = 0 \end{cases}$$

POLARISATION ELLIPSE

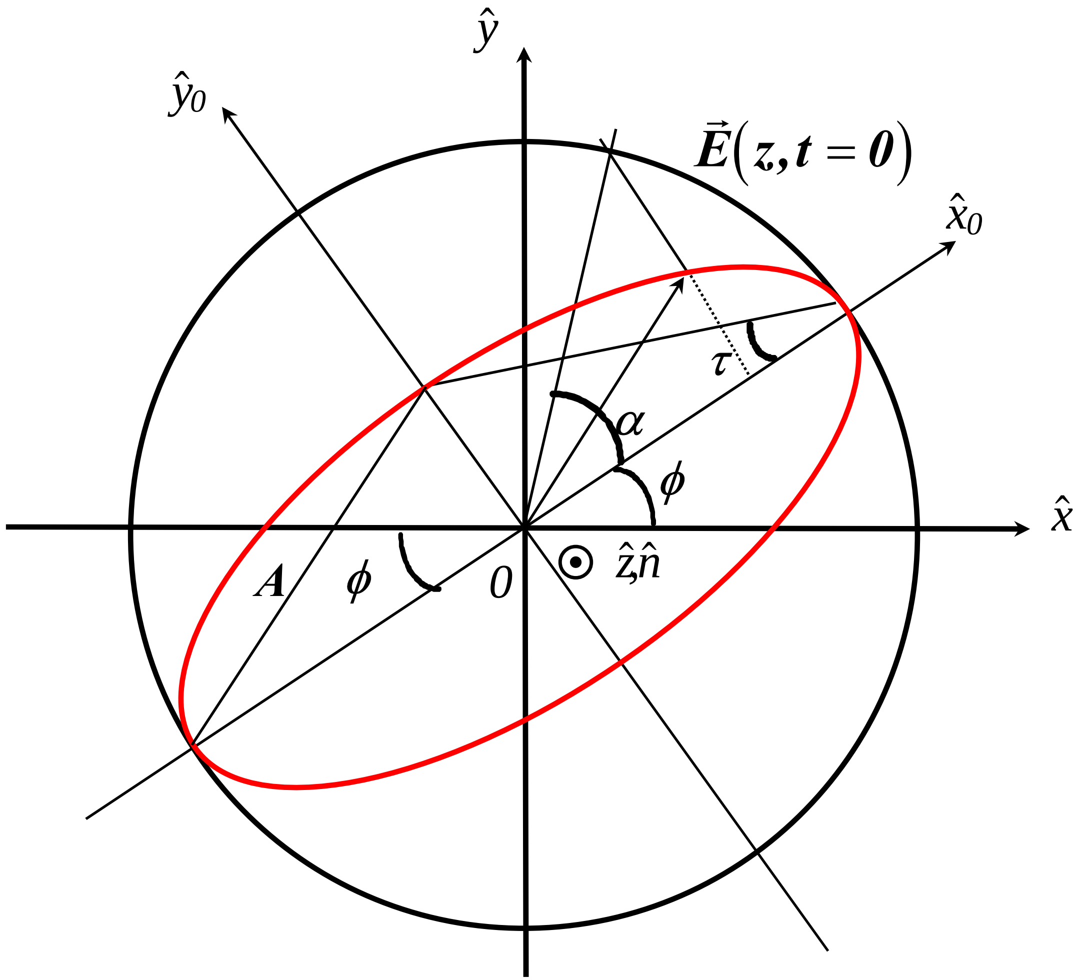


THE REAL ELECTRIC FIELD VECTOR MOVES IN TIME ALONG AN ELLIPSE

$$\left(\frac{E_x}{E_{0x}}\right)^2 - 2\frac{E_xE_y}{E_{0x}E_{0y}}\cos(\delta) + \left(\frac{E_y}{E_{0y}}\right)^2 = \sin^2(\delta)$$

With: $\delta = \delta_y - \delta_x$

POLARISATION ELLIPSE



A : **WAVE AMPLITUDE**

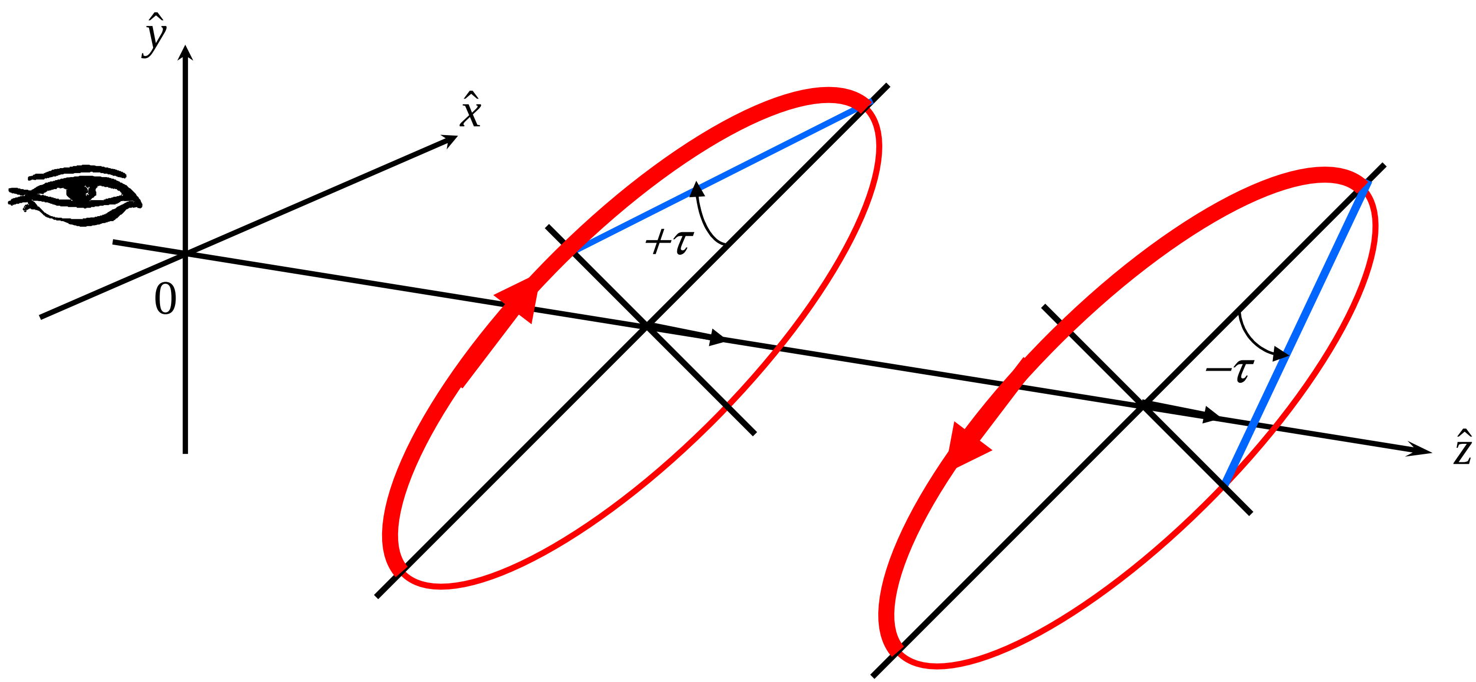
α : **ABSOLUTE PHASE**

ϕ : **ORIENTATION ANGLE** $\quad -\frac{\pi}{2} \leq \phi \leq \frac{\pi}{2}$

τ : **ELLIPTICITY ANGLE** $\quad 0 \leq \tau \leq \frac{\pi}{4}$

POLARISATION HANDENESS

ROTATION SENSE: LOOKING INTO THE DIRECTION OF THE WAVE PROPAGATION



ANTI-CLOCKWISE ROTATION	**CLOCKWISE ROTATION**
LEFT HANDED POLARISATION	**RIGHT HANDED POLARISATION**
⬇	⬇
ELLIPTICITY ANGLE : $\tau > 0$	**ELLIPTICITY ANGLE :** $\tau < 0$

$$-\frac{\pi}{4} \leq \tau \leq \frac{\pi}{4}$$

JONES VECTOR

REAL ELECTRIC FIELD VECTOR

$$\vec{E}(z,t)=\begin{cases} E_x = E_{0x}\cos(\omega t - kz - \delta_x) \\ E_y = E_{0y}\cos(\omega t - kz - \delta_y) \\ E_z = 0 \end{cases}$$

PHASOR = JONES VECTOR

$$\underline{E} = \begin{bmatrix} E_x = E_{ox} e^{j\delta_x} \\ E_y = E_{oy} e^{j\delta_y} \end{bmatrix}$$

With: $\vec{E}(z,t) = \Re\left(\underline{E} e^{j(\omega t - kz)}\right)$

GEOMETRICAL PARAMETERS

ABSOLUTE PHASE

$$\alpha = \delta_x$$

AMPLITUDE

$$A = \sqrt{E_{0x}^2 + E_{0y}^2}$$

ORIENTATION ANGLE

$$\tan 2\phi = 2\frac{E_{0x}E_{0y}}{E_{0x}^2 - E_{0y}^2}\cos\delta$$

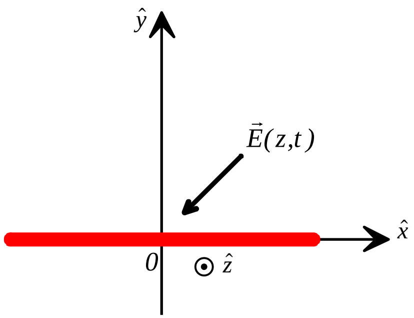
ELLIPTICITY ANGLE

$$\sin 2\tau = 2\frac{E_{0x}E_{0y}}{E_{0x}^2 + E_{0y}^2}\sin\delta$$

POLARISATION HANDENESS: $Sign(\tau)$

JONES VECTOR

HORIZONTAL POLARISATION STATE

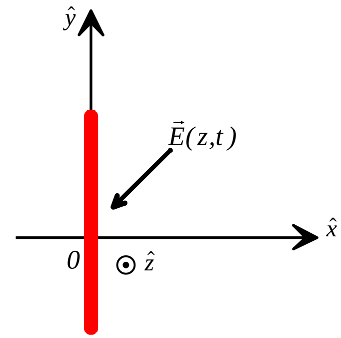


$$\underline{H} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\phi = 0$$

$$\tau = 0$$

VERTICAL POLARISATION STATE

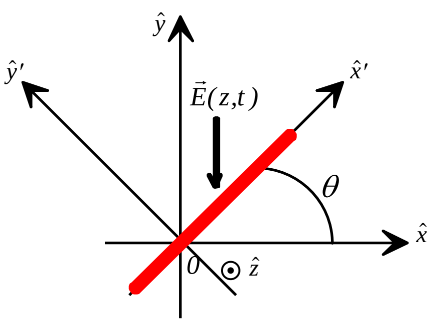


$$\underline{V} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\phi = \frac{\pi}{2}$$

$$\tau = 0$$

LINEAR POLARISATION STATE

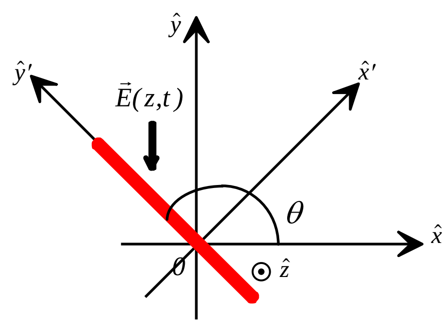


$$\underline{L} = \begin{bmatrix} \cos\theta \\ \sin\theta \end{bmatrix}$$

$$\phi = \theta$$

$$\tau = 0$$

ORTHOGONAL LINEAR POLARISATION STATE



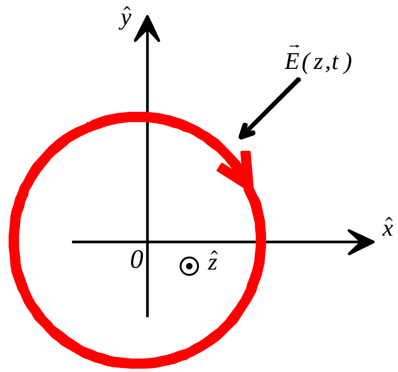
$$\underline{L}_{\perp} = \begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix}$$

$$\phi = \theta + \frac{\pi}{2}$$

$$\tau = 0$$

JONES VECTOR

LEFT CIRCULAR POLARISATION STATE

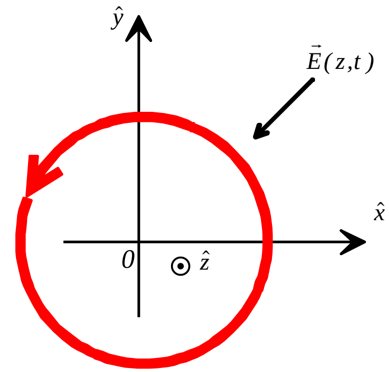


$$\underline{LC} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ j \end{bmatrix}$$

$$-\frac{\pi}{2} \leq \phi \leq +\frac{\pi}{2}$$

$$\tau = +\frac{\pi}{4}$$

RIGHT CIRCULAR POLARISATION STATE

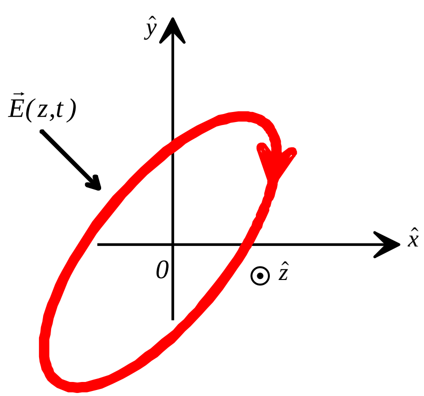


$$\underline{RC} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ -j \end{bmatrix}$$

$$-\frac{\pi}{2} \leq \phi \leq +\frac{\pi}{2}$$

$$\tau = -\frac{\pi}{4}$$

ELLIPTICAL POLARISATION STATE

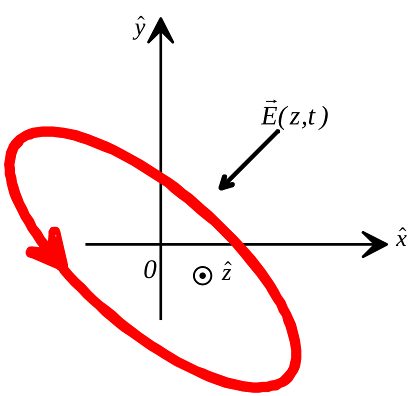


$$\underline{E} = \begin{bmatrix} E_x \\ E_y \end{bmatrix}$$

$$\phi = \theta$$

$$0 \leq \tau \leq +\frac{\pi}{4}$$

ORTHOGONAL ELLIPTICAL POLARISATION STATE

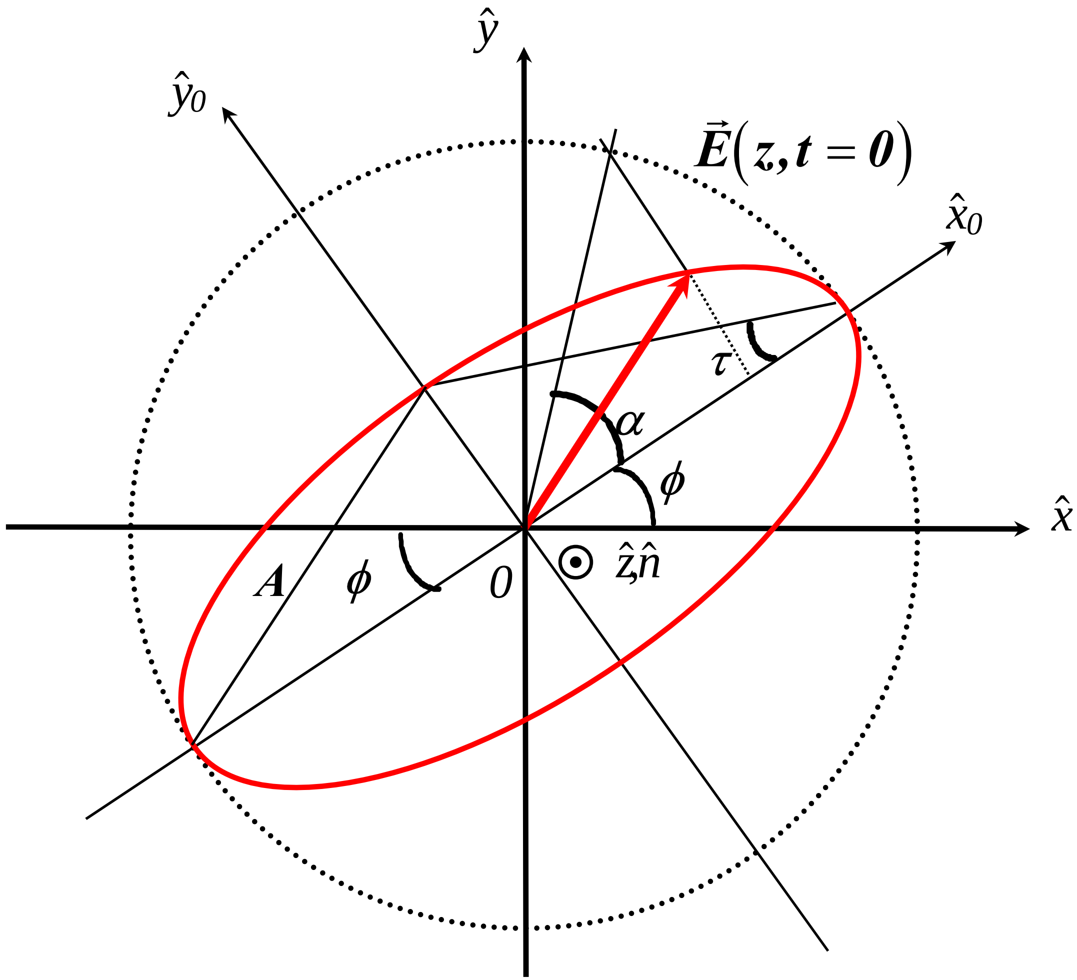


$$\underline{E}_\perp = \begin{bmatrix} E'_x \\ E'_y \end{bmatrix}$$

$$\phi = \theta + \frac{\pi}{2}$$

$$-\frac{\pi}{4} \leq \tau \leq 0$$

JONES VECTOR



$$\underline{E} = A\begin{bmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{bmatrix}\begin{bmatrix} \cos(\tau) & j\sin(\tau) \\ j\sin(\tau) & \cos(\tau) \end{bmatrix}\begin{bmatrix} e^{-j\alpha} & 0 \\ 0 & e^{j\alpha} \end{bmatrix}\hat{u}_x$$

POLARISATION RATIO

JONES VECTOR

$$\underline{E} = \begin{bmatrix} E_x \\ E_y \end{bmatrix} = \begin{bmatrix} E_{ox} e^{j\delta_x} \\ E_{oy} e^{j\delta_y} \end{bmatrix}$$

$$= A \begin{bmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{bmatrix} \begin{bmatrix} \cos(\tau) & j\sin(\tau) \\ j\sin(\tau) & \cos(\tau) \end{bmatrix} \begin{bmatrix} e^{-j\alpha} & 0 \\ 0 & e^{j\alpha} \end{bmatrix} \hat{u}_x$$

COMPLEX POLARISATION RATIO

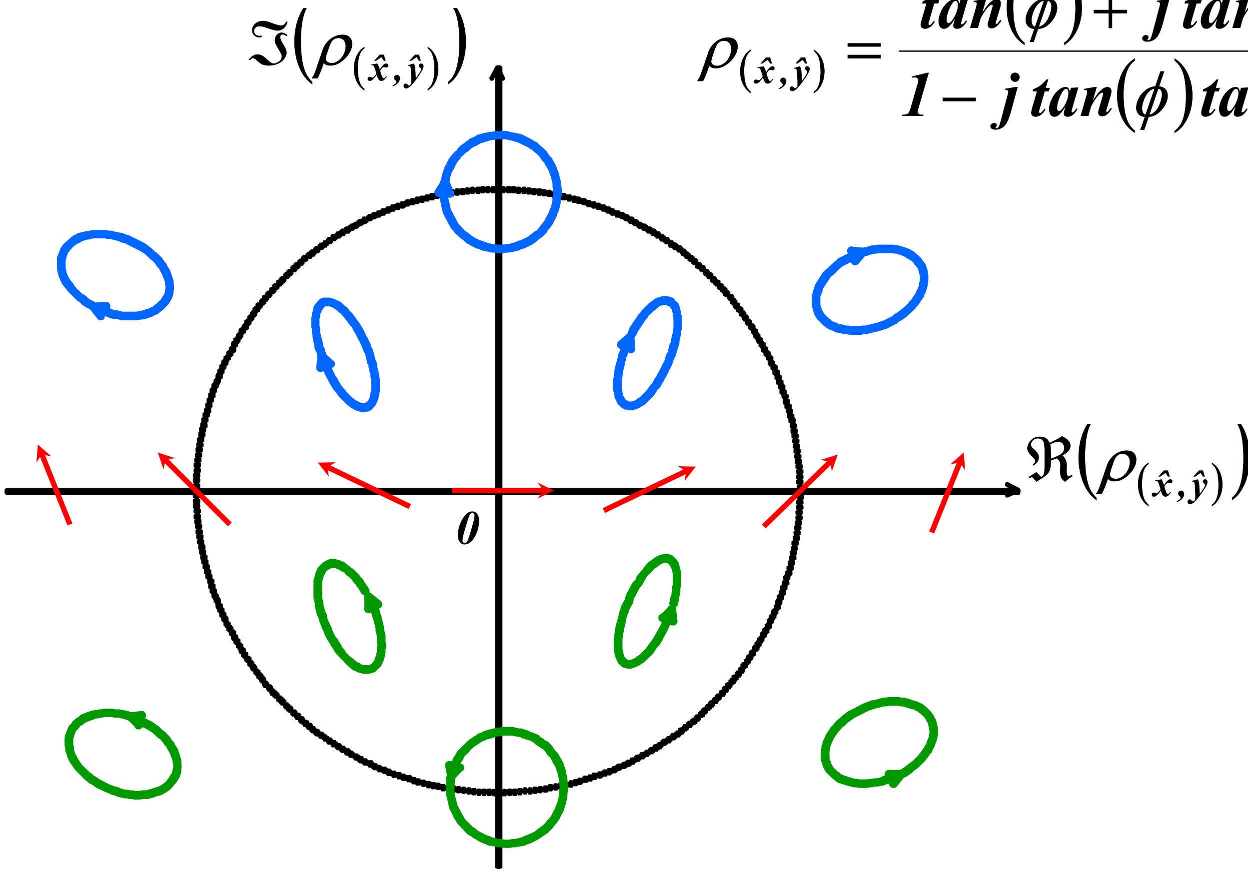
$$\rho_{(\hat{x},\hat{y})} = \frac{E_y}{E_x} = \frac{E_{0y}}{E_{0x}} e^{j(\delta_y - \delta_x)} = \frac{\tan(\phi) + j\tan(\tau)}{1 - j\tan(\phi)\tan(\tau)}$$

INDEPENDENT OF AMPLITUDE AND ABSOLUTE PHASE

COMPLEX POLARISATION PLANE

COMPLEX POLARISATION PLANE

$$\rho_{(\hat{x},\hat{y})} = \frac{\tan(\phi) + j\tan(\tau)}{1 - j\tan(\phi)\tan(\tau)}$$



$$\tan 2\phi = 2\frac{\Re\left(\rho_{(\hat{x},\hat{y})}\right)}{1 - \left|\rho_{(\hat{x},\hat{y})}\right|^2} \qquad \sin 2\tau = 2\frac{\Im\left(\rho_{(\hat{x},\hat{y})}\right)}{1 + \left|\rho_{(\hat{x},\hat{y})}\right|^2}$$

ORTHOGONAL JONES VECTOR

JONES VECTOR

$$\underline{E} = \begin{bmatrix} E_x \\ E_y \end{bmatrix} = \begin{bmatrix} E_{ox} e^{j\delta_x} \\ E_{oy} e^{j\delta_y} \end{bmatrix}$$

$$= A \begin{bmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{bmatrix} \begin{bmatrix} \cos(\tau) & j\sin(\tau) \\ j\sin(\tau) & \cos(\tau) \end{bmatrix} \begin{bmatrix} e^{-j\alpha} & 0 \\ 0 & e^{j\alpha} \end{bmatrix} \hat{u}_x$$

$\Downarrow$

POLARISATION ALGEBRA

NORM OF A JONES VECTOR $\quad \|\underline{E}\| = \sqrt{E_{0x}^2 + E_{0y}^2}$

SCALAR PRODUCT $\quad \langle \underline{A}, \underline{B} \rangle = \underline{A}^{T*} \underline{B}$

ORTHOGONALITY $\quad \langle \underline{A}, \underline{A}_{\perp} \rangle = 0$

ORTHOGONAL JONES VECTOR

JONES VECTOR

$$\underline{E} = A\begin{bmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{bmatrix}\begin{bmatrix} \cos(\tau) & j\sin(\tau) \\ j\sin(\tau) & \cos(\tau) \end{bmatrix}\begin{bmatrix} e^{-j\alpha} & 0 \\ 0 & e^{j\alpha} \end{bmatrix}\hat{u}_x$$

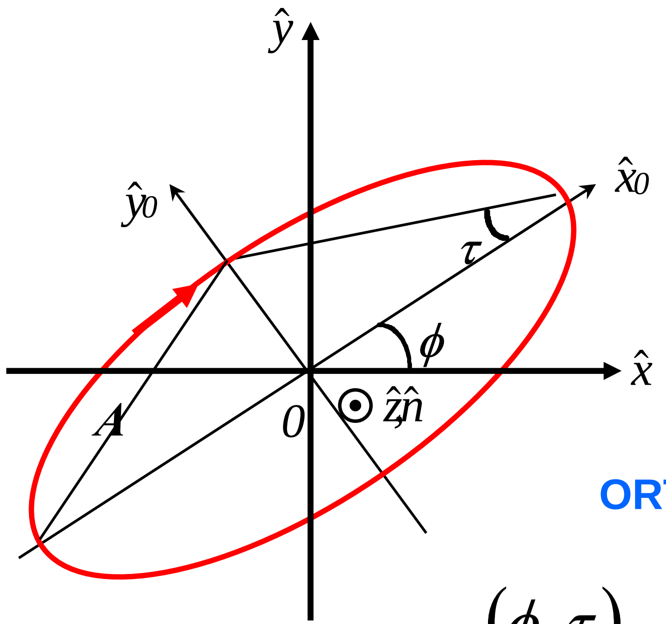
ORTHOGONAL JONES VECTOR

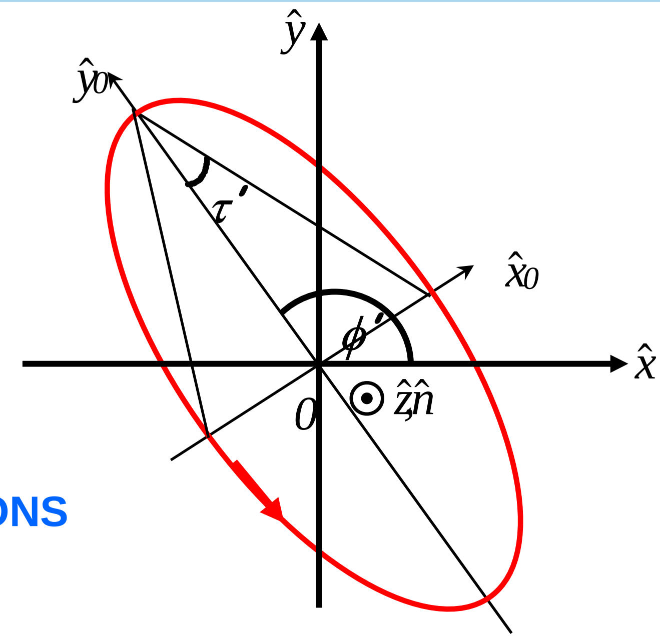
$$\underline{E}_\perp = A\begin{bmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{bmatrix}\begin{bmatrix} \cos(\tau) & j\sin(\tau) \\ j\sin(\tau) & \cos(\tau) \end{bmatrix}\begin{bmatrix} e^{-j\alpha} & 0 \\ 0 & e^{j\alpha} \end{bmatrix}\hat{u}_y$$

$$= A\begin{bmatrix} -\sin(\phi) & -\cos(\phi) \\ \cos(\phi) & -\sin(\phi) \end{bmatrix}\begin{bmatrix} \cos(\tau) & -j\sin(\tau) \\ -j\sin(\tau) & \cos(\tau) \end{bmatrix}\begin{bmatrix} e^{-j\alpha} & 0 \\ 0 & e^{j\alpha} \end{bmatrix}\hat{u}_x$$

$$\underline{E}_\perp = A\begin{bmatrix} \cos\left(\phi+\frac{\pi}{2}\right) & -\sin\left(\phi+\frac{\pi}{2}\right) \\ \sin\left(\phi+\frac{\pi}{2}\right) & \cos\left(\phi+\frac{\pi}{2}\right) \end{bmatrix}\begin{bmatrix} \cos(-\tau) & j\sin(-\tau) \\ j\sin(-\tau) & \cos(-\tau) \end{bmatrix}\begin{bmatrix} e^{-j\alpha} & 0 \\ 0 & e^{j\alpha} \end{bmatrix}\hat{u}_x$$

ORTHOGONAL JONES VECTOR





ORTHOGONALITY CONDITIONS

$$(\phi,\tau) \mapsto \begin{cases} \phi' = \phi + \frac{\pi}{2} \\ \tau' = -\tau \end{cases}$$

➡ CHANGE OF POLARISATION HANDENESS

$$\underline{E} = \begin{bmatrix} E_x \\ E_y \end{bmatrix} \mapsto \rho_{\underline{E}} = \frac{E_y}{E_x} \qquad \underline{E}_\perp = \begin{bmatrix} E'_x \\ E'_y \end{bmatrix} \mapsto \rho_{\underline{E}_\perp} = \frac{E'_y}{E'_x}$$

$$\boxed{\langle \underline{E}, \underline{E}_\perp \rangle = 0 \mapsto \rho_{\underline{E}_\perp} = -\frac{1}{\rho_{\underline{E}}^*}}$$

ELLIPTICAL BASIS TRANSFORMATION

JONES VECTOR

$$\underline{E} = A\begin{bmatrix} cos(\phi) & -sin(\phi) \\ sin(\phi) & cos(\phi) \end{bmatrix}\begin{bmatrix} cos(\tau) & j\,sin(\tau) \\ j\,sin(\tau) & cos(\tau) \end{bmatrix}\begin{bmatrix} e^{-j\alpha} & 0 \\ 0 & e^{j\alpha} \end{bmatrix}\hat{u}_x$$

ORTHOGONAL JONES VECTOR

$$\underline{E}_\perp = A\begin{bmatrix} cos(\phi) & -sin(\phi) \\ sin(\phi) & cos(\phi) \end{bmatrix}\begin{bmatrix} cos(\tau) & j\,sin(\tau) \\ j\,sin(\tau) & cos(\tau) \end{bmatrix}\begin{bmatrix} e^{-j\alpha} & 0 \\ 0 & e^{j\alpha} \end{bmatrix}\hat{u}_y$$

$\Downarrow$

$$[\underline{E}, \underline{E}_\perp] = A\begin{bmatrix} cos(\phi) & -sin(\phi) \\ sin(\phi) & cos(\phi) \end{bmatrix}\begin{bmatrix} cos(\tau) & j\,sin(\tau) \\ j\,sin(\tau) & cos(\tau) \end{bmatrix}\begin{bmatrix} e^{-j\alpha} & 0 \\ 0 & e^{j\alpha} \end{bmatrix}[\hat{u}_x, \hat{u}_y]$$

$\Downarrow$

ELLIPTICAL BASIS TRANSFORMATION

ELLIPTICAL BASIS TRANSFORMATION

ORTHOGONAL JONES VECTORS

$$[\underline{E}, \underline{E}_{\perp}] = A \begin{bmatrix} cos(\phi) & -sin(\phi) \\ sin(\phi) & cos(\phi) \end{bmatrix} \begin{bmatrix} cos(\tau) & j\,sin(\tau) \\ j\,sin(\tau) & cos(\tau) \end{bmatrix} \begin{bmatrix} e^{-j\alpha} & 0 \\ 0 & e^{j\alpha} \end{bmatrix} [\hat{u}_x, \hat{u}_y]$$

$$\Downarrow$$

SU(2) : SPECIAL UNITARY TRANSFORMATION MATRIX

$$[U(\phi,\tau,\alpha)] = \underbrace{\begin{bmatrix} cos(\phi) & -sin(\phi) \\ sin(\phi) & cos(\phi) \end{bmatrix}}_{[U_2(\phi)]} \underbrace{\begin{bmatrix} cos(\tau) & j\,sin(\tau) \\ j\,sin(\tau) & cos(\tau) \end{bmatrix}}_{[U_2(\tau)]} \underbrace{\begin{bmatrix} e^{-j\alpha} & 0 \\ 0 & e^{j\alpha} \end{bmatrix}}_{[U_2(\alpha)]}$$

$[U_2][U_2]^{T*} = [I_{D2}]$ CONSERVATION OF THE WAVE ENERGY

$det([U_2]) = +1$ ENSURES THE CORRECT PHASE DEFINITION

ELLIPTICAL BASIS TRANSFORMATION

SU(2) : SPECIAL UNITARY TRANSFORMATION MATRIX

$$[U(\phi,\tau,\alpha)] = \begin{bmatrix} cos(\phi) & -sin(\phi) \\ sin(\phi) & cos(\phi) \end{bmatrix} \begin{bmatrix} cos(\tau) & j\,sin(\tau) \\ j\,sin(\tau) & cos(\tau) \end{bmatrix} \begin{bmatrix} e^{-j\alpha} & 0 \\ 0 & e^{j\alpha} \end{bmatrix}$$

$$\Downarrow$$

ELLIPTICAL BASIS TRANSFORMATION MATRIX

$$\begin{aligned} \left[U_{(\underline{A},\underline{A}_\perp)\mapsto(\underline{B},\underline{B}_\perp)}\right] &= [U(\phi,\tau,\alpha)]^{-1} \\ &= \begin{bmatrix} e^{j\alpha} & 0 \\ 0 & e^{-j\alpha} \end{bmatrix} \begin{bmatrix} cos(\tau) & -j\,sin(\tau) \\ -j\,sin(\tau) & cos(\tau) \end{bmatrix} \begin{bmatrix} cos(\phi) & sin(\phi) \\ -sin(\phi) & cos(\phi) \end{bmatrix} \end{aligned}$$

ELLIPTICAL BASIS TRANSFORMATION

FROM LINEAR TO CIRCULAR BASIS

$$\underline{E} = E_H \underline{H} + E_V \underline{V} = E_{LC} \underline{LC} + E_{RC} \underline{RC}$$

With:

$$\underline{H} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad \underline{V} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad \underline{LC} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ j \end{bmatrix} \quad \underline{RC} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -j \end{bmatrix}$$

$$\Downarrow$$

$$[\underline{LC}, \underline{RC}] = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ j & -j \end{bmatrix}$$

ELLIPTICAL BASIS TRANSFORMATION

FROM LINEAR TO CIRCULAR BASIS

$$\underline{E} = E_H \underline{H} + E_V \underline{V} = E_{LC} \underline{LC} + E_{RC} \underline{RC}$$

With:

$$\underline{H} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad \underline{V} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad \underline{LC} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ j \end{bmatrix} \quad \underline{RC} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ -j \end{bmatrix}$$

$$\Downarrow$$

$$\cancel{\left[\underline{LC}, \underline{RC}\right] = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ j & -j \end{bmatrix}} \qquad \notin SU(2)$$

Ernst LÜNEBURG
(PIERS95 - Pasadena)

ELLIPTICAL BASIS TRANSFORMATION

FROM LINEAR TO CIRCULAR BASIS

$$\underline{E} = E_H \underline{H} + E_V \underline{V} = E_{LC} \underline{LC} + E_{LC_\perp} \underline{LC}_\perp$$

With:

$$\underline{H} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad \underline{V} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad \underline{LC} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ j \end{bmatrix} \quad \underline{LC}_\perp = \frac{1}{\sqrt{2}} \begin{bmatrix} j \\ 1 \end{bmatrix}$$

$$\Downarrow$$

$$\left[\underline{LC}, \underline{LC}_\perp\right] = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & j \\ j & 1 \end{bmatrix} \qquad \in SU(2)$$

ELLIPTICAL BASIS TRANSFORMATION

FROM LINEAR TO CIRCULAR BASIS

$$\underline{E} = E_H \underline{H} + E_V \underline{V} = E_{LC} \underline{LC} + E_{LC_\perp} \underline{LC}_\perp$$

$$\Downarrow$$

$$[\underline{LC}, \underline{LC}_\perp] = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & j \\ j & 1 \end{bmatrix} = \left[U\left(0, \tfrac{\pi}{4}\right)\right] \Rightarrow \left[U_{(\underline{H},\underline{V}) \mapsto (\underline{LC},\underline{LC}_\perp)}\right] = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -j \\ -j & 1 \end{bmatrix}$$

$$\Downarrow$$

$$\begin{bmatrix} E_{LC} \\ E_{LC_\perp} \end{bmatrix} = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -j \\ -j & 1 \end{bmatrix} \begin{bmatrix} E_H \\ E_V \end{bmatrix}$$

STOKES VECTOR

REAL REPRESENTATION OF THE POLARISATION STATE OF A MONOCHROMATIC WAVE

$$\underline{E}\cdot\underline{E}^{T*} = \begin{bmatrix} E_x E_x^* & E_x E_y^* \\ E_y E_x^* & E_y E_y^* \end{bmatrix}$$

PAULI MATRICES GROUP

$$\sigma_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \sigma_1 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad \sigma_2 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \quad \sigma_3 = \begin{bmatrix} 0 & -j \\ j & 0 \end{bmatrix}$$

$$\Downarrow$$

$$\underline{E}\cdot\underline{E}^{T*} = \frac{1}{2}\left\{g_0\sigma_0 + g_1\sigma_1 + g_2\sigma_2 + g_3\sigma_3\right\} = \frac{1}{2}\begin{bmatrix} g_0 + g_1 & g_2 - jg_3 \\ g_2 + jg_3 & g_0 - g_1 \end{bmatrix}$$

$\{g_0, g_1, g_2, g_3\}$ STOKES PARAMETERS

STOKES VECTOR

JONES VECTOR

$$\underline{E} = \begin{bmatrix} E_x = E_{ox} e^{j\delta_x} \\ E_y = E_{oy} e^{j\delta_y} \end{bmatrix}$$

$$\Downarrow$$

STOKES VECTOR

$$\underline{g}_{\underline{E}} = \begin{bmatrix} g_0 = |E_x|^2 + |E_y|^2 \\ g_1 = |E_x|^2 - |E_y|^2 \\ g_2 = 2\Re\left(E_x E_y^*\right) \\ g_3 = -2\Im\left(E_x E_y^*\right) \end{bmatrix}$$

WAVE POLARISATION STATE ESTIMATION FROM INTENSITIES MEASUREMENTS

STOKES VECTOR

STOKES VECTOR

$$\underline{g}_E = \begin{bmatrix} g_0 = E_{0x}^2 + E_{0y}^2 \\ g_1 = E_{0x}^2 - E_{0y}^2 \\ g_2 = 2E_{0x}E_{0y}\cos(\delta) \\ g_3 = 2E_{0x}E_{0y}\sin(\delta) \end{bmatrix} = \begin{bmatrix} g_0 = A^2 \\ g_1 = A^2 \cos 2\phi \cos 2\tau \\ g_2 = A^2 \sin 2\phi \cos 2\tau \\ g_3 = A^2 \sin 2\tau \end{bmatrix}$$

GEOMETRICAL PARAMETERS

ORIENTATION ANGLE

$$\tan 2\phi = 2\frac{E_{0x}E_{0y}}{E_{0x}^2 - E_{0y}^2}\cos\delta = \frac{g_2}{g_1}$$

ELLIPTICITY ANGLE

$$\sin 2\tau = 2\frac{E_{0x}E_{0y}}{E_{0x}^2 + E_{0y}^2}\sin\delta = \frac{g_3}{g_0}$$

STOKES VECTOR

JONES VECTOR

$$\underline{E} = A\begin{bmatrix} cos(\phi) & -sin(\phi) \\ sin(\phi) & cos(\phi) \end{bmatrix}\begin{bmatrix} cos(\tau) & j\,sin(\tau) \\ j\,sin(\tau) & cos(\tau) \end{bmatrix}\begin{bmatrix} e^{-j\alpha} & 0 \\ 0 & e^{j\alpha} \end{bmatrix}\hat{u}_x$$

$[U_2(\phi)]$ $[U_2(\tau)]$ $[U_2(\alpha)]$

HOMOMORPHISM SU(2) - O(3)

$$[O_3(2\theta)]_{p,q} = \frac{1}{2}Tr\left([U_2(\theta)]^{T*}\sigma_p[U_2(\theta)]\sigma_q\right)$$

(σ_p, σ_q) : Pauli Matrices

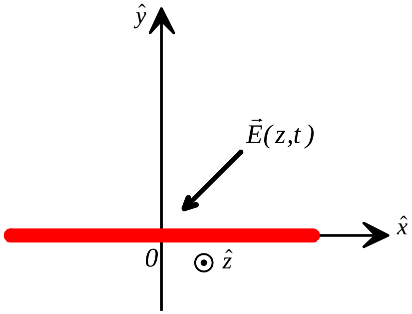
STOKES VECTOR

$$\underline{g}_{\underline{E}} = A^2\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & cos(2\phi) & -sin(2\phi) & 0 \\ 0 & sin(2\phi) & cos(2\phi) & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & cos(2\tau) & 0 & -sin(2\tau) \\ 0 & 0 & 1 & 0 \\ 0 & sin(2\tau) & 0 & cos(2\tau) \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & cos(2\alpha) & -sin(2\alpha) \\ 0 & 0 & sin(2\alpha) & cos(2\alpha) \end{bmatrix}\underline{g}_{\hat{u}_x}$$

$[O_4(2\phi)]$ $[O_4(2\tau)]$ $[O_4(2\alpha)]$

STOKES VECTOR

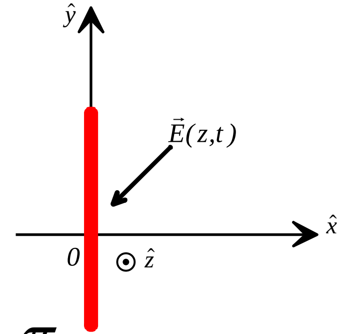
HORIZONTAL POLARISATION STATE



$$\underline{g}_{\underline{H}} = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\phi = 0 \qquad \tau = 0$$

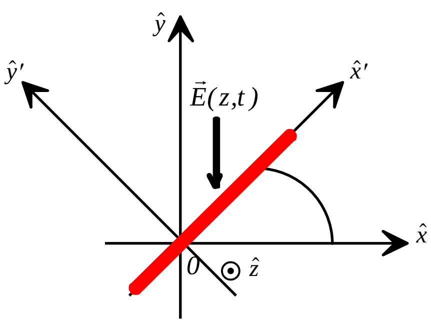
VERTICAL POLARISATION STATE



$$\underline{g}_{\underline{V}} = \begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}$$

$$\phi = \frac{\pi}{2} \qquad \tau = 0$$

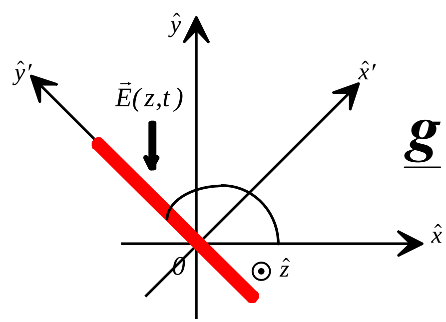
LINEAR POLARISATION STATE



$$\underline{g}_{\underline{L}} = \begin{bmatrix} 1 \\ \cos(2\theta) \\ \sin(2\theta) \\ 0 \end{bmatrix}$$

$$\phi = \theta \qquad \tau = 0$$

ORTHOGONAL LINEAR POLARISATION STATE

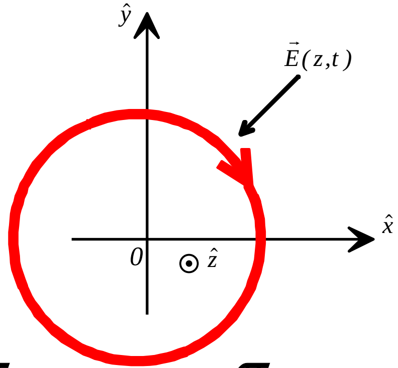


$$\underline{g}_{\underline{L}} = \begin{bmatrix} 1 \\ -\cos(2\theta) \\ -\sin(2\theta) \\ 0 \end{bmatrix}$$

$$\phi = \theta + \frac{\pi}{2} \qquad \tau = 0$$

STOKES VECTOR

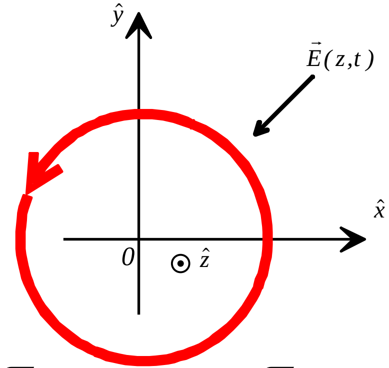
LEFT CIRCULAR POLARISATION STATE



$$\underline{g}_{\underline{LC}} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$-\frac{\pi}{2} \le \phi \le +\frac{\pi}{2} \qquad \tau = +\frac{\pi}{4}$$

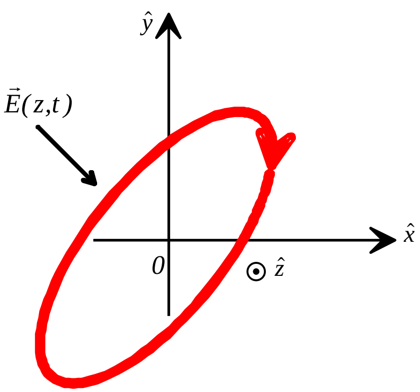
RIGHT CIRCULAR POLARISATION STATE



$$\underline{g}_{\underline{RC}} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}$$

$$-\frac{\pi}{2} \le \phi \le +\frac{\pi}{2} \qquad \tau = -\frac{\pi}{4}$$

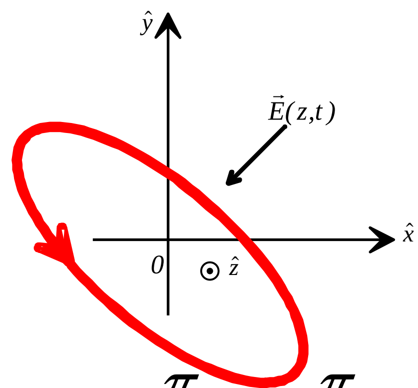
ELLIPTICAL POLARISATION STATE



$$\underline{g}_{\underline{E}} = \begin{bmatrix} 1 \\ g_1 \\ g_2 \\ g_3 \end{bmatrix}$$

$$\phi = \theta \qquad 0 \le \tau \le +\frac{\pi}{4}$$

ORTHOGONAL ELLIPTICAL POLARISATION STATE



$$\underline{g}_{\underline{E}_\perp} = \begin{bmatrix} 1 \\ -g_1 \\ -g_2 \\ -g_3 \end{bmatrix}$$

$$\phi = \theta + \frac{\pi}{2} \qquad -\frac{\pi}{4} \le \tau \le 0$$

ELLIPTICAL BASIS TRANSFORMATION

SPECIAL UNITARY SU(2) GROUP

$$[U_2] = \begin{bmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{bmatrix} \begin{bmatrix} \cos(\tau) & j\sin(\tau) \\ j\sin(\tau) & \cos(\tau) \end{bmatrix} \begin{bmatrix} e^{-j\alpha} & 0 \\ 0 & e^{j\alpha} \end{bmatrix}$$

$$[U_2(\phi)] \qquad [U_2(\tau)] \qquad [U_2(\alpha)]$$

HOMOMORPHISM SU(2) - O(3)

$$[O_3(2\theta)]_{p,q} = \frac{1}{2} Tr\left([U_2(\theta)]^{T*} \sigma_p [U_2(\theta)] \sigma_q\right)$$

(σ_p, σ_q) : Pauli Matrices

O(4) UNITARY GROUP

$$[O_4] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(2\phi) & -\sin(2\phi) & 0 \\ 0 & \sin(2\phi) & \cos(2\phi) & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(2\tau) & 0 & -\sin(2\tau) \\ 0 & 0 & 1 & 0 \\ 0 & \sin(2\tau) & 0 & \cos(2\tau) \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos(2\alpha) & -\sin(2\alpha) \\ 0 & 0 & \sin(2\alpha) & \cos(2\alpha) \end{bmatrix}$$

$$[O_4(2\phi)] \qquad [O_4(2\tau)] \qquad [O_4(2\alpha)]$$

POINCARE SPHERE

STOKES VECTOR

$$\underline{g}_{\underline{E}} = \begin{bmatrix} g_0 \\ g_1 \\ g_2 \\ g_3 \end{bmatrix} = \begin{bmatrix} |E_x|^2 + |E_y|^2 \\ |E_x|^2 - |E_y|^2 \\ 2\Re\left(E_x E_y^*\right) \\ -2\Im\left(E_x E_y^*\right) \end{bmatrix} = \begin{bmatrix} E_{0x}^2 + E_{0y}^2 \\ E_{0x}^2 - E_{0y}^2 \\ 2E_{0x}E_{0y}\cos(\delta) \\ 2E_{0x}E_{0y}\sin(\delta) \end{bmatrix} = \begin{bmatrix} A^2 \\ A^2 \cos 2\phi \cos 2\tau \\ A^2 \sin 2\phi \cos 2\tau \\ A^2 \sin 2\tau \end{bmatrix}$$

$\{g_0\}$ TOTAL WAVE INTENSITY

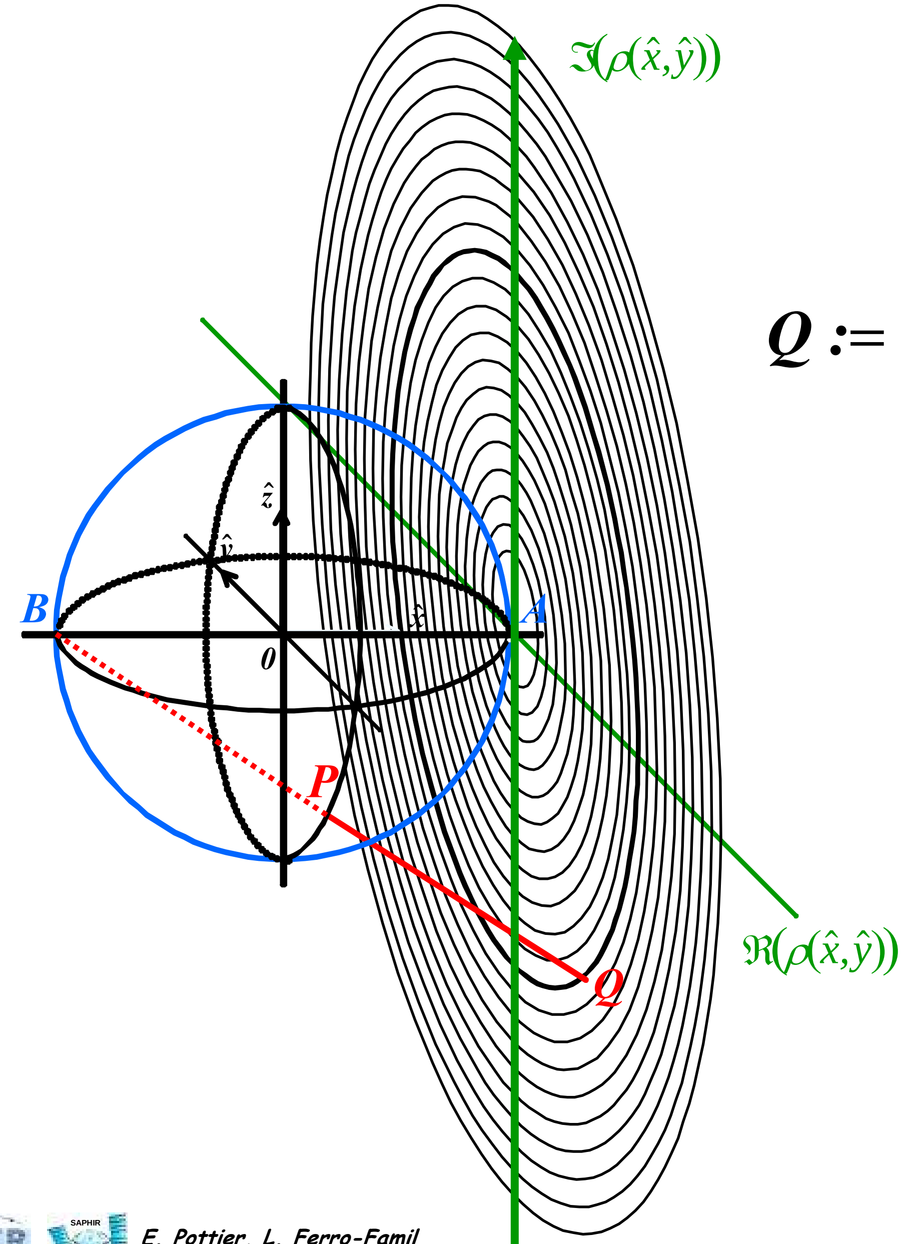
$\{g_1, g_2, g_3\}$ POLARISED WAVE INTENSITIES

$$\boxed{g_0^2 = g_1^2 + g_2^2 + g_3^2}$$

WAVE FULLY POLARISED

$\{g_1, g_2, g_3\}$ Spherical Coordinates of a point P on a sphere with radius g_0

POINCARE SPHERE



$$Q := \begin{cases} \Re e\left(\rho_{(\hat{x},\hat{y})}\right) = \dfrac{\cos(2\tau)\sin(2\phi)}{1+\cos(2\tau)\cos(2\phi)} \\ \Im m\left(\rho_{(\hat{x},\hat{y})}\right) = \dfrac{\sin(2\tau)}{1+\cos(2\tau)\cos(2\phi)} \end{cases}$$

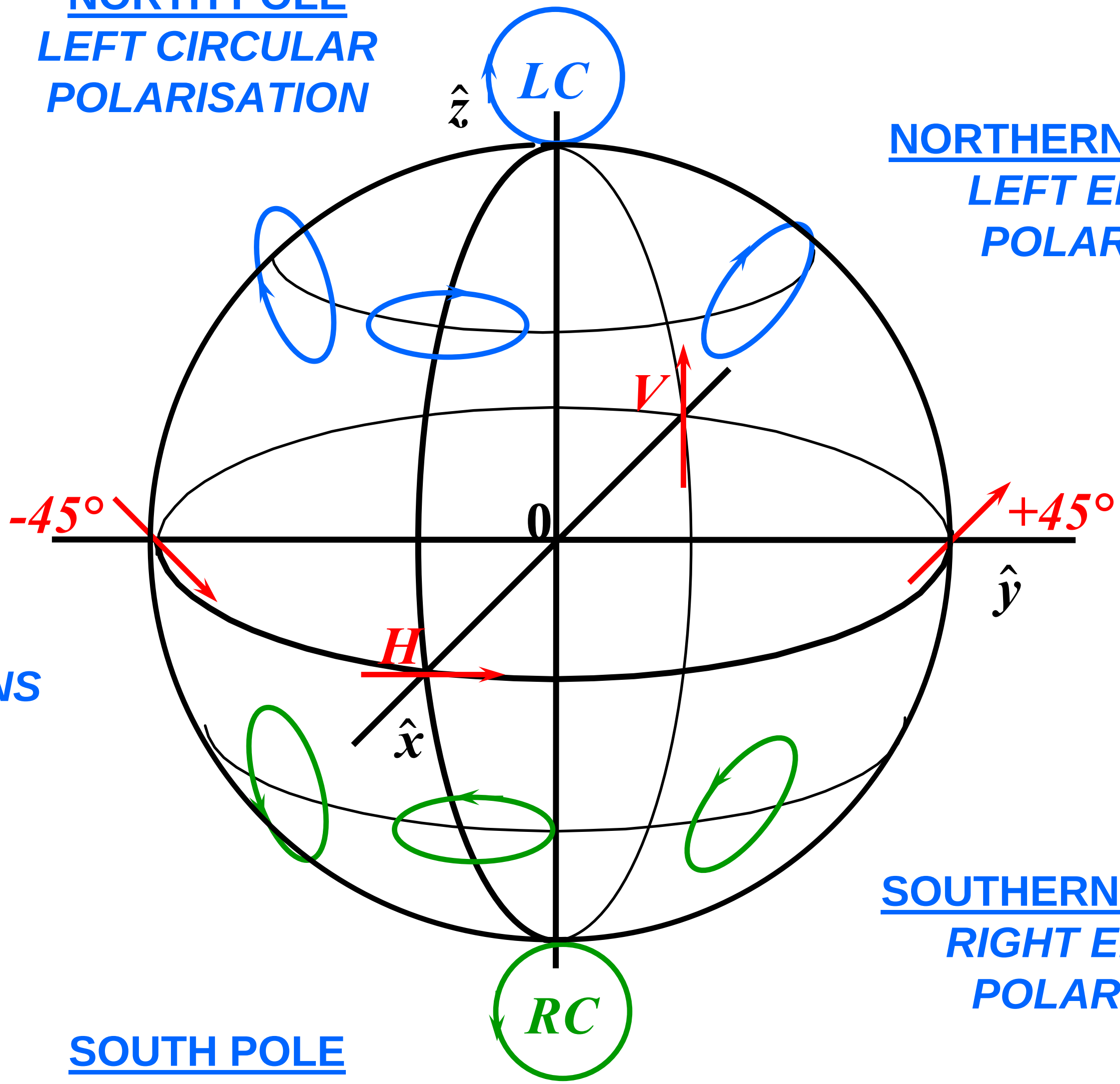
STEREOGRAPHIC PROJECTION

$$P := \begin{cases} \cos 2\phi \cos 2\tau \\ \sin 2\phi \cos 2\tau \\ \sin 2\tau \end{cases}$$

POINCARE SPHERE

NORTH POLE
LEFT CIRCULAR POLARISATION

NORTHERN HEMISPHERE
LEFT ELLIPTICAL POLARISATIONS

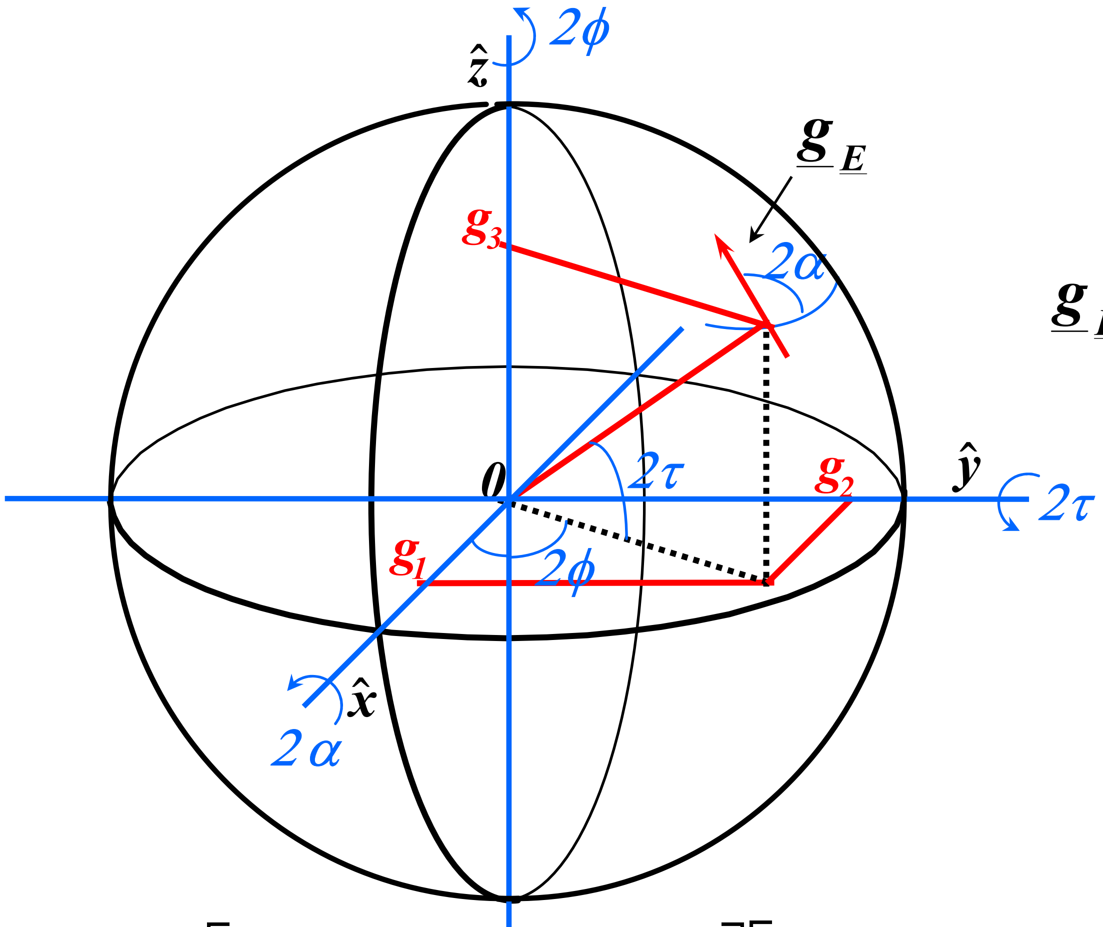


EQUATOR
LINEAR POLARISATIONS

SOUTHERN HEMISPHERE
RIGHT ELLIPTICAL POLARISATIONS

SOUTH POLE
RIGHT CIRCULAR POLARISATION

POINCARE SPHERE



$$\underline{g}_E = \begin{bmatrix} g_0 \\ g_1 \\ g_2 \\ g_3 \end{bmatrix} = \begin{bmatrix} A^2 \\ A^2 \cos 2\phi \cos 2\tau \\ A^2 \sin 2\phi \cos 2\tau \\ A^2 \sin 2\tau \end{bmatrix}$$

$$\underline{g}_E = A^2 \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(2\phi) & -\sin(2\phi) & 0 \\ 0 & \sin(2\phi) & \cos(2\phi) & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}}_{[O_4(2\phi)]} \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(2\tau) & 0 & -\sin(2\tau) \\ 0 & 0 & 1 & 0 \\ 0 & \sin(2\tau) & 0 & \cos(2\tau) \end{bmatrix}}_{[O_4(2\tau)]} \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos(2\alpha) & -\sin(2\alpha) \\ 0 & 0 & \sin(2\alpha) & \cos(2\alpha) \end{bmatrix}}_{[O_4(2\alpha)]} \underline{g}_{\hat{u}}$$

POINCARE SPHERE

JONES VECTOR

$$\underline{E} = \begin{bmatrix} E_x \\ E_y \end{bmatrix}$$

ORTHOGONAL JONES VECTOR

$$\underline{E}_\perp = \begin{bmatrix} E'_x \\ E'_y \end{bmatrix}$$

ORTHOGONALITY CONDITIONS

$$(\phi, \tau) \mapsto \begin{cases} \phi' = \phi + \frac{\pi}{2} \\ \tau' = -\tau \end{cases}$$

STOKES VECTOR

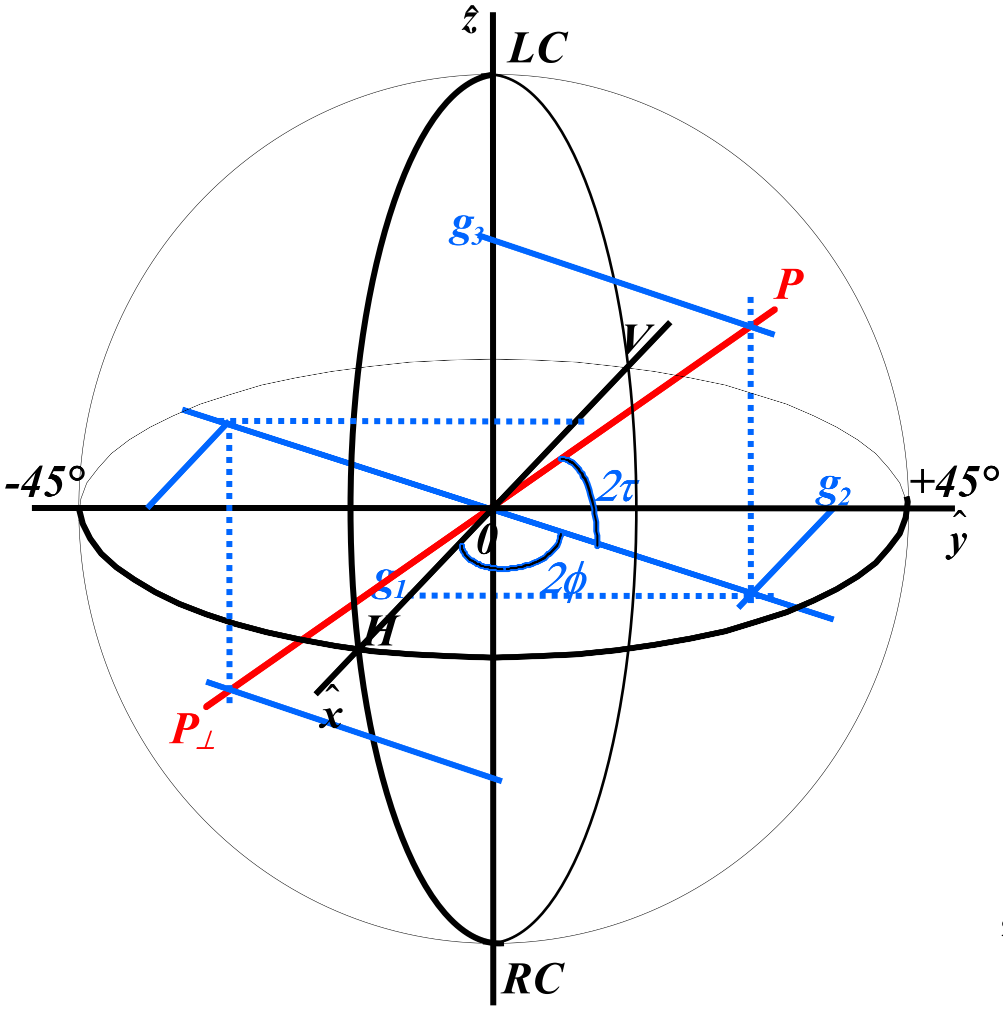
$$\underline{g}_{\underline{E}} = \begin{bmatrix} g_0 \\ g_1 \\ g_2 \\ g_3 \end{bmatrix} = \begin{bmatrix} A \\ A\cos 2\phi \cos 2\tau \\ A\sin 2\phi \cos 2\tau \\ A\sin 2\tau \end{bmatrix}$$

ORTHOGONAL STOKES VECTOR

$$\underline{g}_{\underline{E}_\perp} = \begin{bmatrix} g_0 \\ g_1 \\ g_2 \\ g_3 \end{bmatrix} = \begin{bmatrix} A \\ -A\cos 2\phi \cos 2\tau \\ -A\sin 2\phi \cos 2\tau \\ -A\sin 2\tau \end{bmatrix}$$

ORTHOGONALITY = ANTIPODALITY

POINCARE SPHERE



STOKES VECTOR

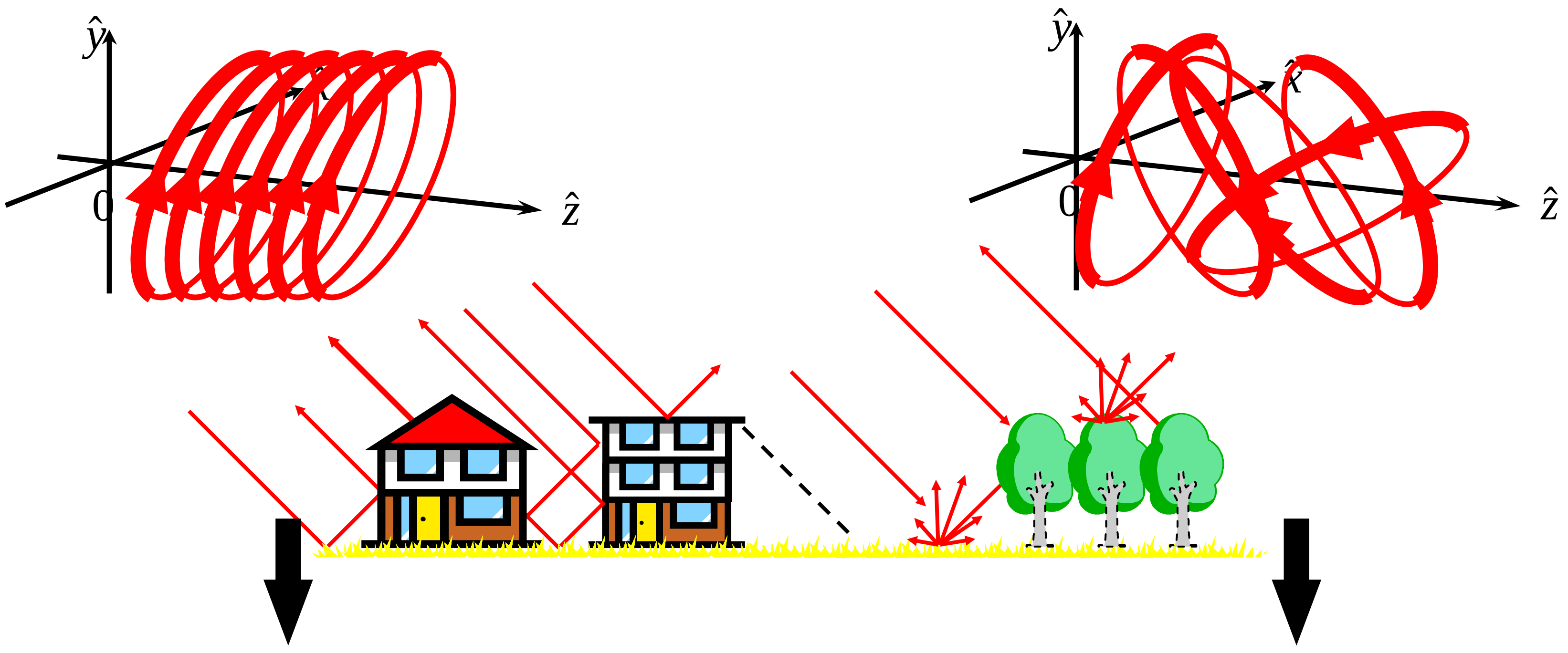
$$\underline{g}_E = \begin{bmatrix} g_0 \\ g_1 \\ g_2 \\ g_3 \end{bmatrix} = \begin{bmatrix} A \\ A\cos 2\phi \cos 2\tau \\ A \sin 2\phi \cos 2\tau \\ A \sin 2\tau \end{bmatrix}$$

ORTHOGONAL STOKES VECTOR

$$\underline{g}_{E_\perp} = \begin{bmatrix} g_0 \\ g_1 \\ g_2 \\ g_3 \end{bmatrix} = \begin{bmatrix} A \\ -A\cos 2\phi \cos 2\tau \\ -A \sin 2\phi \cos 2\tau \\ -A \sin 2\tau \end{bmatrix}$$

ORTHOGONALITY = ANTIPODALITY

PARTIALLY POLARISED WAVES



DETERMINISTIC SCATTERING

COMPLETELY POLARISED WAVE

RANDOM SCATTERING

PARTIALLY POLARISED WAVE

Polarisation Ellipse varies in time
Amplitude, Phase: Random processes

STATISTICAL DESCRIPTION

PARTIALLY POLARISED WAVES

JONES VECTORS $\{\underline{E}\}$

$\Downarrow$

WAVE COVARIANCE MATRIX

$$\langle[J]\rangle = \left\langle \underline{E}\,\underline{E}^{T*} \right\rangle = \begin{bmatrix} \left\langle |E_x|^2 \right\rangle & \left\langle E_x E_y^* \right\rangle \\ \left\langle E_y E_x^* \right\rangle & \left\langle |E_y|^2 \right\rangle \end{bmatrix}$$

$\Updownarrow$

$$\langle[J]\rangle = \frac{1}{2}\begin{bmatrix} \langle g_0\rangle + \langle g_1\rangle & \langle g_2\rangle - j\langle g_3\rangle \\ \langle g_2\rangle + j\langle g_3\rangle & \langle g_0\rangle - \langle g_1\rangle \end{bmatrix}$$

$$\boxed{\langle g_0\rangle^2 \geq \langle g_1\rangle^2 + \langle g_2\rangle^2 + \langle g_3\rangle^2}$$

PARTIALLY POLARISED WAVES

PARTIALLY POLARISED WAVES

WAVE COVARIANCE MATRIX

$$\langle[J]\rangle = \left\langle \underline{E}\,\underline{E}^{T*} \right\rangle = \begin{bmatrix} \left\langle |E_x|^2 \right\rangle & \left\langle E_x E_y^* \right\rangle \\ \left\langle E_y E_x^* \right\rangle & \left\langle |E_y|^2 \right\rangle \end{bmatrix}$$

<u>DIAGONAL ELEMENTS</u> : INTENSITIES ON EACH OF THE 2 ORTHOGONAL COMPONENTS OF THE WAVE

<u>OFF-DIAGONAL ELEMENTS</u> : CROSS-CORRELATIONS BETWEEN THE 2 ORTHOGONAL COMPONENTS OF THE WAVE

$$Trace([J]) = \left\langle |E_x|^2 \right\rangle + \left\langle |E_y|^2 \right\rangle = A^2$$

TOTAL WAVE INTENSITY

THE WAVE COVARIANCE MATRIX IS A
2x2 HERMITIAN POSITIVE SEMI-DEFINITE MATRIX

PARTIALLY POLARISED WAVES

EIGENVALUES DECOMPOSITION

$$\langle [J] \rangle = [U_2] \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} [U_2]^{-1} = \lambda_1 \underline{u}_1 \underline{u}_1^{T*} + \lambda_2 \underline{u}_2 \underline{u}_2^{T*}$$

➡ 2 ORTHOGONAL EIGENVECTORS $[U_2] = [\underline{u}_1, \underline{u}_2]$

➡ 2 REAL EIGENVALUES

$$\lambda_1 = \frac{1}{2} \left\{ \langle g_0 \rangle + \sqrt{\langle g_1 \rangle^2 + \langle g_2 \rangle^2 + \langle g_3 \rangle^2} \right\}$$

$$\lambda_2 = \frac{1}{2} \left\{ \langle g_0 \rangle - \sqrt{\langle g_1 \rangle^2 + \langle g_2 \rangle^2 + \langle g_3 \rangle^2} \right\}$$

PARTIALLY POLARISED WAVES

PARTIALLY POLARISED WAVES DESCRIPTORS

Degree of Polarisation

$$DoP = \frac{\sqrt{\langle g_1 \rangle^2 + \langle g_2 \rangle^2 + \langle g_3 \rangle^2}}{\langle g_0 \rangle} = \frac{\lambda_1 - \lambda_2}{\lambda_1 + \lambda_2} = \left(1 - \frac{4\, det([J])}{Trace^2([J])} \right)$$

Polarised Wave Power

Total Wave Power

Anisotropy

Wave Entropy

$$H = -\sum_{i=1}^{i=2} p_i \log_2(p_i)$$

With: $$p_i = \frac{\lambda_i}{\lambda_1 + \lambda_2}$$

Degree of randomness, statistical disorder

PARTIALLY POLARISED WAVES

COMPLETELY POLARISED WAVES

Maximum Correlation Between E_x and E_y

$$\left\langle |E_x|^2 \right\rangle \left\langle |E_y|^2 \right\rangle = \left\langle E_x E_y^* \right\rangle \left\langle E_y E_x^* \right\rangle \Rightarrow det([J]) = 0 \Rightarrow \begin{cases} \lambda_1 \neq 0 \\ \lambda_2 = 0 \end{cases} \Rightarrow \begin{cases} DoP = 1 \\ H = 0 \end{cases}$$

COMPLETELY UNPOLARISED WAVES

Absence of any Polarised Structure in the Wave

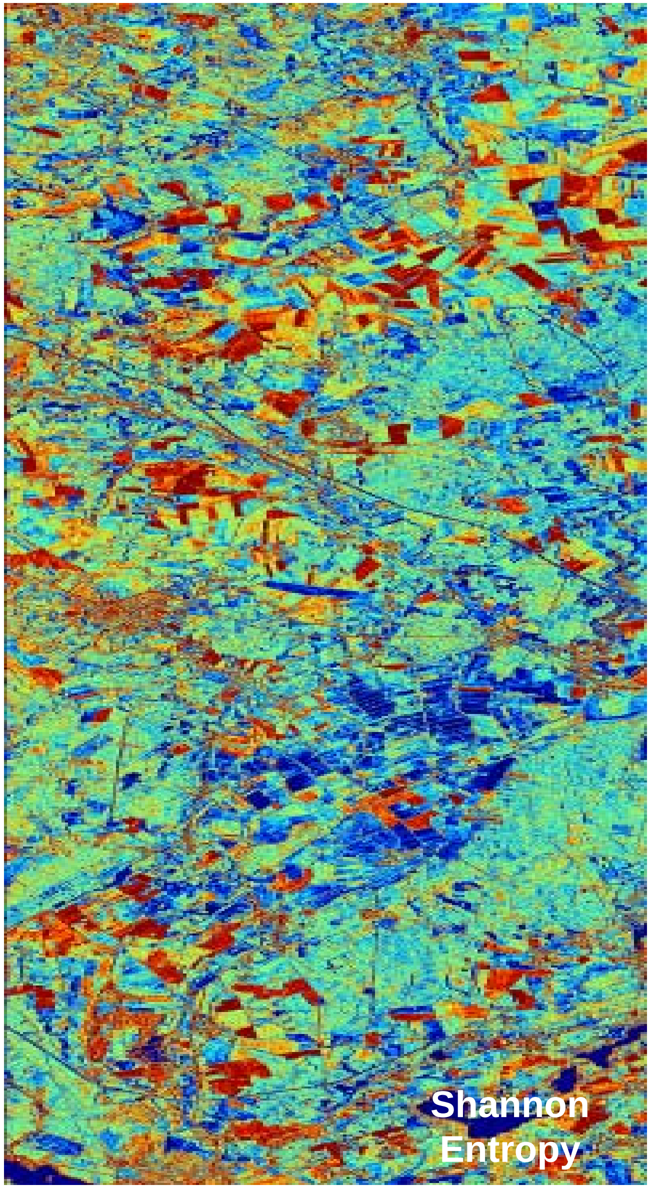
$$\begin{cases} \left\langle |E_x|^2 \right\rangle = \left\langle |E_y|^2 \right\rangle \\ \left\langle E_x E_y^* \right\rangle = \left\langle E_y E_x^* \right\rangle = 0 \end{cases} \Rightarrow det([J]) = \frac{Trace^2([J])}{4} \Rightarrow \lambda_1 = \lambda_2 \Rightarrow \begin{cases} DoP = 0 \\ H = 1 \end{cases}$$

PARTIALLY POLARISED WAVES

Correlation between E_x and E_y

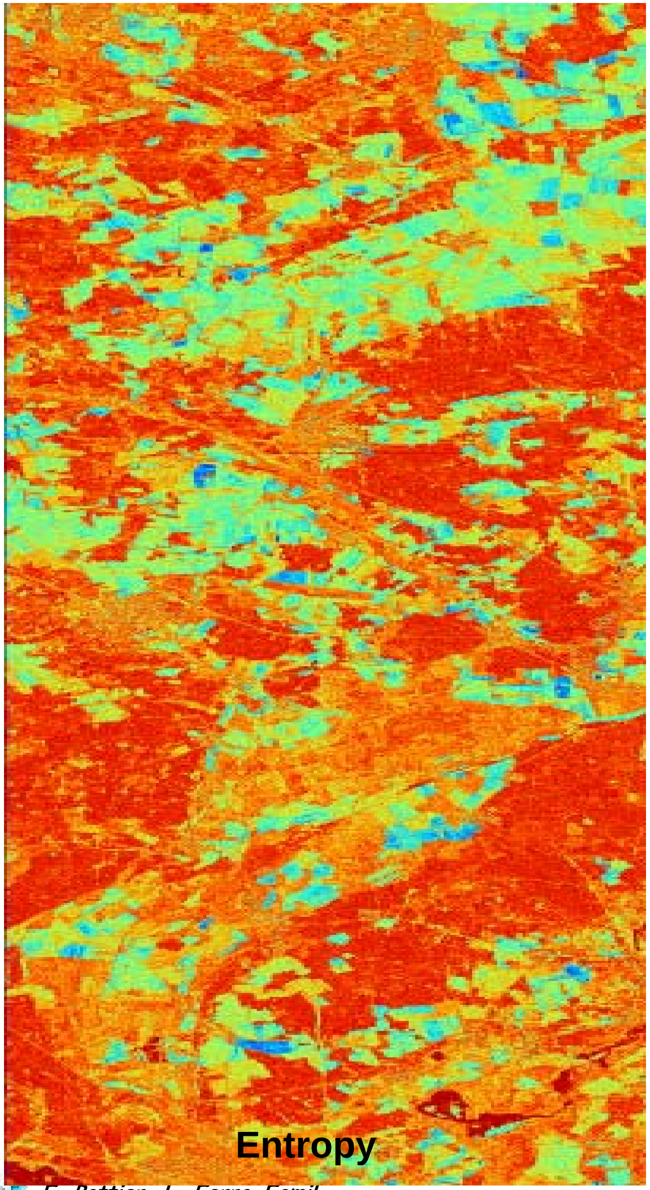
$$\langle [J] \rangle = \begin{bmatrix} \left\langle |E_x|^2 \right\rangle & \left\langle E_x E_y^* \right\rangle \\ \left\langle E_y E_x^* \right\rangle & \left\langle |E_y|^2 \right\rangle \end{bmatrix} \Rightarrow \begin{cases} det([J]) \geq 0 \\ \lambda_1 \neq \lambda_2 \geq 0 \end{cases} \Rightarrow \begin{cases} 0 \leq DoP \leq 1 \\ 0 \leq H \leq 1 \end{cases}$$

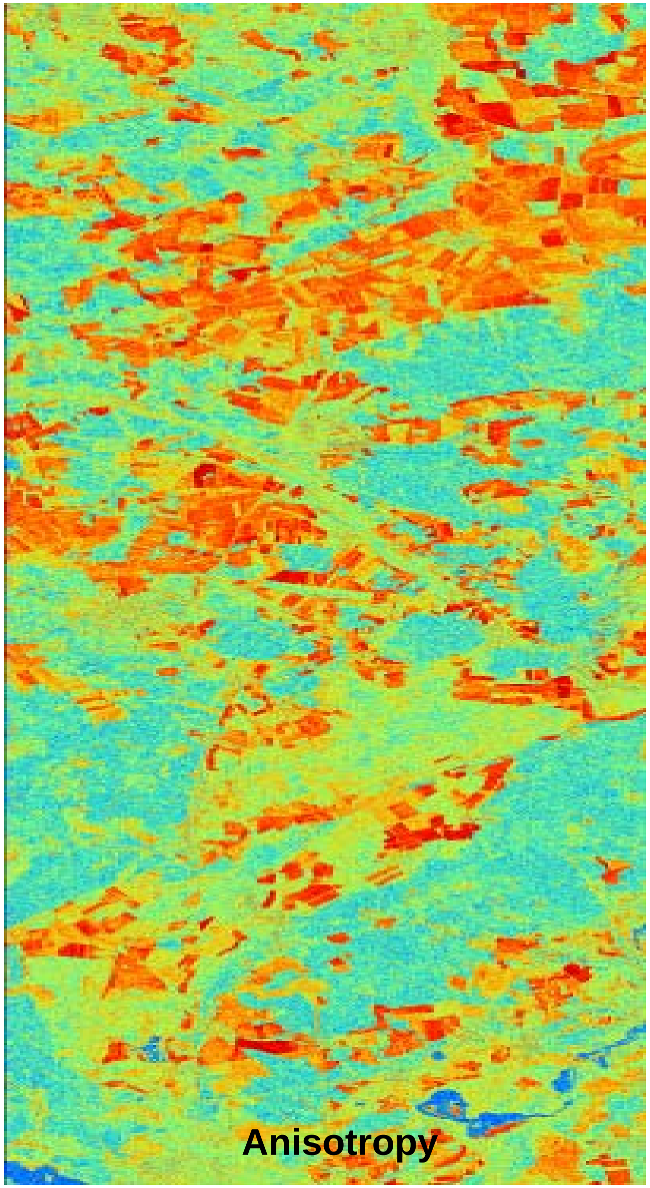
RADAR POLARIMETRY



IGARSS 2008 BOSTON

RADAR POLARIMETRY





WAVE DESCRIPTORS

MONOCHROMATIC PLANE WAVES

COMPLEX DOMAIN

JONES VECTOR $\underline{E} = \begin{bmatrix} E_x \\ E_y \end{bmatrix}$

REAL DOMAIN

STOKES VECTOR $\underline{g}_{\underline{E}} = \begin{bmatrix} g_0 \\ g_1 \\ g_2 \\ g_3 \end{bmatrix}$

PLANE WAVES FULLY DESCRIBED
BY 3 INDEPENDANT PARAMETERS

- $E_{0x}, E_{0x}, \delta = \delta_y - \delta_x$
- (A, ϕ, τ) or (A, γ, δ)
- $\{g_1, g_2, g_3\}$

WAVE POLARIMETRIC DIMENSION = 3

WAVE DESCRIPTORS

PARTIALLY POLARISED PLANE WAVES

COMPLEX DOMAIN

COVARIANCE MATRIX $\langle [J] \rangle = \langle \underline{E}\,\underline{E}^{T*} \rangle$

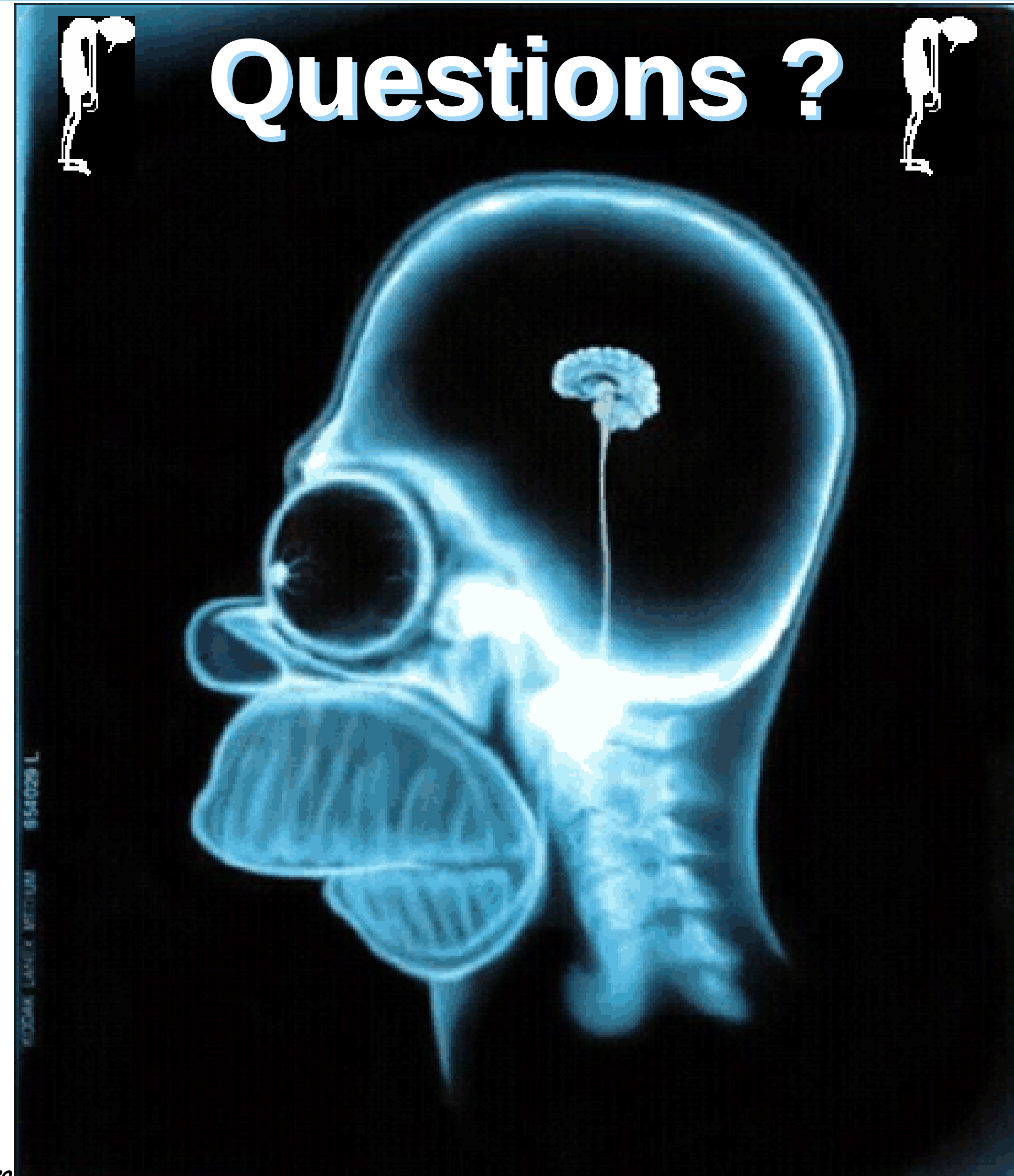
REAL DOMAIN

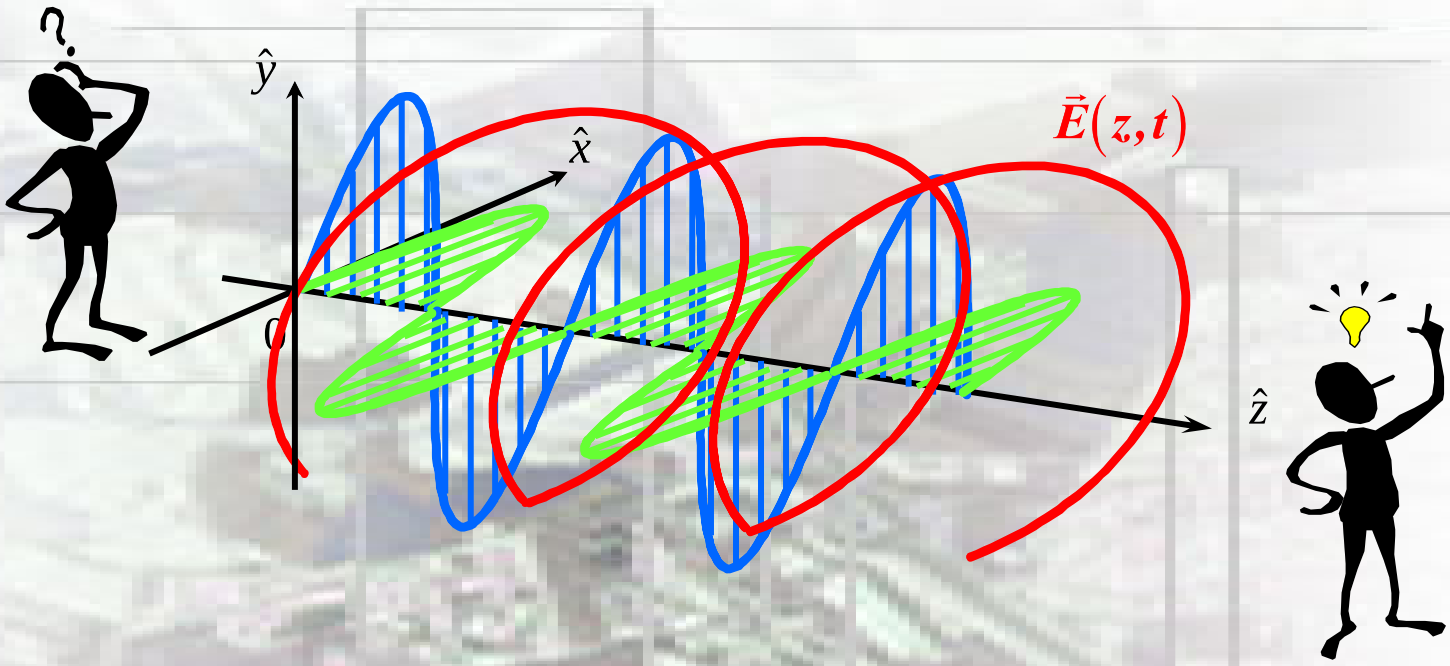
STOKES VECTOR $\langle \underline{g}_E \rangle = \begin{bmatrix} \langle g_0 \rangle \\ \langle g_1 \rangle \\ \langle g_2 \rangle \\ \langle g_3 \rangle \end{bmatrix}$

PLANE WAVES FULLY DESCRIBED
BY 4 INDEPENDANT PARAMETERS

- $\langle |E_x|^2 \rangle, \langle E_x E_y^* \rangle, \langle E_y E_x^* \rangle, \langle |E_y|^2 \rangle$
- $\{ \langle g_0 \rangle, \langle g_1 \rangle, \langle g_2 \rangle, \langle g_3 \rangle \}$

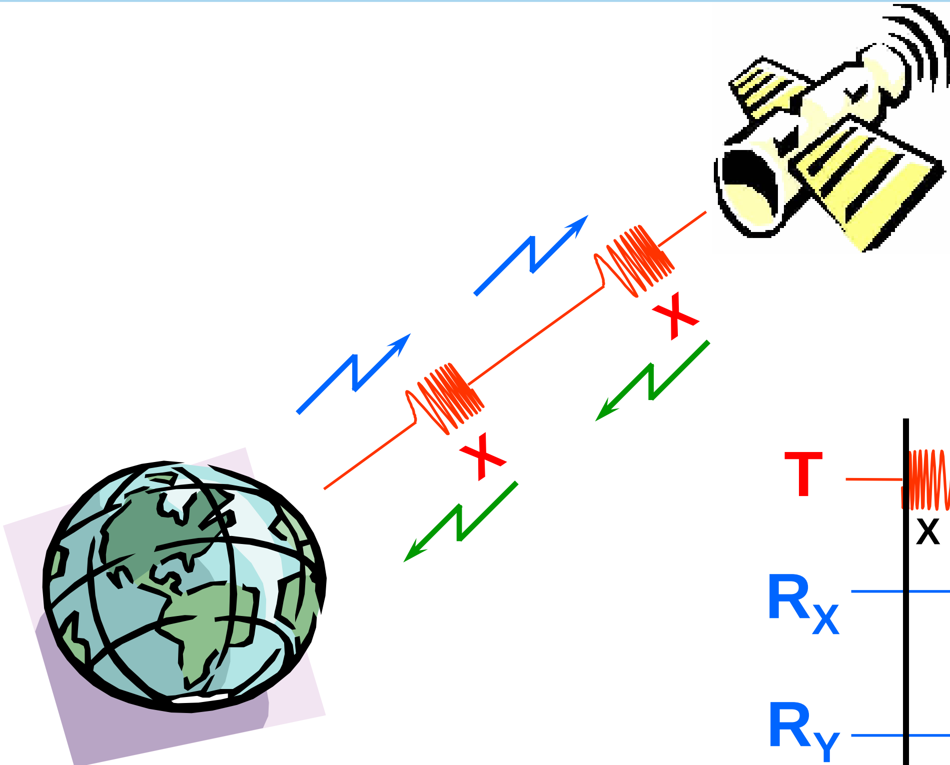
WAVE POLARIMETRIC DIMENSION = 4

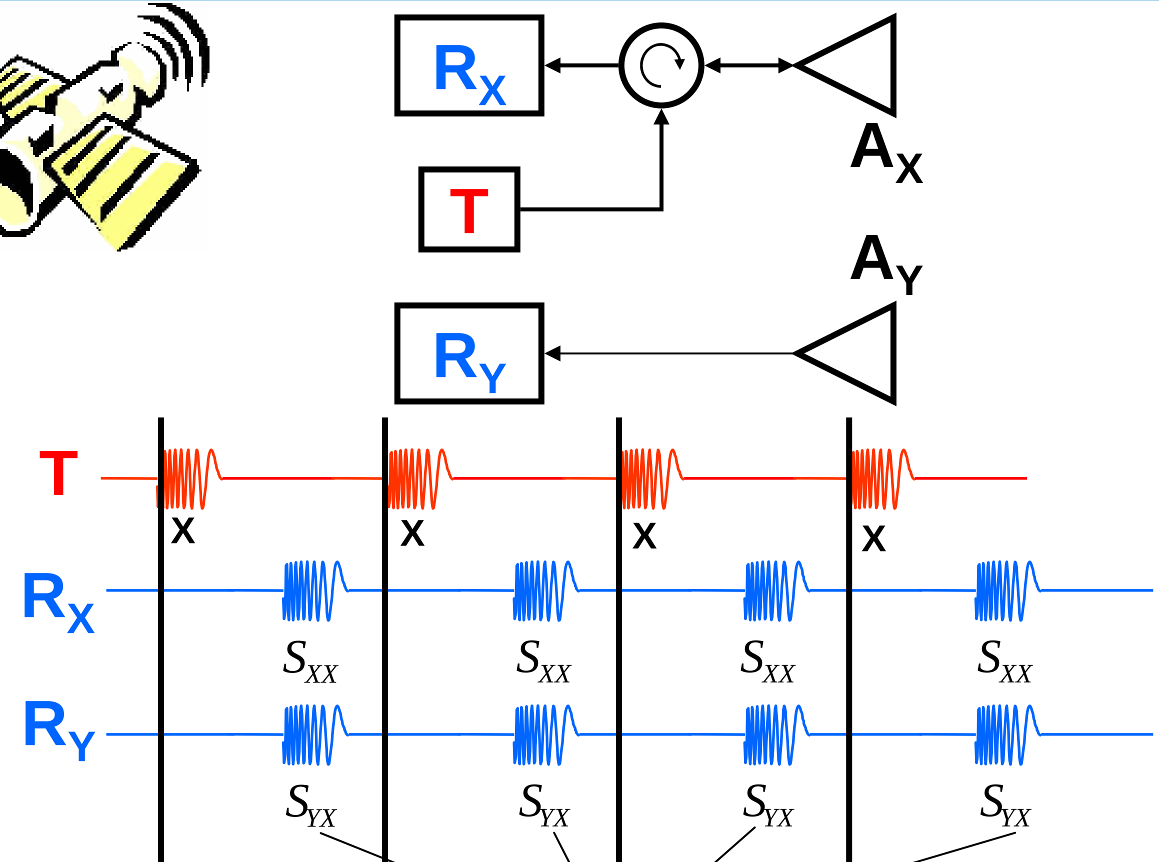

Questions ?



SCATTERING POLARIMETRY

WAVE POLARIMETRY





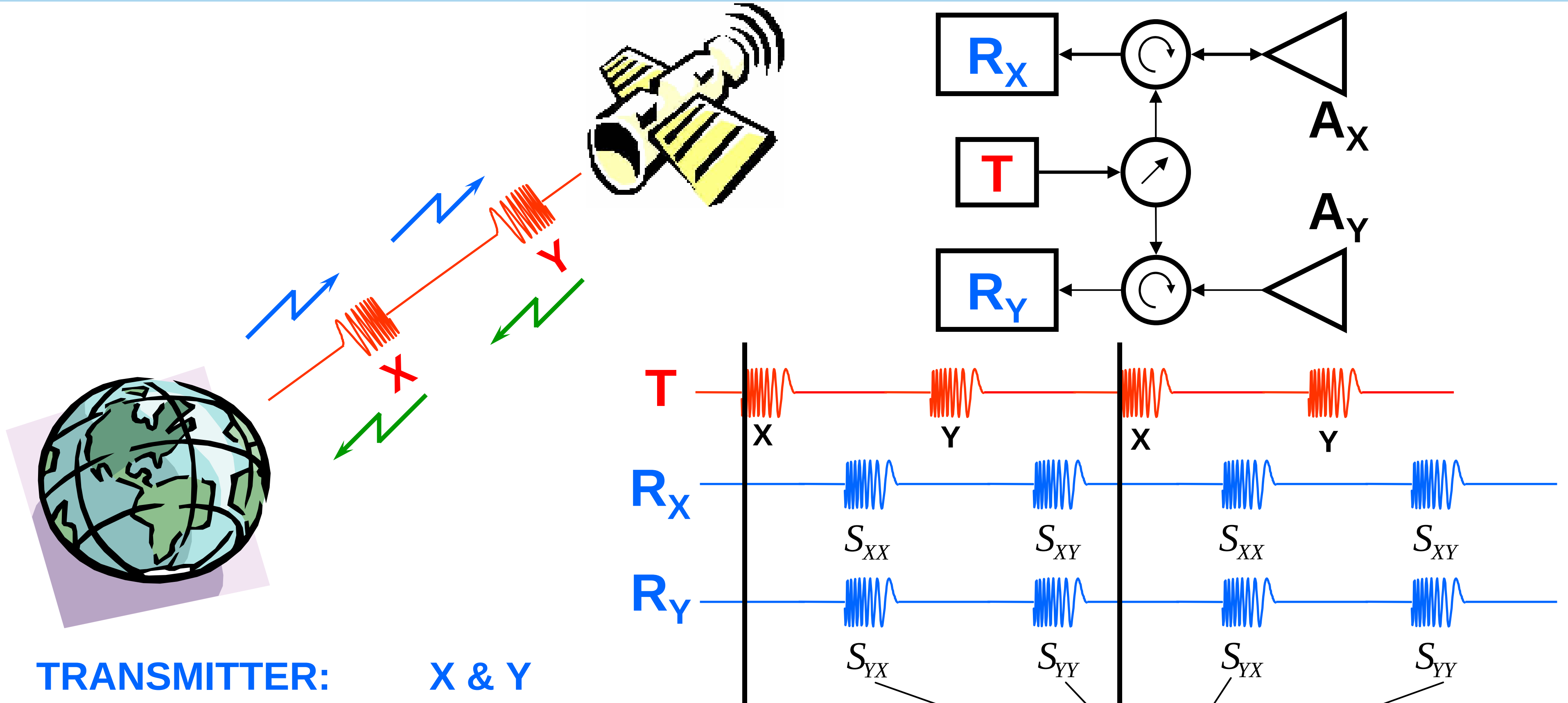
TRANSMITTER: X

RECEIVERS: X & Y

JONES VECTORS $\left\{ \underline{E}_S = \begin{bmatrix} S_{XX} \\ S_{YX} \end{bmatrix} \right\}$

WAVE POLARIMETRY

SCATTERING POLARIMETRY



TRANSMITTER: X & Y

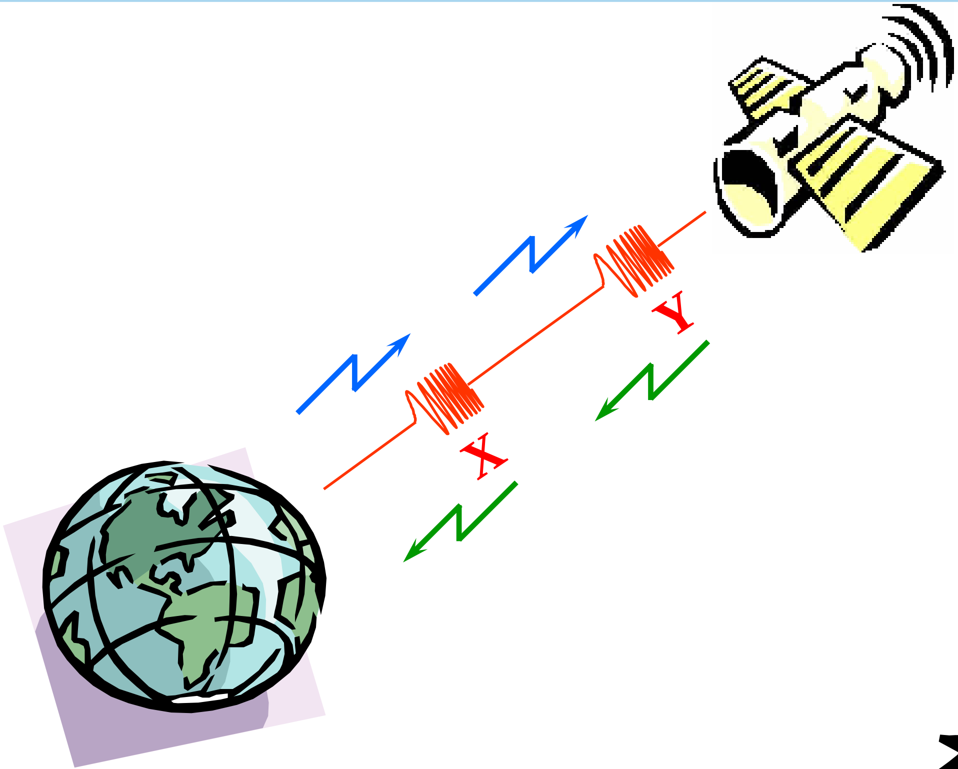
RECEIVERS: X & Y

SINCLAIR MATRICES

$$\left\{ [S] = \begin{bmatrix} S_{XX} & S_{XY} \\ S_{YX} & S_{YY} \end{bmatrix} \right\}$$

SCATTERING POLARIMETRY

POLARIMETRIC DESCRIPTORS



TRANSMITTER: X & Y
RECEIVERS: X & Y

THE DIFFERENT TARGET POLARIMETRIC DESCRIPTORS

- [S] SINCLAIR Matrix
- $\underline{k}$, $\underline{\Omega}$ Target Vectors
- [K] KENNAUGH Matrix
- [T] Coherency Matrix
- [C] Covariance Matrix

SCATTERING MATRIX

BISTATIC CASE

SCATTERING MATRIX or JONES MATRIX

$$\begin{bmatrix} E_X^s \\ E_Y^s \end{bmatrix} = \frac{e^{jkr}}{r} \begin{bmatrix} S_{XX} & S_{XY} \\ S_{YX} & S_{YY} \end{bmatrix} \begin{bmatrix} E_X^i \\ E_Y^i \end{bmatrix}$$

DEFINED IN THE LOCAL COORDINATES SYSTEM

$[S]$ IS INDEPENDENT OF THE POLARISATION STATE OF THE INCIDENCE WAVE

$[S]$ IS DEPENDENT ON THE FREQUENCY AND THE GEOMETRICAL AND ELECTRICAL PROPERTIES OF THE SCATTERER

TOTAL SCATTERED POWER

$$Span([S]) = Trace\left([S][S]^{T*}\right) = |S_{XX}|^2 + |S_{XY}|^2 + |S_{YX}|^2 + |S_{YY}|^2$$

SCATTERING MATRIX

$$[S] = \frac{e^{jkr}}{r}\begin{bmatrix} S_{XX} & S_{XY} \\ S_{YX} & S_{YY} \end{bmatrix} = \underbrace{\frac{e^{jkr}}{r}\begin{bmatrix} |S_{XX}|e^{j\varphi_{XX}} & |S_{XY}|e^{j\varphi_{XY}} \\ |S_{YX}|e^{j\varphi_{YX}} & |S_{YY}|e^{j\varphi_{YY}} \end{bmatrix}}_{\text{ABSOLUTE SCATTERING MATRIX}}$$

ABSOLUTE SCATTERING MATRIX

$$[S] = \underbrace{\frac{e^{jkr}e^{j\phi_{XX}}}{r}}_{\text{Absolute Phase Factor}} \underbrace{\begin{bmatrix} |S_{XX}| & |S_{XY}|e^{j(\varphi_{XY}-\varphi_{XX})} \\ |S_{YX}|e^{j(\varphi_{YX}-\varphi_{XX})} & |S_{YY}|e^{j(\varphi_{YY}-\varphi_{XX})} \end{bmatrix}}_{\text{RELATIVE SCATTERING MATRIX}}$$

Absolute Phase Factor

RELATIVE SCATTERING MATRIX

<u>Seven Parameters:</u> 4 Amplitudes and 3 Phases

SCATTERER POLARIMETRIC DIMENSION = 7

BACKSCATTERING MATRIX

MONOSTATIC CASE

BACKSCATTERING MATRIX or SINCLAIR MATRIX

$\Downarrow$

In the case of Backscattering from Reciprocal Scatterers:

RECIPROCITY THEOREM $S_{XY}^{BSA} = S_{YX}^{BSA} \quad \Leftrightarrow \quad S_{XY}^{FSA} = -S_{YX}^{FSA}$

$\Downarrow$

$$\begin{bmatrix} E_X^s \\ E_Y^s \end{bmatrix} = \frac{e^{jkr}}{r} \begin{bmatrix} S_{XX} & S_{XY} \\ S_{XY} & S_{YY} \end{bmatrix} \begin{bmatrix} E_X^i \\ E_Y^i \end{bmatrix}$$

(BSA CONVENTION)

TOTAL SCATTERED POWER

$$Span([S]) = Trace\left([S][S]^{T*}\right) = |S_{XX}|^2 + 2|S_{XY}|^2 + |S_{YY}|^2$$

BACKSCATTERING MATRIX

$$[S] = \frac{e^{jkr}}{r}\begin{bmatrix} S_{XX} & S_{XY} \\ S_{XY} & S_{YY} \end{bmatrix} = \frac{e^{jkr}}{r}\underbrace{\begin{bmatrix} |S_{XX}|e^{j\varphi_{XX}} & |S_{XY}|e^{j\varphi_{XY}} \\ |S_{XY}|e^{j\varphi_{XY}} & |S_{YY}|e^{j\varphi_{YY}} \end{bmatrix}}_{\text{ABSOLUTE BACKSCATTERING MATRIX}}$$

$$[S] = \underbrace{\frac{e^{jkr}e^{j\phi_{XX}}}{r}}_{\text{Absolute Phase Factor}}\underbrace{\begin{bmatrix} |S_{XX}| & |S_{XY}|e^{j(\varphi_{XY}-\varphi_{XX})} \\ |S_{XY}|e^{j(\varphi_{XY}-\varphi_{XX})} & |S_{YY}|e^{j(\varphi_{YY}-\varphi_{XX})} \end{bmatrix}}_{\text{RELATIVE BACKSCATTERING MATRIX}}$$

RELATIVE BACKSCATTERING MATRIX

<u>Five Parameters:</u> 3 Amplitudes and 2 Phases

$$\boxed{\text{SCATTERER POLARIMETRIC DIMENSION} = 5}$$

ELLIPTICAL BASIS TRANSFORMATION

$$\left[S_{(B,B_\perp)}\right] = \left[U_{(A,A_\perp)\mapsto(B,B_\perp)}\right]^T \left[S_{(A,A_\perp)}\right]\left[U_{(A,A_\perp)\mapsto(B,B_\perp)}\right]$$

CON-SIMILARITY TRANSFORMATION

$$\left[U_{(A,A_\perp)\mapsto(B,B_\perp)}\right]$$

SU(2) SPECIAL UNITARY ELLIPTICAL BASIS TRANSFORMATION MATRIX

$$\begin{aligned}\left[U_{(\underline{A},\underline{A}_\perp)\mapsto(\underline{B},\underline{B}_\perp)}\right] &= \left[U(\phi,\tau,\alpha)\right]^{-1} \\ &= \underbrace{\begin{bmatrix} e^{j\alpha} & 0 \\ 0 & e^{-j\alpha}\end{bmatrix}}_{\left[U_2(-\alpha)\right]}\underbrace{\begin{bmatrix} \cos(\tau) & -j\sin(\tau) \\ -j\sin(\tau) & \cos(\tau)\end{bmatrix}}_{\left[U_2(-\tau)\right]}\underbrace{\begin{bmatrix} \cos(\phi) & \sin(\phi) \\ -\sin(\phi) & \cos(\phi)\end{bmatrix}}_{\left[U_2(-\phi)\right]}\end{aligned}$$

ELLIPTICAL BASIS TRANSFORMATION

FROBENIUS NORM OF $\left[S_{(A,A_\perp)}\right]$

$$Span\left(\left[S_{(A,A_\perp)}\right]\right) = Trace\left(\left[S_{(A,A_\perp)}\right]\left[S_{(A,A_\perp)}\right]^{T*}\right) = |S_{AA}|^2 + 2|S_{AA_\perp}|^2 + |S_{A_\perp A_\perp}|^2$$

FROBENIUS NORM OF $\left[S_{(B,B_\perp)}\right]$

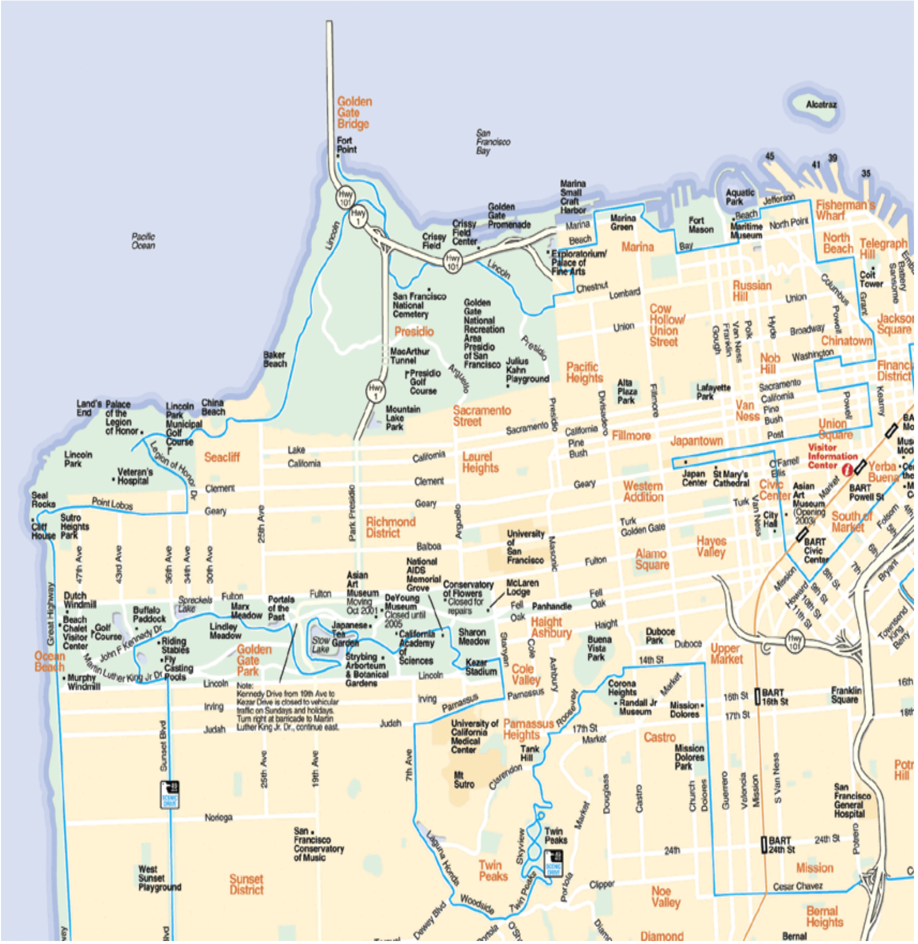
$$Span\left(\left[S_{(B,B_\perp)}\right]\right) = Trace\left(\left[S_{(B,B_\perp)}\right]\left[S_{(B,B_\perp)}\right]^{T*}\right) = |S_{BB}|^2 + 2|S_{BB_\perp}|^2 + |S_{B_\perp B_\perp}|^2$$

SPECIAL UNITARY SU(2) GROUP $\quad [U][U]^{T*} = [I_{D2}] \quad det([U]) = +1$

$$Span\left(\left[S_{(A,A_\perp)}\right]\right) = Span\left(\left[S_{(B,B_\perp)}\right]\right)$$

FROBENIUS NORM OF A SCATTERING MATRIX
IS INVARIANT UNDER BASIS ELLIPTICAL TRANSFORMATION

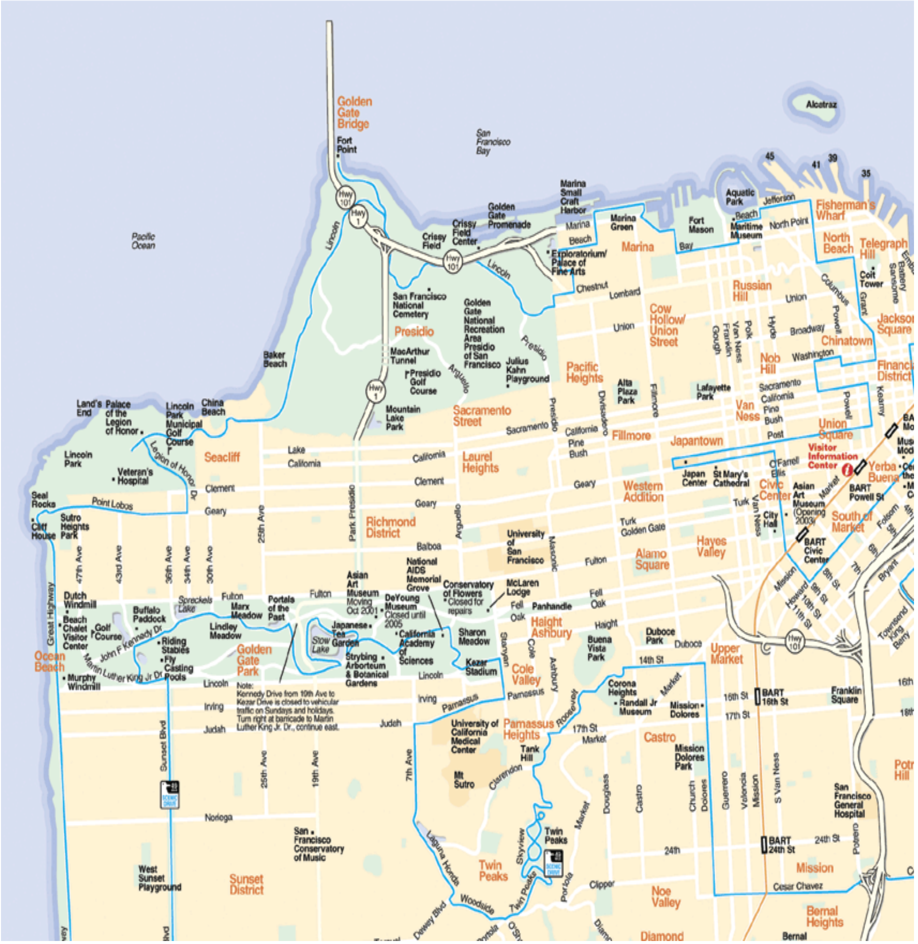
ELLIPTICAL BASIS TRANSFORMATION



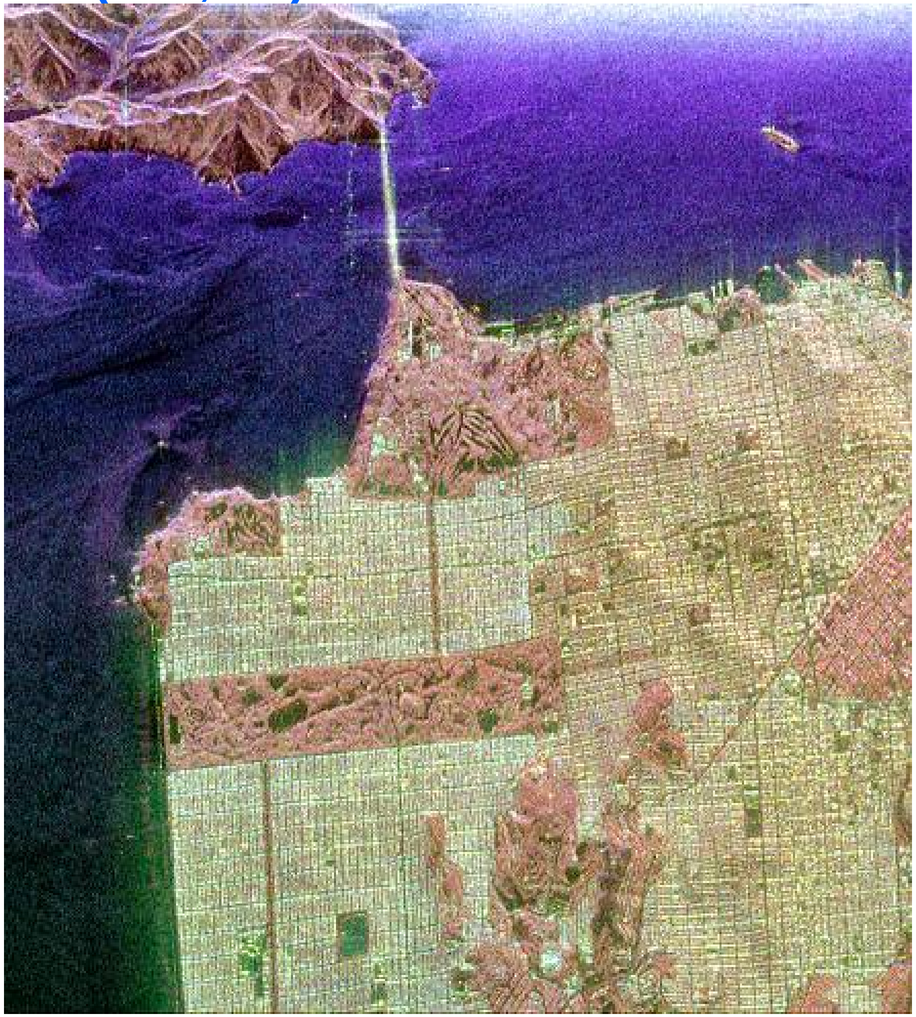
(H,V) POLARISATION BASIS

|HH+VV| |HV | |HH-VV|

ELLIPTICAL BASIS TRANSFORMATION



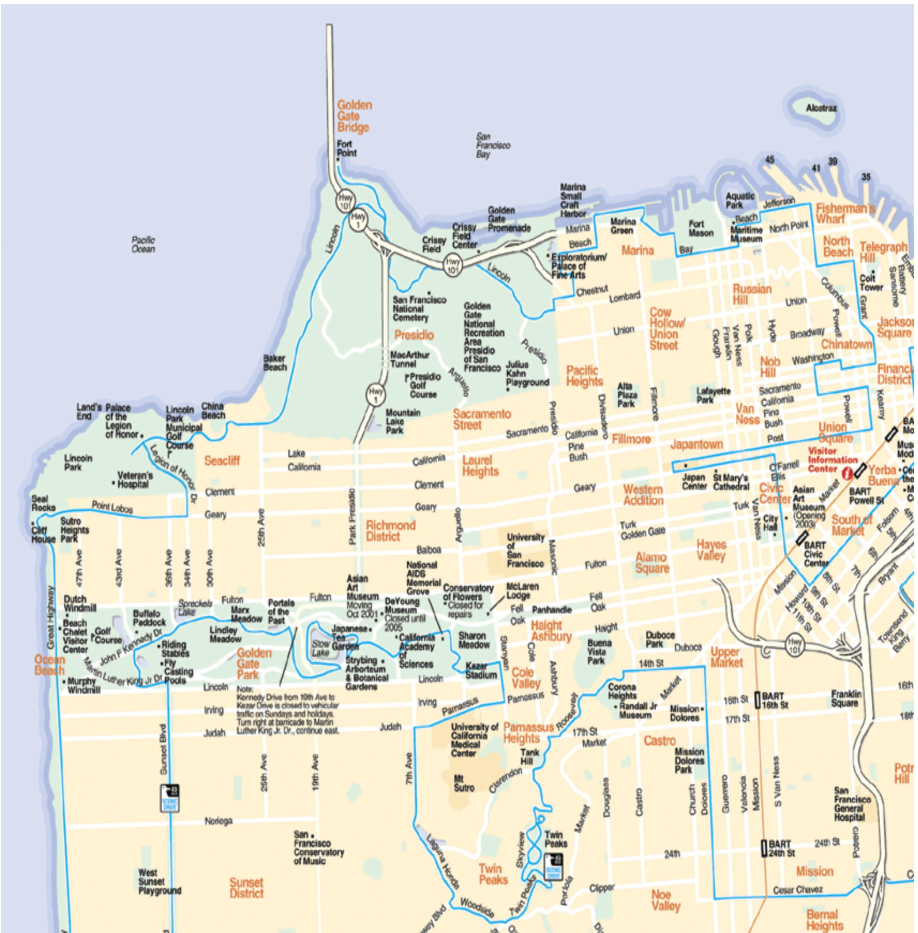
(+45°,-45°) POLARISATION BASIS



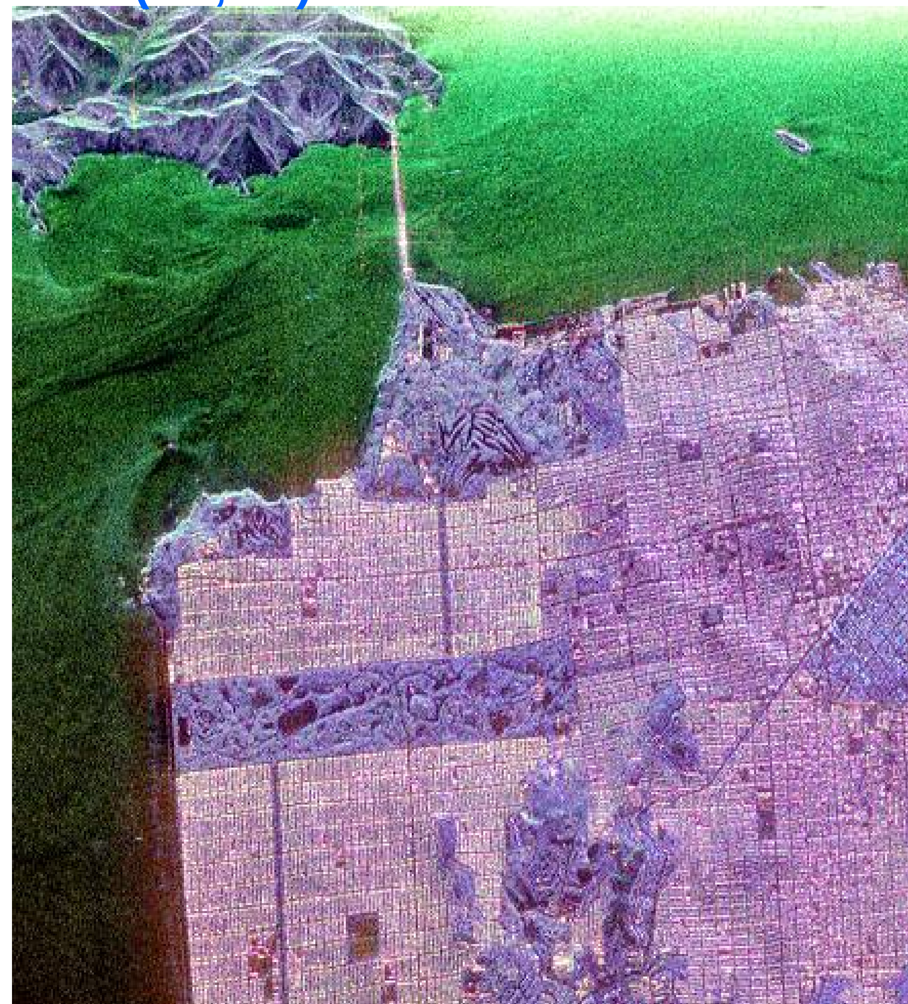
|AA+BB| **|AB |** **|AA-BB|**

With: A=Linear +45°, B=Linear –45°

ELLIPTICAL BASIS TRANSFORMATION

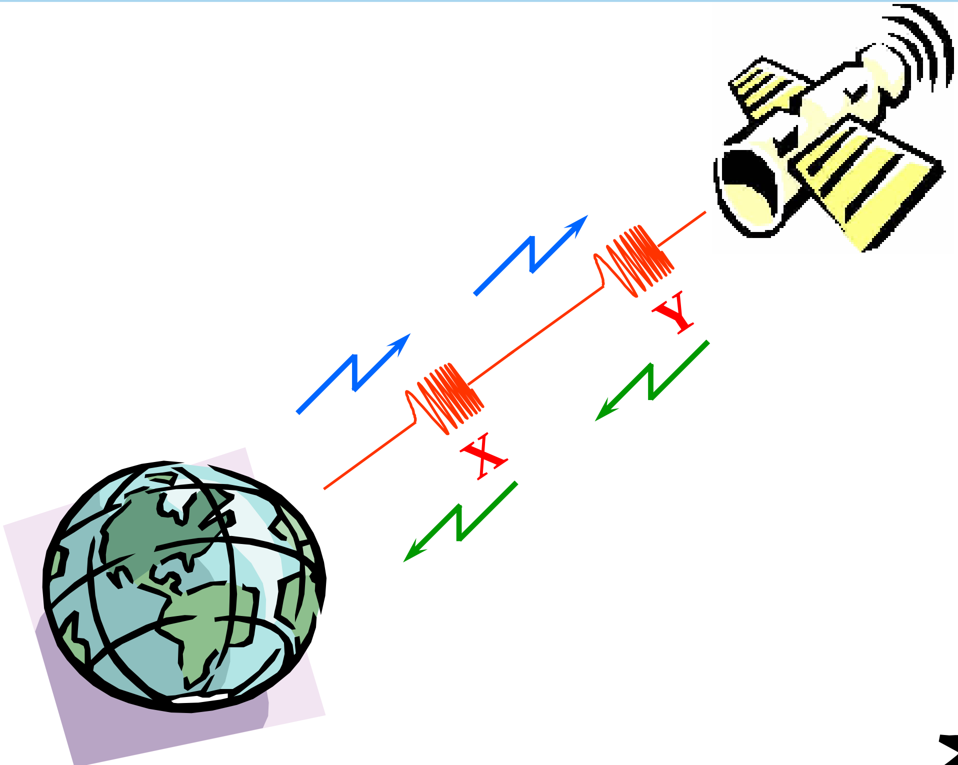


(LC,RC) POLARISATION BASIS



|LL+RR| |LR | |LL-RR|

POLARIMETRIC DESCRIPTORS



TRANSMITTER: X & Y
RECEIVERS: X & Y

THE DIFFERENT TARGET POLARIMETRIC DESCRIPTORS

[S]	SINCLAIR Matrix
k, Ω	**Target Vectors**
[K]	KENNAUGH Matrix
[T]	Coherency Matrix
[C]	Covariance Matrix

TARGET VECTORS

VECTORIAL FORMULATION OF THE SCATTERING PROBLEM

SCATTERING MATRIX

$$[S] = \begin{bmatrix} S_{XX} & S_{XY} \\ S_{YX} & S_{YY} \end{bmatrix}$$

$$\Downarrow$$

SCATTERING VECTOR

$$\vec{S} := V([S]) = \frac{1}{2} Trace([S][\Psi]) = \begin{bmatrix} S1 \\ S2 \\ S3 \\ S4 \end{bmatrix} \in C_4$$

With: $V([S])$ MATRIX VECTORISATION OPERATOR

$[\Psi]$ SET OF ORTHOGONAL 2x2 MATRICES

FROBENIOUS NORM OF $\vec{S}$

$$\|\vec{S}\|^2 = \vec{S}^{T*} \cdot \vec{S} = |S_1|^2 + |S_2|^2 + |S_3|^2 + |S_4|^2$$

$$= Span([S]) = |S_{XX}|^2 + |S_{YX}|^2 + |S_{XY}|^2 + |S_{YY}|^2$$

TARGET VECTORS

PAULI SCATTERING VECTOR $$\underline{k} = V([S]) = \frac{1}{2} Trace([S][\psi_P])$$

SET OF 2x2 COMPLEX MATRICES
FROM THE PAULI MATRICES GROUP

$$[\psi_P] = \left\{ \sqrt{2}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \sqrt{2}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \sqrt{2}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \sqrt{2}\begin{bmatrix} 0 & -j \\ j & 0 \end{bmatrix} \right\}$$

$$\Downarrow$$

$$\boxed{\underline{k} = \frac{1}{\sqrt{2}}\begin{bmatrix} S_{XX} + S_{YY} & S_{XX} - S_{YY} & S_{XY} + S_{YX} & j(S_{XY} - S_{YX}) \end{bmatrix}^T}$$

<u>**Advantage:**</u> **Closer related to physical properties of the scatterer**

<u>**Note:**</u> **Also known as $\underline{k}_{4P}$**

TARGET VECTORS

LEXICOGRAPHIC SCATTERING VECTOR $\underline{\Omega} = V([S]) = \frac{1}{2} Trace([S][\psi_L])$

SET OF 2x2 COMPLEX MATRICES
FROM THE LEXICOGRAPHIC MATRICES GROUP

$$[\psi_L] = \left\{ 2\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, 2\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, 2\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, 2\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right\}$$

$$\Downarrow$$

$$\boxed{\underline{\Omega} = [S_{XX} \quad S_{XY} \quad S_{YX} \quad S_{YY}]^T}$$

<u>Advantage:</u> **Directly related to the system measurables**

<u>Note:</u> **Also known as** $\underline{k}_{4L}$

TARGET VECTORS

SCATTERING VECTOR TRANSFORMATIONS

Pauli Scattering Vector:

$$\underline{k} = \frac{1}{\sqrt{2}} \begin{bmatrix} S_{XX} + S_{YY} \\ S_{XX} - S_{YY} \\ S_{XY} + S_{YX} \\ j\left(S_{XY} - S_{YX}\right) \end{bmatrix}$$

Lexicographic Scattering Vector:

$$\underline{\Omega} = \begin{bmatrix} S_{XX} \\ S_{XY} \\ S_{YX} \\ S_{YY} \end{bmatrix}$$

UNITARY TRANSFORMATION

$$\underline{k} = [D_4]\underline{\Omega} \quad and \quad \underline{\Omega} = [D_4]^{-1}\underline{k} = [D_4]^{T*}\underline{k}$$

WHERE $[D_4]$ IS A SU(4) MATRIX
IN ORDER TO PRESERVE THE NORM
OF THE SCATTERING VECTOR

$$[D_4] = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & -1 \\ 0 & 1 & 1 & 0 \\ 0 & j & -j & 0 \end{bmatrix}$$

TARGET VECTORS

MONOSTATIC CASE

Pauli Scattering Vector:

$$\underline{k} = \frac{1}{\sqrt{2}}\begin{bmatrix} S_{XX}+S_{YY} \\ S_{XX}-S_{YY} \\ S_{XY}+S_{YX} \\ j\left(S_{XY}-S_{YX}\right) \end{bmatrix} \Rightarrow \underline{k} = \frac{1}{\sqrt{2}}\begin{bmatrix} S_{XX}+S_{YY} \\ S_{XX}-S_{YY} \\ 2S_{XY} \end{bmatrix}$$

Note: Also known as $\underline{k}_{3P}$

Lexicographic Scattering Vector:

$$\underline{\Omega} = \begin{bmatrix} S_{XX} \\ S_{XY} \\ S_{YX} \\ S_{YY} \end{bmatrix} \Rightarrow \underline{\Omega} = \begin{bmatrix} S_{XX} \\ \sqrt{2}S_{XY} \\ S_{YY} \end{bmatrix}$$

Note: Also known as $\underline{k}_{3L}$

TARGET VECTORS

SCATTERING VECTOR TRANSFORMATIONS

Pauli Scattering Vector:

$$\underline{k} = \frac{1}{\sqrt{2}}\begin{bmatrix} S_{XX} + S_{YY} \\ S_{XX} - S_{YY} \\ 2S_{XY} \end{bmatrix}$$

Lexicographic Scattering Vector:

$$\underline{\Omega} = \begin{bmatrix} S_{XX} \\ \sqrt{2}S_{XY} \\ S_{YY} \end{bmatrix}$$

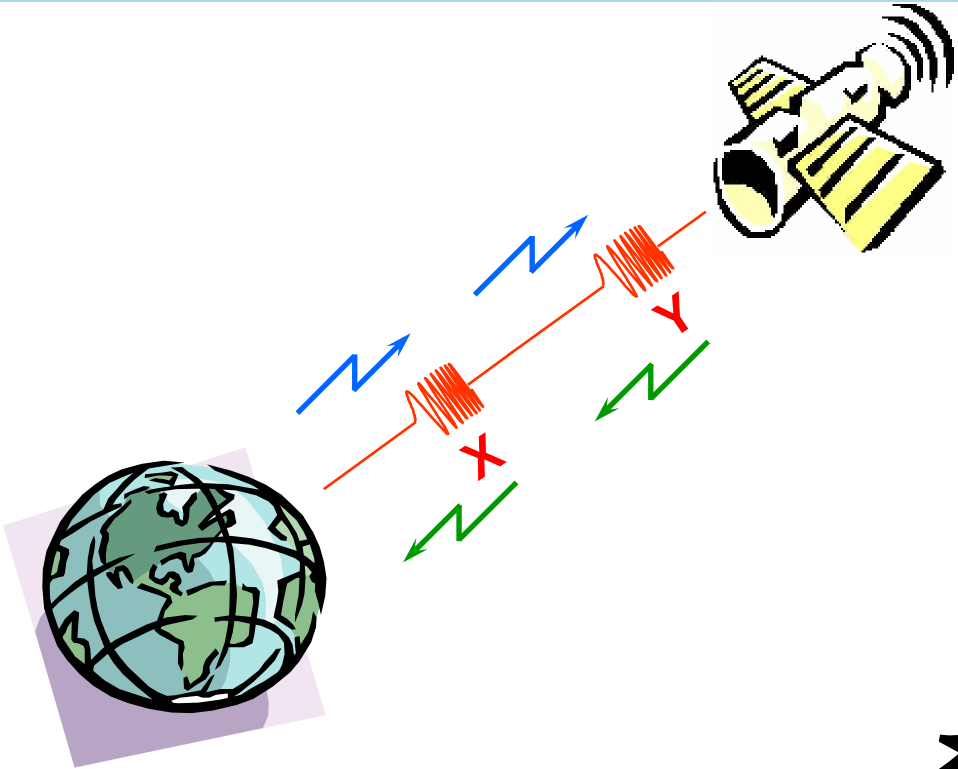
UNITARY TRANSFORMATION

$$\underline{k} = [D_3]\underline{\Omega} \quad \text{and} \quad \underline{\Omega} = [D_3]^{-1}\underline{k} = [D_3]^T\underline{k}$$

WHERE $[D_3]$ IS A SU(3) MATRIX
IN ORDER TO PRESERVE THE NORM
OF THE SCATTERING VECTOR

$$[D_3] = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 0 & 1 \\ 1 & 0 & -1 \\ 0 & \sqrt{2} & 0 \end{bmatrix}$$

POLARIMETRIC DESCRIPTORS



TRANSMITTER: X & Y
RECEIVERS: X & Y

THE DIFFERENT TARGET POLARIMETRIC DESCRIPTORS

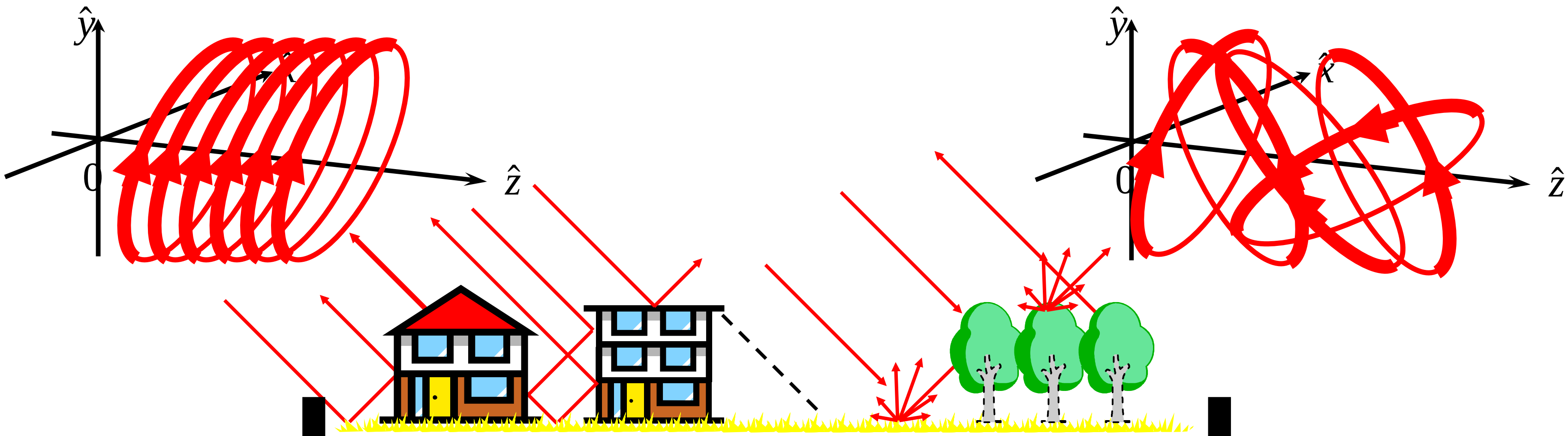
[S]	SINCLAIR Matrix
$\underline{k}$, $\underline{\Omega}$	Target Vectors
[K]	**KENNAUGH Matrix**
[T]	Coherency Matrix
[C]	Covariance Matrix

STATISTICAL DESCRIPTION

PARTIAL SCATTERING POLARIMETRY

PARTIAL SCATTERING POLARIMETRY

DETERMINISTIC SCATTERERS | **PARTIAL SCATTERERS**



DETERMINISTIC SCATTERING

COMPLETELY POLARISED SCATTERING

COMPLETELY DESCRIBED BY $[S]$

RANDOM SCATTERING
(Variation in Time / Space)

PARTIALLY POLARISED SCATTERING

CAN NOT BE DESCRIBED BY $[S]$

→ **STATISTICAL DESCRIPTION**

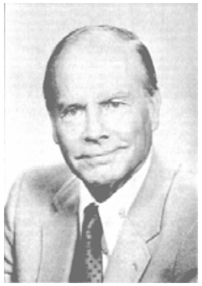
MUELLER MATRIX

BISTATIC CASE

JONES MATRIX : $\underline{E}_s = [S]\underline{E}_i$

MUELLER MATRIX : $\underline{g}_{\underline{E}_s} = [M]\underline{g}_{\underline{E}_i}$

$$[M] = \frac{1}{2}\left([V]^T\left[[S]\otimes[S]^*\right][V]\right) \qquad [V] = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -j \\ 0 & 0 & 1 & +j \\ 1 & -1 & 0 & 0 \end{bmatrix}$$



HUYNEN PARAMETERS

$$[M] = \begin{bmatrix} A_0 + B_0 + A & C + I & H + N & F + L \\ C - I & A_0 + B - A & E + K & G + M \\ H - N & E - K & A_0 - B - A & D + J \\ F - L & G - M & D - J & -A_0 + B_0 - A \end{bmatrix}$$

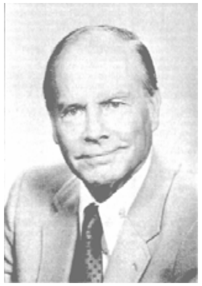
KENNAUGH MATRIX

MONOSTATIC CASE

SINCLAIR MATRIX : $\underline{E}_s = [S]\underline{E}_i$

KENNAUGH MATRIX : $\underline{g}_{\underline{E}_s} = [K]\underline{g}_{\underline{E}_i}$

$$[K] = \frac{1}{2}\left([V]^T\left[[S]\otimes[S]^*\right][V]\right) \qquad [V] = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -j \\ 0 & 0 & 1 & +j \\ 1 & -1 & 0 & 0 \end{bmatrix}$$



HUYNEN PARAMETERS

$$[K] = \begin{bmatrix} A_0 + B_0 & C & H & F \\ C & A_0 + B & E & G \\ H & E & A_0 - B & D \\ F & G & D & -A_0 + B_0 \end{bmatrix}$$

HUYNEN PARAMETERS

PHYSICAL INTERPRETATION

MAN-MADE TARGET DECOMPOSITION
IDENTIFICATION and ANALYSIS

« *PHENOMENOLOGICAL THEORY OF RADAR TARGETS* » (1970)

A0 : GENERATOR OF TARGET SYMMETRY

B0+B : GENERATOR OF TARGET NON-SYMMETRY

B0-B : GENERATOR OF TARGET IRREGULARITY

C : GENERATOR OF TARGET GLOBAL SHAPE (LINEAR)

D : GENERATOR OF TARGET LOCAL SHAPE (CURVATURE)

E : GENERATOR OF TARGET LOCAL TWIST (TORSION)

F : GENERATOR OF TARGET GLOBAL TWIST (HELICITY)

G : GENERATOR OF TARGET LOCAL COUPLING (GLUE)

H : GENERATOR OF TARGET GLOBAL COUPLING (ORIENTATION)

STOKES VECTOR

JONES VECTOR

$$\underline{E} = A\begin{bmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{bmatrix}\begin{bmatrix} \cos(\tau) & j\sin(\tau) \\ j\sin(\tau) & \cos(\tau) \end{bmatrix}\begin{bmatrix} e^{-j\alpha} & 0 \\ 0 & e^{j\alpha} \end{bmatrix}\hat{u}_x$$

$[U_2(\phi)]$ $[U_2(\tau)]$ $[U_2(\alpha)]$

HOMOMORPHISM SU(2) - O(3)

$$[O_3(2\theta)]_{p,q} = \frac{1}{2} Tr\left([U_2(\theta)]^{T*}\sigma_p[U_2(\theta)]\sigma_q\right)$$

(σ_p, σ_q) : Pauli Matrices

STOKES VECTOR

$$\underline{g}_{\underline{E}} = A^2\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(2\phi) & -\sin(2\phi) & 0 \\ 0 & \sin(2\phi) & \cos(2\phi) & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(2\tau) & 0 & -\sin(2\tau) \\ 0 & 0 & 1 & 0 \\ 0 & \sin(2\tau) & 0 & \cos(2\tau) \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos(2\alpha) & -\sin(2\alpha) \\ 0 & 0 & \sin(2\alpha) & \cos(2\alpha) \end{bmatrix}\underline{g}_{\hat{u}_x}$$

$[O_4(2\phi)]$ $[O_4(2\tau)]$ $[O_4(2\alpha)]$

ELLIPTICAL BASIS TRANSFORMATION

SINCLAIR MATRIX

$$\underline{E}^{s}_{(A,A_\perp)} = \left[S_{(A,A_\perp)}\right]\underline{E}^{i}_{(A,A_\perp)} \qquad \underline{E}^{s}_{(B,B_\perp)} = \left[S_{(B,B_\perp)}\right]\underline{E}^{i}_{(B,B_\perp)}$$

$$\left[S_{(B,B_\perp)}\right] = \left[U_{(A,A_\perp)\mapsto(B,B_\perp)}\right]^{T}\left[S_{(A,A_\perp)}\right]\left[U_{(A,A_\perp)\mapsto(B,B_\perp)}\right]$$

CON-SIMILARITY TRANSFORMATION

KENNAUGH MATRIX

$$\underline{g}_{\underline{E}^{s}_{(A,A_\perp)}} = \left[K_{(A,A_\perp)}\right]\underline{g}_{\underline{E}^{i}_{(A,A_\perp)}} \qquad \underline{g}_{\underline{E}^{s}_{(B,B_\perp)}} = \left[K_{(B,B_\perp)}\right]\underline{g}_{\underline{E}^{i}_{(B,B_\perp)}}$$

$$\left[K_{(B,B_\perp)}\right] = \left[O_{4(A,A_\perp)\mapsto(B,B_\perp)}\right]\left[K_{(A,A_\perp)}\right]\left[O_{4(A,A_\perp)\mapsto(B,B_\perp)}\right]^{-1}$$

SIMILARITY TRANSFORMATION

$$\left[O_{4(A,A_\perp)\mapsto(B,B_\perp)}\right]$$

O(4) SPECIAL UNITARY ELLIPTICAL BASIS TRANSFORMATION MATRIX

ELLIPTICAL BASIS TRANSFORMATION

SPECIAL UNITARY SU(2) GROUP

$$[U_2] = \begin{bmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{bmatrix} \begin{bmatrix} \cos(\tau) & j\sin(\tau) \\ j\sin(\tau) & \cos(\tau) \end{bmatrix} \begin{bmatrix} e^{-j\alpha} & 0 \\ 0 & e^{j\alpha} \end{bmatrix}$$

$[U_2(\phi)]$ $[U_2(\tau)]$ $[U_2(\alpha)]$

HOMOMORPHISM SU(2) - O(3)

$$[O_3(2\theta)]_{p,q} = \frac{1}{2} Tr\left([U_2(\theta)]^{T*} \sigma_p [U_2(\theta)] \sigma_q\right)$$

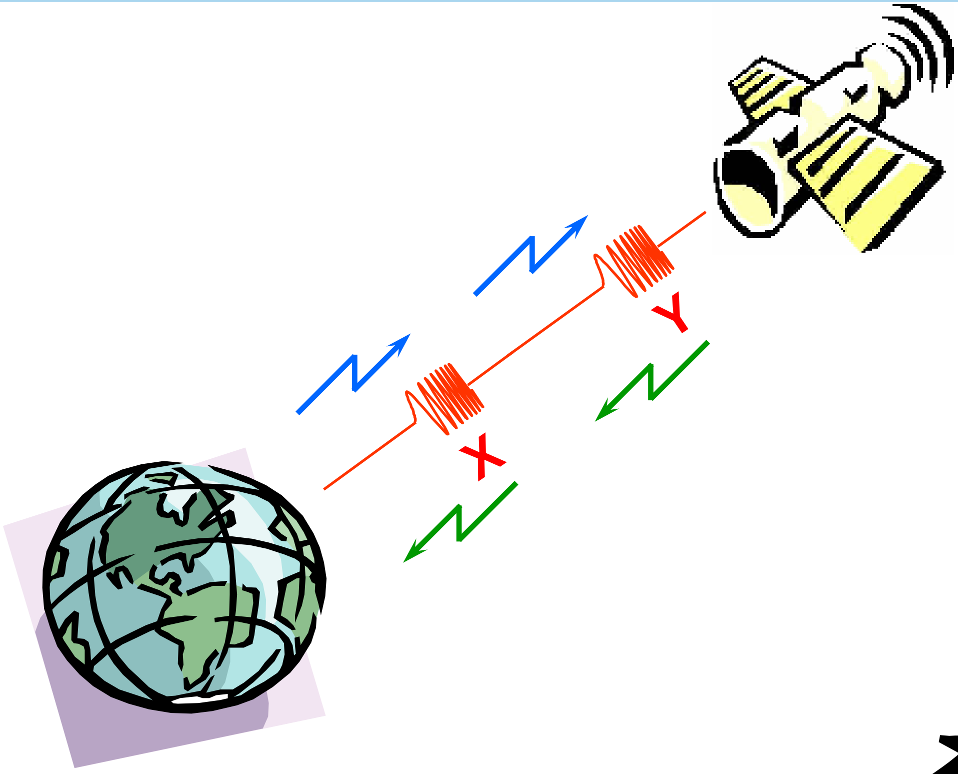
(σ_p, σ_q) : Pauli Matrices

O(4) UNITARY GROUP

$$[O_4] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(2\phi) & -\sin(2\phi) & 0 \\ 0 & \sin(2\phi) & \cos(2\phi) & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(2\tau) & 0 & -\sin(2\tau) \\ 0 & 0 & 1 & 0 \\ 0 & \sin(2\tau) & 0 & \cos(2\tau) \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos(2\alpha) & -\sin(2\alpha) \\ 0 & 0 & \sin(2\alpha) & \cos(2\alpha) \end{bmatrix}$$

$[O_4(2\phi)]$ $[O_4(2\tau)]$ $[O_4(2\alpha)]$

POLARIMETRIC DESCRIPTORS



TRANSMITTER: X & Y
RECEIVERS: X & Y

THE DIFFERENT TARGET POLARIMETRIC DESCRIPTORS

[S]	SINCLAIR Matrix
$\underline{k}$, $\underline{\Omega}$	Target Vectors
[K]	KENNAUGH Matrix
[T]	**Coherency Matrix**
[C]	Covariance Matrix

STATISTICAL DESCRIPTION

PARTIAL SCATTERING POLARIMETRY

COHERENCY MATRIX

BISTATIC CASE

PAULI SCATTERING VECTOR $\underline{k}$

$$\underline{k} = \frac{1}{\sqrt{2}}\left[S_{XX} + S_{YY} \quad S_{XX} - S_{YY} \quad S_{XY} + S_{YX} \quad j(S_{XY} - S_{YX})\right]^T$$

$$\Downarrow$$

COHERENCY MATRIX $[T]$

$$[T] = \underline{k} \cdot \underline{k}^{*T} = \begin{bmatrix} 2A_0 & C - jD & H + jG & L - jK \\ C + jD & B_0 + B & E + jF & M - jN \\ H - jG & E - jF & B_0 - B & J + jI \\ L + jK & M + jN & J - jI & 2A \end{bmatrix}$$

HERMITIAN POSITIVE SEMI DEFINITE MATRIX - RANK 1

COHERENCY MATRIX

MONOSTATIC CASE

PAULI SCATTERING VECTOR $\underline{k}$

$$\underline{k} = \frac{1}{\sqrt{2}}\left[S_{XX}+S_{YY} \quad S_{XX}-S_{YY} \quad 2S_{XY}\right]^T$$

$$\Downarrow$$

COHERENCY MATRIX $[T]$

$$[T] = \underline{k}\cdot\underline{k}^{*T} = \begin{bmatrix} 2A_0 & C-jD & H+jG \\ C+jD & B_0+B & E+jF \\ H-jG & E-jF & B_0-B \end{bmatrix}$$

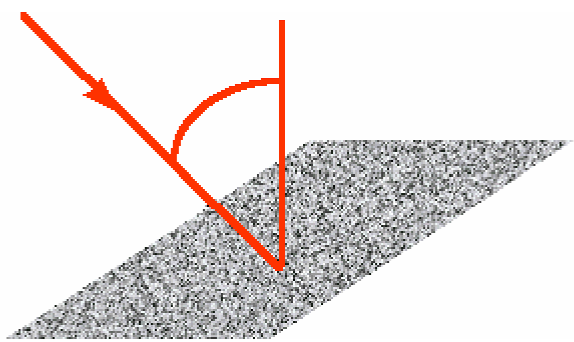
HERMITIAN MATRIX - RANK 1

A0, B0+B, B0-B : HUYNEN TARGET GENERATORS

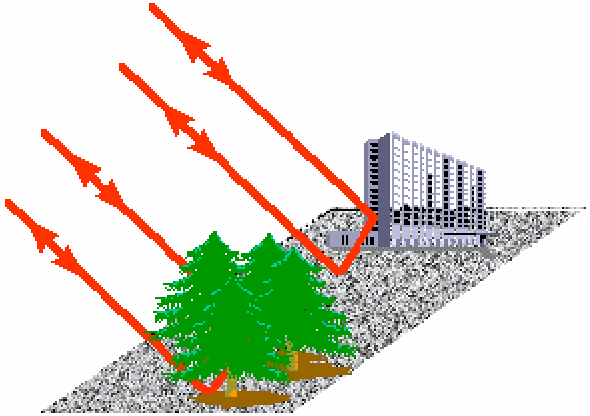
TARGET GENERATORS

PHYSICAL INTERPRETATION

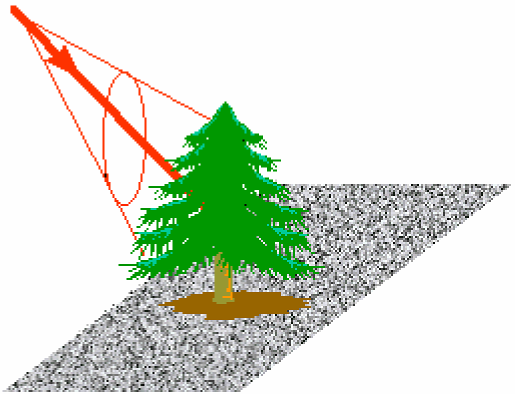
SINGLE BOUNCE SCATTERING (ROUGH SURFACE)



DOUBLE BOUNCE SCATTERING



VOLUME SCATTERING

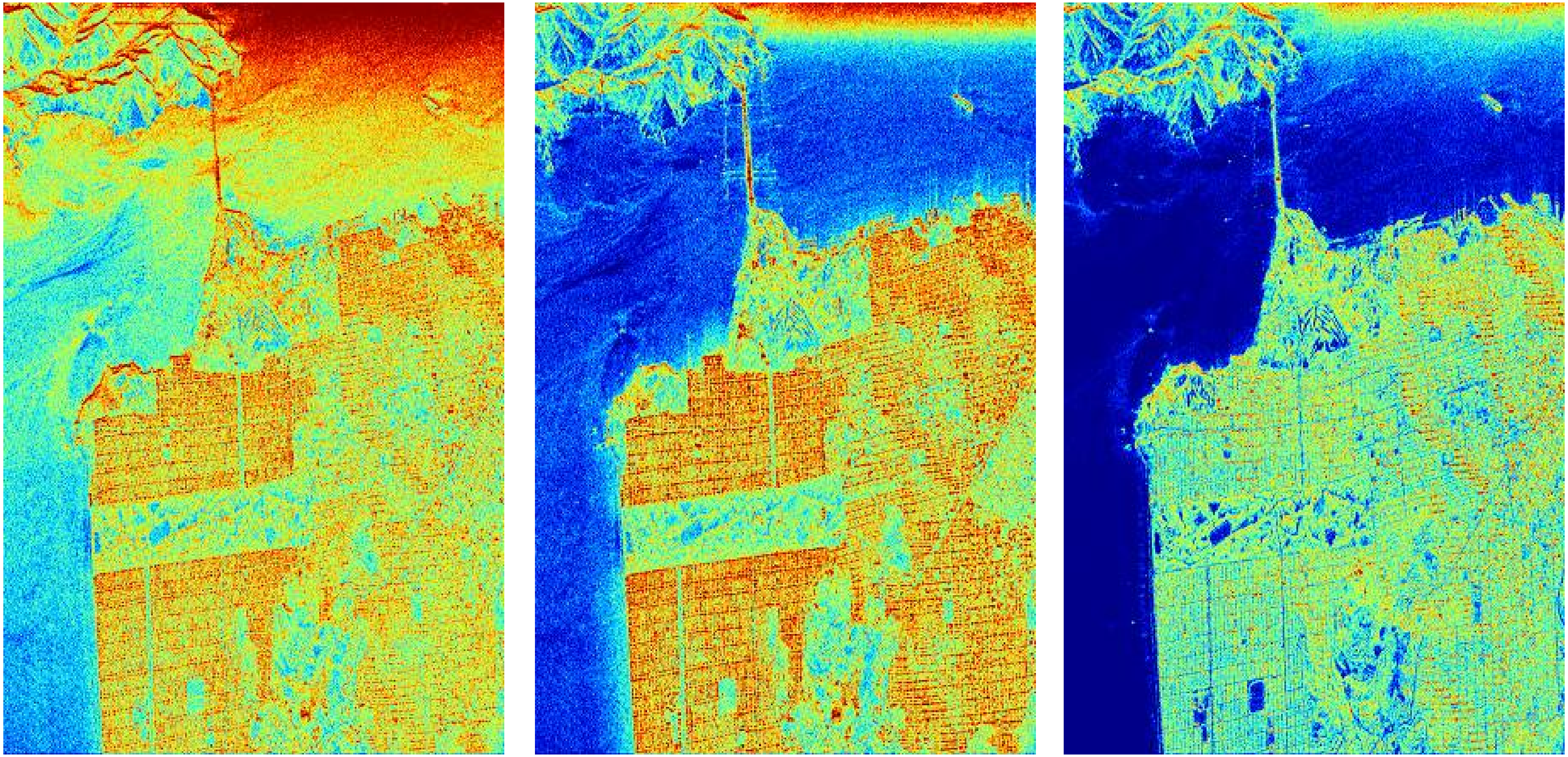


$$T_{11} = 2A_0 = \left|S_{XX} + S_{YY}\right|^2$$

$$T_{33} = B_0 - B = 2\left|S_{XY}\right|^2$$

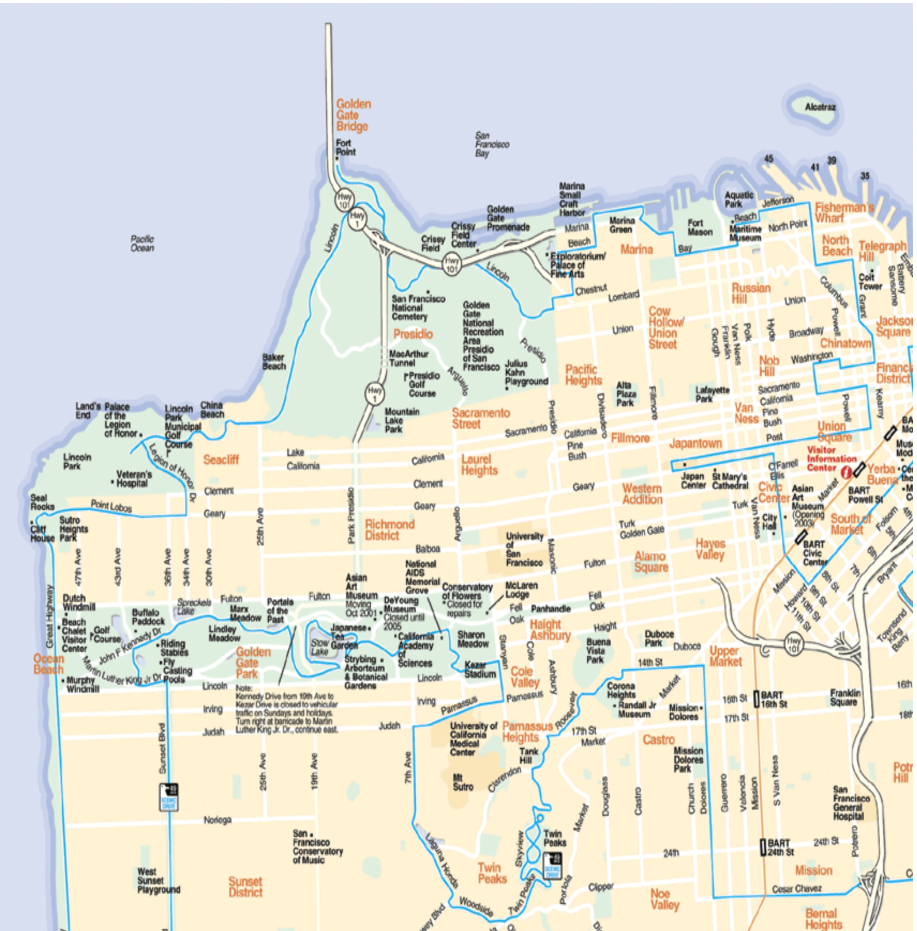
$$T_{22} = B_0 + B = \left|S_{XX} - S_{YY}\right|^2$$

TARGET GENERATORS



$\left(2A_0\right)_{dB}$ $\left(B_0 + B\right)_{dB}$ $\left(B_0 - B\right)_{dB}$

TARGET GENERATORS



(H,V) POLARISATION BASIS

|HH+VV| |HV | |HH-VV|

ELLIPTICAL BASIS TRANSFORMATION

SINCLAIR MATRIX

$$\underline{E}^{s}_{(A,A_\perp)} = \left[S_{(A,A_\perp)}\right]\underline{E}^{i}_{(A,A_\perp)}$$

$$\underline{E}^{s}_{(B,B_\perp)} = \left[S_{(B,B_\perp)}\right]\underline{E}^{i}_{(B,B_\perp)}$$

$$\left[S_{(B,B_\perp)}\right] = \left[U_{(A,A_\perp)\mapsto(B,B_\perp)}\right]^T\left[S_{(A,A_\perp)}\right]\left[U_{(A,A_\perp)\mapsto(B,B_\perp)}\right]$$

CON-SIMILARITY TRANSFORMATION

COHERENCY MATRIX

$$\left[T_{(B,B_\perp)}\right] = \left[U_{3(A,A_\perp)\mapsto(B,B_\perp)}\right]\left[T_{(A,A_\perp)}\right]\left[U_{3(A,A_\perp)\mapsto(B,B_\perp)}\right]^{-1}$$

SIMILARITY TRANSFORMATION

$$\left[U_{3(A,A_\perp)\mapsto(B,B_\perp)}\right]$$

U(3) SPECIAL UNITARY ELLIPTICAL BASIS TRANSFORMATION MATRIX

ELLIPTICAL BASIS TRANSFORMATION

SPECIAL UNITARY SU(2) GROUP

$$[U_2] = \begin{bmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{bmatrix} \begin{bmatrix} \cos(\tau) & j\sin(\tau) \\ j\sin(\tau) & \cos(\tau) \end{bmatrix} \begin{bmatrix} e^{-j\alpha} & 0 \\ 0 & e^{j\alpha} \end{bmatrix}$$

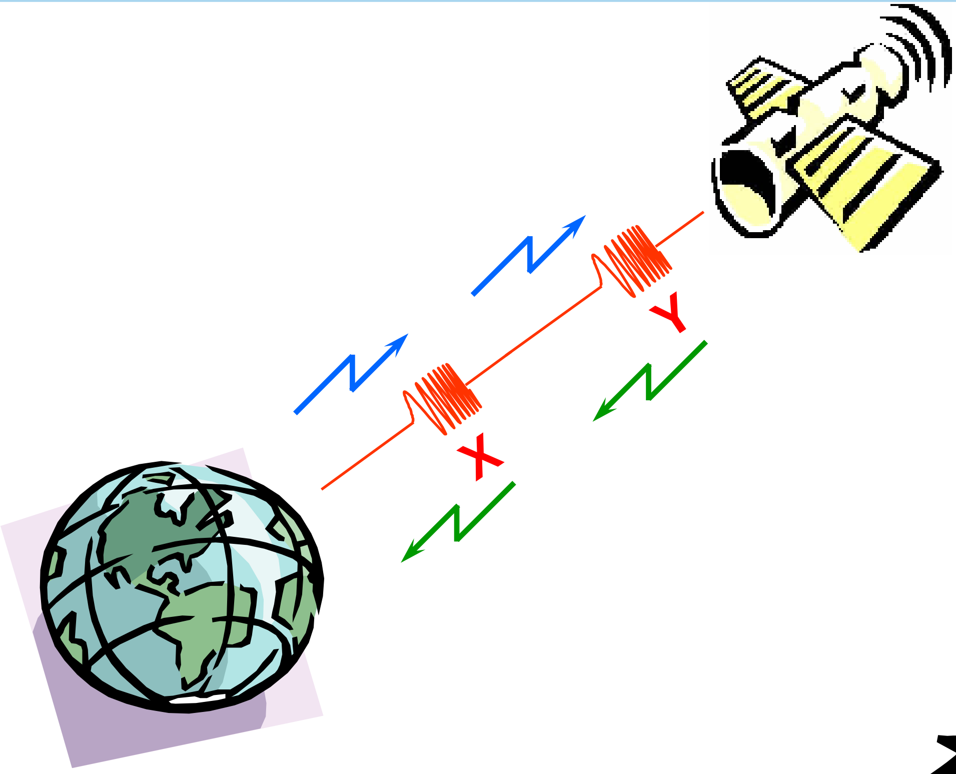
$$[U_2(\phi)] \qquad [U_2(\tau)] \qquad [U_2(\alpha)]$$

SPECIAL UNITARY SU(3) GROUP

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(2\phi) & \sin(2\phi) \\ 0 & -\sin(2\phi) & \cos(2\phi) \end{bmatrix} \begin{bmatrix} \cos(2\tau) & 0 & j\sin(2\tau) \\ 0 & 1 & 0 \\ j\sin(2\tau) & 0 & \cos(2\tau) \end{bmatrix} \begin{bmatrix} \cos(2\alpha) & -j\sin(2\alpha) & 0 \\ -j\sin(2\alpha) & \cos(2\alpha) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[U_3(2\phi)] \qquad [U_3(2\tau)] \qquad [U_3(2\alpha)]$$

POLARIMETRIC DESCRIPTORS



TRANSMITTER: X & Y
RECEIVERS: X & Y

THE DIFFERENT TARGET POLARIMETRIC DESCRIPTORS

[S] SINCLAIR Matrix
k, Ω Target Vectors
[K] KENNAUGH Matrix
[T] Coherency Matrix
[C] Covariance Matrix

STATISTICAL DESCRIPTION

PARTIAL SCATTERING POLARIMETRY

E. Pottier, L. Ferro-Famil

COVARIANCE MATRIX

BISTATIC CASE

LEXICOGRAPHIC SCATTERING VECTOR $\underline{\Omega}$

$$\underline{\Omega} = \begin{bmatrix} S_{XX} & S_{XY} & S_{YX} & S_{YY} \end{bmatrix}^T$$

$$\Downarrow$$

COVARIANCE MATRIX $[C]$

$$[C] = \underline{\Omega} \cdot \underline{\Omega}^{*T} = \begin{bmatrix} S_{XX}S_{XX}^* & S_{XX}S_{XY}^* & S_{XX}S_{YX}^* & S_{XX}S_{YY}^* \\ S_{XY}S_{XX}^* & S_{XY}S_{XY}^* & S_{XY}S_{YX}^* & S_{XY}S_{YY}^* \\ S_{YX}S_{XX}^* & S_{YX}S_{XY}^* & S_{YX}S_{YX}^* & S_{YX}S_{YY}^* \\ S_{YY}S_{XX}^* & S_{YY}S_{XY}^* & S_{YY}S_{YX}^* & S_{YY}S_{YY}^* \end{bmatrix}$$

HERMITIAN POSITIVE SEMI DEFINITE MATRIX - RANK 1

COVARIANCE MATRIX

MONOSTATIC CASE

LEXICOGRAPHIC SCATTERING VECTOR $\underline{\Omega}$

$$\underline{\Omega} = \begin{bmatrix} S_{XX} & \sqrt{2}S_{XY} & S_{YY} \end{bmatrix}^T$$

COVARIANCE MATRIX $[C]$

$$[C] = \underline{\Omega} \cdot \underline{\Omega}^{*T} = \begin{bmatrix} S_{XX}S_{XX}^* & \sqrt{2}S_{XX}S_{XY}^* & S_{XX}S_{YY}^* \\ \sqrt{2}S_{XY}S_{XX}^* & 2S_{XY}S_{XY}^* & \sqrt{2}S_{XY}S_{YY}^* \\ S_{YY}S_{XX}^* & \sqrt{2}S_{YY}S_{XY}^* & S_{YY}S_{YY}^* \end{bmatrix}$$

HERMITIAN POSITIVE SEMI DEFINITE MATRIX - RANK 1

COVARIANCE-COHERENCY MATRICES

COHERENCY MATRIX

$$[T] = \underline{k} \cdot \underline{k}^{*T}$$

$$\boxed{\underline{k} = [D_{3 or 4}]\underline{\Omega}}$$

COVARIANCE MATRIX

$$[C] = \underline{\Omega} \cdot \underline{\Omega}^{*T}$$

UNITARY TRANSFORMATION

$$[T] = [D_{3 or 4}][C][D_{3 or 4}]^{T*}$$

$[T]$ and $[C]$ **HAVE THE SAME EIGENVALUES**

Both contain the same information about Polarimetric Scattering Amplitudes, Phase Angles and Correlations

$[T]$ is closer related to Physical and Geometrical Properties of the Scattering Process, and thus allows a better and direct physical interpretation

$[C]$ is directly related to the system measurables

$[T]$ is directly related to the Kennaugh matrix and the Huynen parameters

POLARIMETRIC DESCRIPTORS

SINCLAIR MATRIX

$$[S] = \begin{bmatrix} S_{XX} & S_{XY} \\ S_{YX} & S_{YY} \end{bmatrix}$$

KENNAUGH MATRIX

$$[K] = \frac{1}{2}\left([V]^T \left[[S] \otimes [S]^*\right][V]\right)$$

EQUIVALENCE ?

SCATTERING VECTOR $\underline{k}$

$$\underline{k} = \frac{1}{\sqrt{2}}\left[S_{XX} + S_{YY} \quad S_{XX} - S_{YY} \quad 2S_{XY}\right]^T$$

$\Downarrow$

COHERENCY MATRIX $[T]$

$$[T] = \underline{k} \cdot \underline{k}^{*T}$$

SCATTERING VECTOR $\underline{\Omega}$

$$\underline{\Omega} = \left[S_{XX} \quad \sqrt{2}S_{XY} \quad S_{YY}\right]^T$$

$\Downarrow$

COVARIANCE MATRIX $[C]$

$$[C] = \underline{\Omega}\,\underline{\Omega}^{T*}$$

POLARIMETRIC DESCRIPTORS

$$[\,S'\,] = [\,U_2\,]^T [\,S\,]\,[\,U_2\,]$$

$[\,S\,]$
SINCLAIR
SU(2)

$$[\,C'\,] = [\,U_3\,]\,[\,C\,]\,[\,U_3\,]^{-1}$$

$[\,C\,]$
COVARIANCE
SU(3)

$[\,T\,]$
COHERENCY
SU(3)

$[\,K\,]$
KENNAUGH
O(4)

$$[\,T'\,] = [\,U_3\,]\,[\,T\,]\,[\,U_3\,]^{-1}$$

$$[\,K'\,] = [\,O_4\,]\,[\,K\,]\,[\,O_4\,]^{-1}$$

POLARIMETRIC DESCRIPTORS

SPECIAL UNITARY SU(2) GROUP

$$\underbrace{\begin{bmatrix} cos(\phi) & -sin(\phi) \\ sin(\phi) & cos(\phi) \end{bmatrix}}_{[U_2(\phi)]} \underbrace{\begin{bmatrix} cos(\tau) & j\,sin(\tau) \\ j\,sin(\tau) & cos(\tau) \end{bmatrix}}_{[U_2(\tau)]} \underbrace{\begin{bmatrix} e^{-j\alpha} & 0 \\ 0 & e^{j\alpha} \end{bmatrix}}_{[U_2(\alpha)]}$$

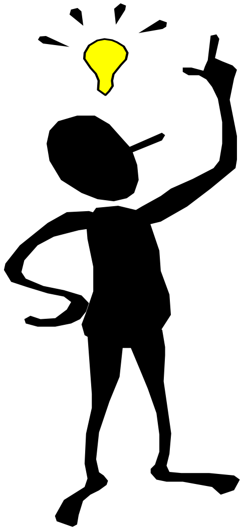
SPECIAL UNITARY SU(3) GROUP (T Matrix)

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & cos(2\phi) & sin(2\phi) \\ 0 & -sin(2\phi) & cos(2\phi) \end{bmatrix}}_{[U_3(2\phi)]} \underbrace{\begin{bmatrix} cos(2\tau) & 0 & j\,sin(2\tau) \\ 0 & 1 & 0 \\ j\,sin(2\tau) & 0 & cos(2\tau) \end{bmatrix}}_{[U_3(2\tau)]} \underbrace{\begin{bmatrix} cos(2\alpha) & -j\,sin(2\alpha) & 0 \\ -j\,sin(2\alpha) & cos(2\alpha) & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{[U_3(2\alpha)]}$$

O(4) UNITARY GROUP

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & cos(2\phi) & -sin(2\phi) & 0 \\ 0 & sin(2\phi) & cos(2\phi) & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}}_{[O_4(2\phi)]} \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & cos(2\tau) & 0 & -sin(2\tau) \\ 0 & 0 & 1 & 0 \\ 0 & sin(2\tau) & 0 & cos(2\tau) \end{bmatrix}}_{[O_4(2\tau)]} \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & cos(2\alpha) & -sin(2\alpha) \\ 0 & 0 & sin(2\alpha) & cos(2\alpha) \end{bmatrix}}_{[O_4(2\alpha)]}$$

TARGET EQUATIONS



POLARIMETRIC GOLDEN NUMBER

POLARIMETRIC TARGET DIMENSION

TARGET EQUATIONS

$$[S] = \begin{bmatrix} S_{XX} & S_{XY} \\ S_{YX} & S_{YY} \end{bmatrix}$$

5 DEGREES OF FREEDOM

$$|S_{XX}|, |S_{XY}|, |S_{YY}|$$

$$\phi_{XY-XX}, \phi_{YY-XX}$$

TARGET MONOSTATIC POLARIMETRIC « *DIMENSION* »

=

5

KENNAUGH MATRIX $[K]$

COHERENCY MATRIX $[T]$

9 HUYNEN REAL PARAMETERS

(*A0, B0, B, C, D, E, F, G, H*)

COVARIANCE MATRIX $[C]$

9 REAL PARAMETERS

|*XX*|, |XY|, |*YY*|,

Re(*XXXY**), Im(*XXXY**)

Re(*XXYY**), Im(*XXYY**)

Re(*XYYY**), Im(*XYYY**)

9 - 5 = 4 TARGET EQUATIONS

TARGET EQUATIONS

PURE TARGET – MONOSTATIC CASE

$$[T] = \underline{k} \cdot \underline{k}^{*T} = \begin{bmatrix} 2A_0 & C - jD & H + jG \\ C + jD & B_0 + B & E + jF \\ H - jG & E - jF & B_0 - B \end{bmatrix}$$

3x3 HERMITIAN MATRIX - RANK 1

↓

9 PRINCIPAL MINORS = 0

$$2A_0(B_0 + B) - C^2 - D^2 = 0 \qquad 2A_0(B_0 - B) - G^2 - H^2 = 0$$

$$-2A_0 E + CH - DG = 0 \qquad B_0^2 - B^2 - E^2 - F^2 = 0$$

$$C(B_0 - B) - EH - GF = 0 \qquad -D(B_0 - B) + FH - GE = 0$$

$$2A_0 F - CG - DH = 0 \qquad -G(B_0 + B) + FC - ED = 0$$

$$H(B_0 + B) - CE - DF = 0$$

TARGET EQUATIONS

$$[S]=\begin{bmatrix} S_{XX} & S_{XY} \\ S_{YX} & S_{YY} \end{bmatrix}$$

5 DEGREES OF FREEDOM

$$|S_{XX}|,|S_{XY}|,|S_{YY}|$$

$$\phi_{XY-XX},\phi_{YY-XX}$$

COHERENCY MATRIX $[T]$

9 HUYNEN REAL PARAMETERS

(*A0, B0, B, C, D, E, F, G, H*)

TARGET MONOSTATIC POLARIMETRIC « *DIMENSION* »

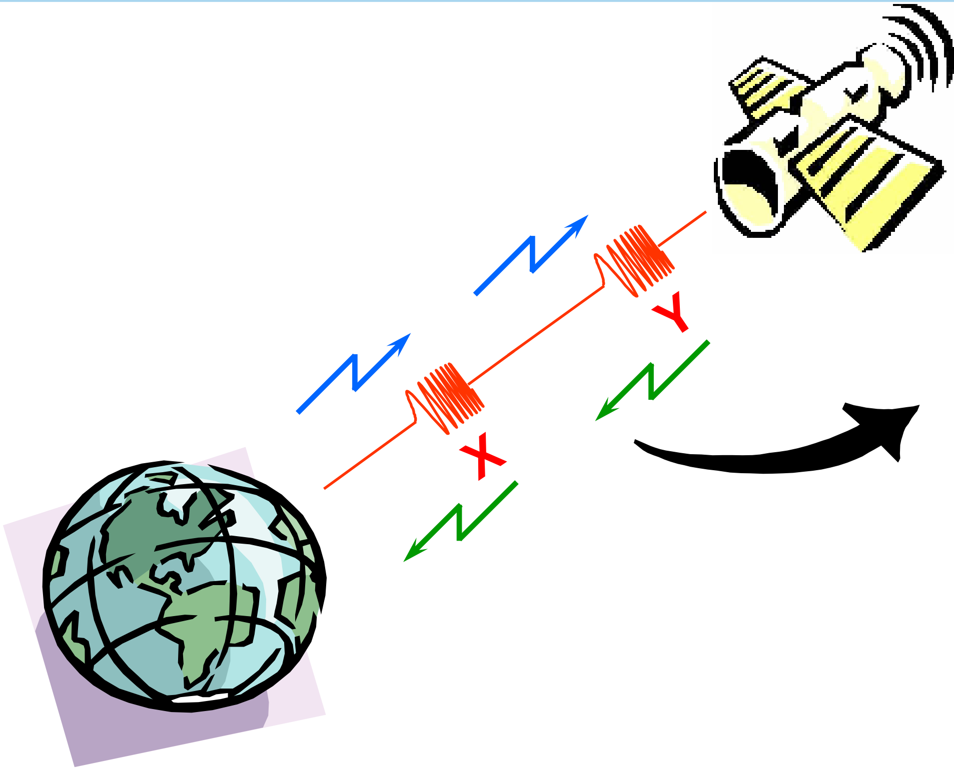
=

5

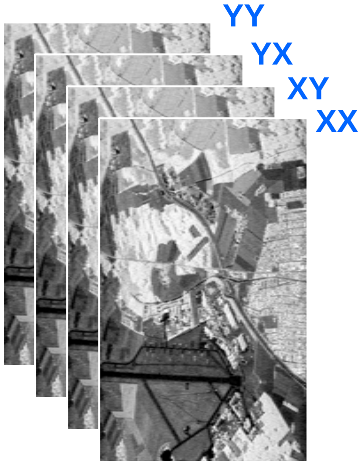
9 - 5 = 4 TARGET EQUATIONS

$$\begin{aligned} 2A_0(B_0+B) &= C^2+D^2 \\ 2A_0(B_0-B) &= G^2+H^2 \\ 2A_0E &= CH-DG \\ 2A_0F &= CG+DH \end{aligned}$$

SCATTERING POLARIMETRY



SINCLAIR MATRIX $[S]$



THE DIFFERENT TARGET POLARIMETRIC DESCRIPTORS

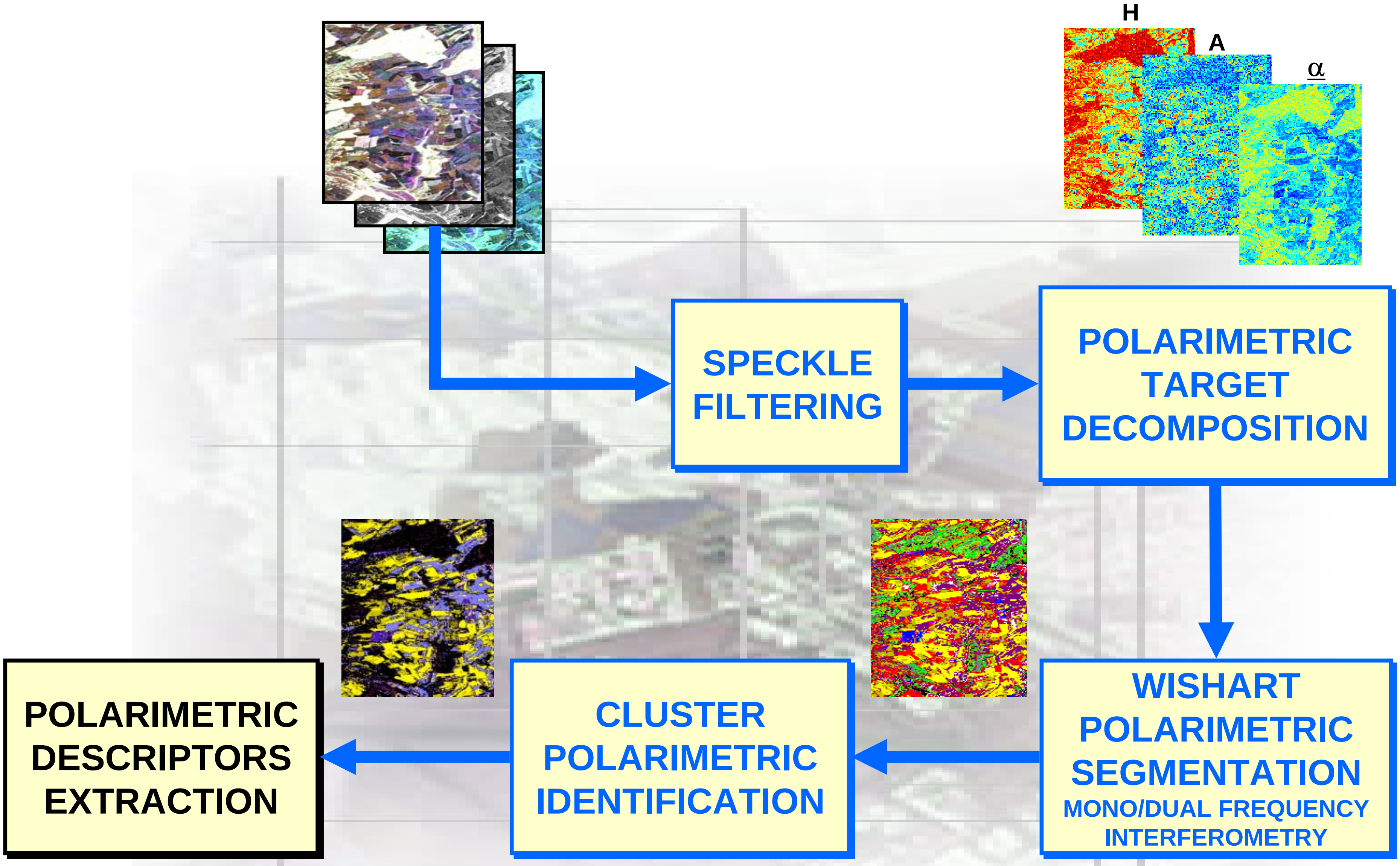
[S]	SINCLAIR Matrix
$\underline{k}$, $\underline{\Omega}$	Target Vectors
[K]	KENNAUGH Matrix
[T]	Coherency Matrix
[C]	Covariance Matrix

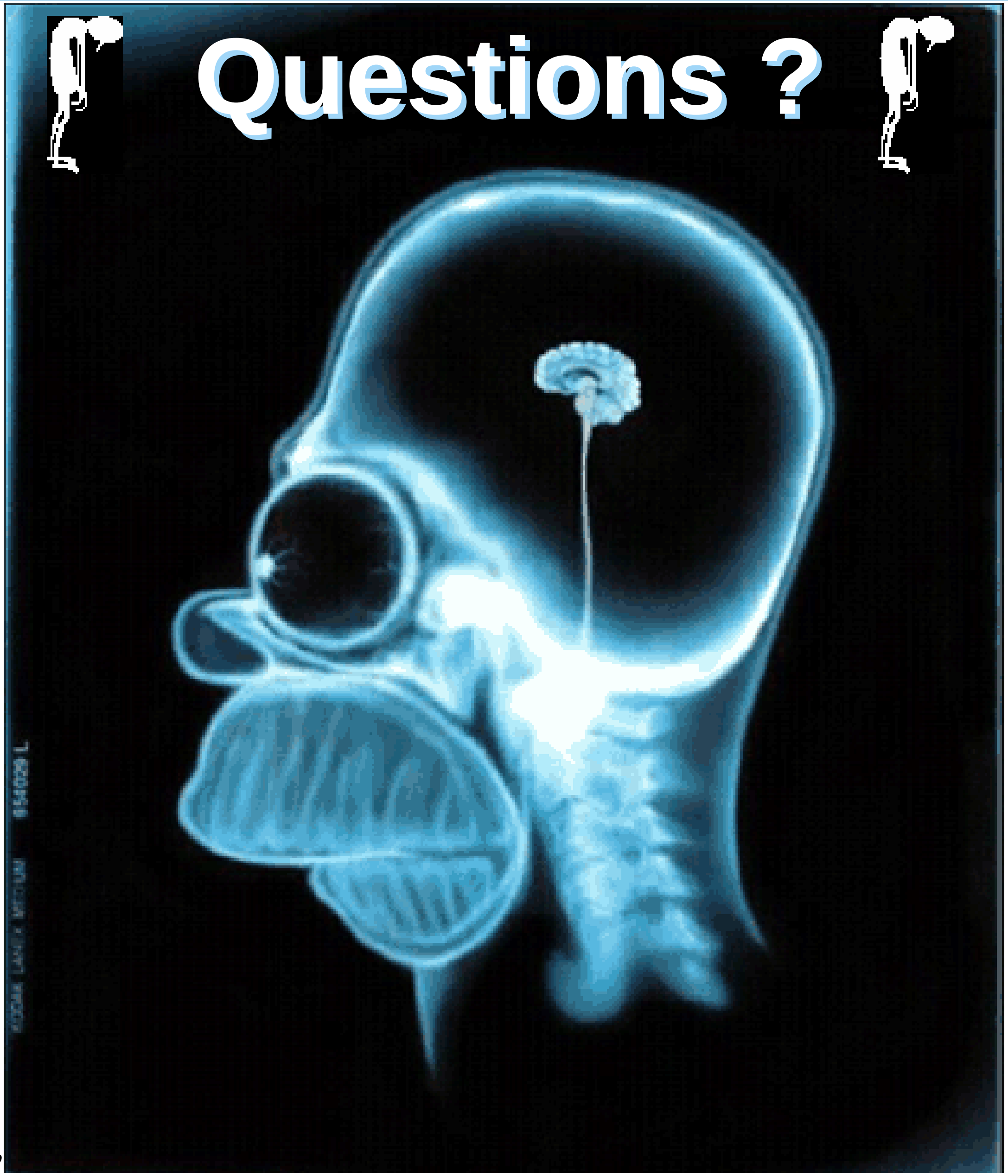
TRANSMITTER: X & Y

RECEIVERS: X & Y

POLARIMETRIC REMOTE SENSING

QUALITATIVE ANALYSIS




Questions ?