Models and Information
ation in Multichannel Radar Remote Sensing Imagery

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IGARSS 2007 Tutorial

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Summary

- Introduction
- One Dimensional SAR Systems
  - Synthetic Aperture Radar Principles
  - Scatterer Models
  - Wave Scattering Models. Interaction with Matter
  - SAR Data Models and Speckle Noise
- Multidimensional SAR Systems
- Multidimensional SAR Data Models
- Multidimensional SAR Speckle Noise Models
  - Coherence Modelling and Estimation
  - Polarimetric Information Estimation
  - Multidimensional SAR Data Estimation
  - PolInSAR Data Estimation

Part 1: One Dimensional SAR Systems and its sub-items

Part 2: Multidimensional SAR Systems through PolInSAR Data Estimation

Introduction

Synthetic Aperture Radar

- Why to use Synthetic Aperture Radar to perform remote sensing?
  - Active system providing its own illumination source. Day/night imaging capability (x2)
  - Imaging capability independent of weather conditions (~ x5)
  - High spatial resolution
  - Sensitive to a wide range of Earth surface properties, especially in the case of multichannel or multidimensional SAR systems
    - Interferometry, differential interferometry, polarimetry, polarimetric interferometry, multifequency, multitime, etc…
- SAR technology has been considered in different applications
    - Topography, agriculture, forestry, hydrology, oceanography, glaciology, environment monitoring, MTI, etc…
- Complementary to optical remote sensing systems

Synthetic Aperture Radar

ERS 2 (Composite Image)

?

Band 1: 5.66 cm

Landsat Thematic Mapper (TM)

?

Band 1: 0.45 - 0.52 µm
Band 2: 0.52 - 0.60 µm
Band 3: 0.63 - 0.69 µm
Band 4: 0.76 - 0.90 µm
Band 5: 1.55 - 1.75 µm
Band 6: 10.40 - 12.50 µm
Band 7: 2.08 - 2.35 µm

Barcelona (Spain)

Introduction

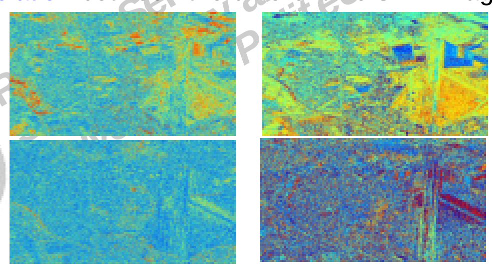
Synthetic Aperture Radar

Multidimensional SAR systems exploit diversity to increase the amount of information

- Multiple channels of information, i.e., PolSAR images

$|S_{hh}|$ $|S_{hv}|$ $|S_{vv}|$

- Correlation between the different PolSAR images



$|r_{hhvv}|$ φ_{hhvv}

$|r_{hhhv}|$ φ_{hhhv}

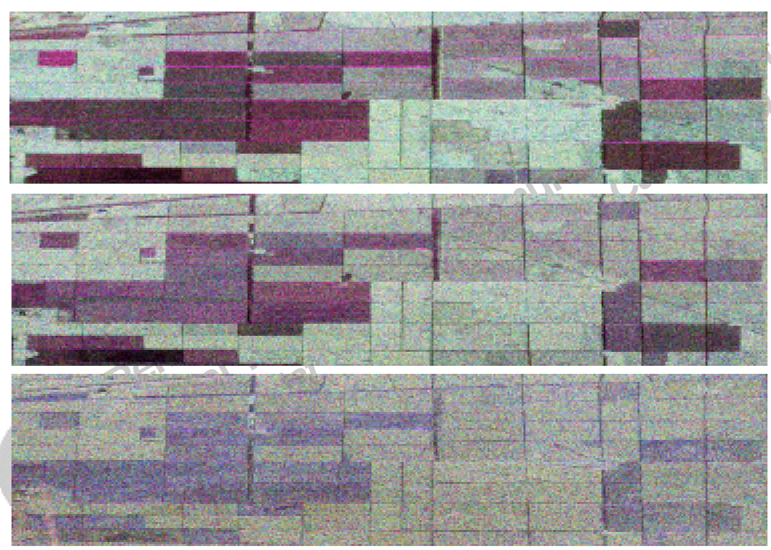
$$r_{k,l} = \frac{E\{S_k \cdot S_l^*\}}{\sqrt{E\{|S_k|^2\} \cdot E\{|S_l|^2\}}} = |r_{k,l}| e^{j\varphi_{k,l}}$$

L-band (1.3 GHz) fully PolSAR data
E-SAR system. Oberpfaffenhofen test area (D)

Introduction

Synthetic Aperture Radar

Multifrequency configurations sensitive to different properties of the scatterers



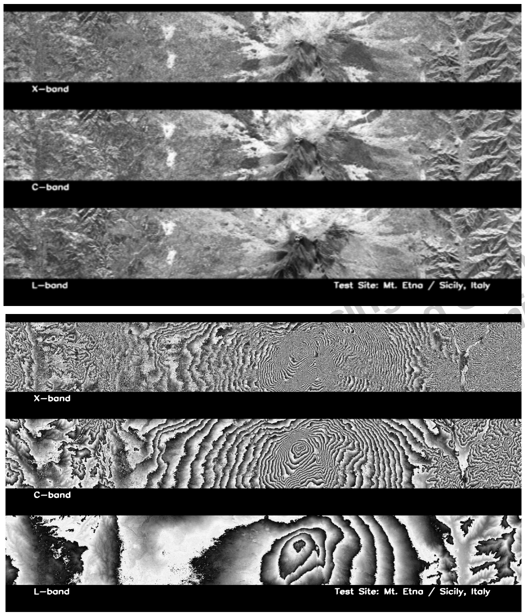
P-Band

L-Band

C-Band

|Shh| |Shv| |Svv| JPL AIRSAR Nezer Forest Data

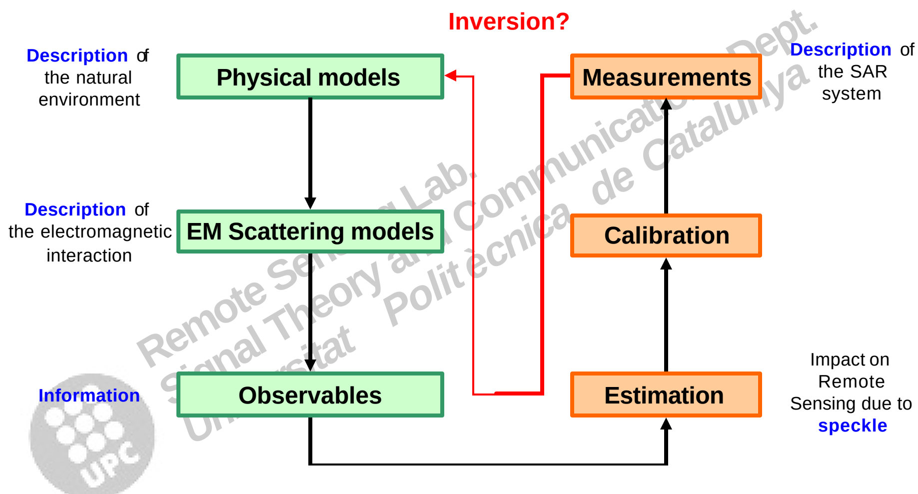
Synthetic Aperture Radar



Interferometric configurations sensitive to the terrain's topography

Information Extraction

What do we mean by information extraction ?



Introduction

Information Extraction

Nature of the extracted information

- Qualitative information: Refers to relative information. The interest focuses on retrieving information concerning physical parameters that may describe, or even to distinguish different areas of the SAR data
  - Classification techniques
- Quantitative information: Refers to absolute information. The interest is on the retrieval of geophysical and biophysical parameters to describe the Earth surface and its dynamics
  - Inverse Problem
  - Electromagnetic modelling

Introduction

Data Models

What do they mean and which advantages provide data models ?

- A (better) description of the data acquired by the SAR system, making possible the (better) extraction of useful information
- Data can be systematically interpreted
- Allow a generalization of the observations
- Make possible to deal with the complexity associated with the scattering process
- In the lack of data models, only a phenomenological interpretation is possible

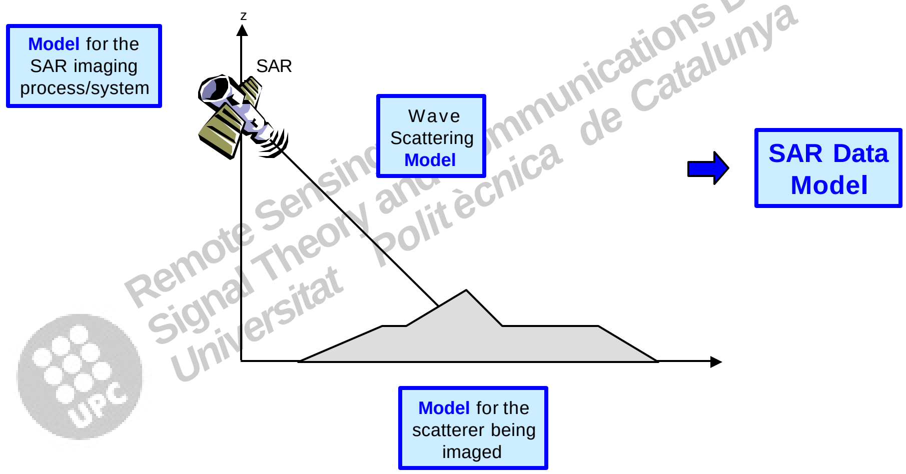
SAR data models are:

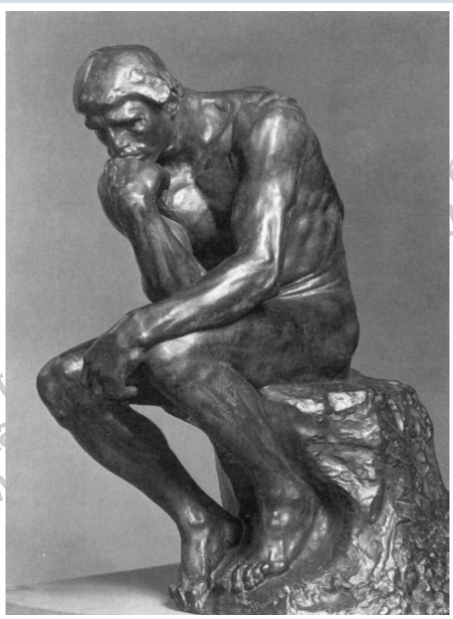
- Controlled by a set of parameters
- Stochastic in nature

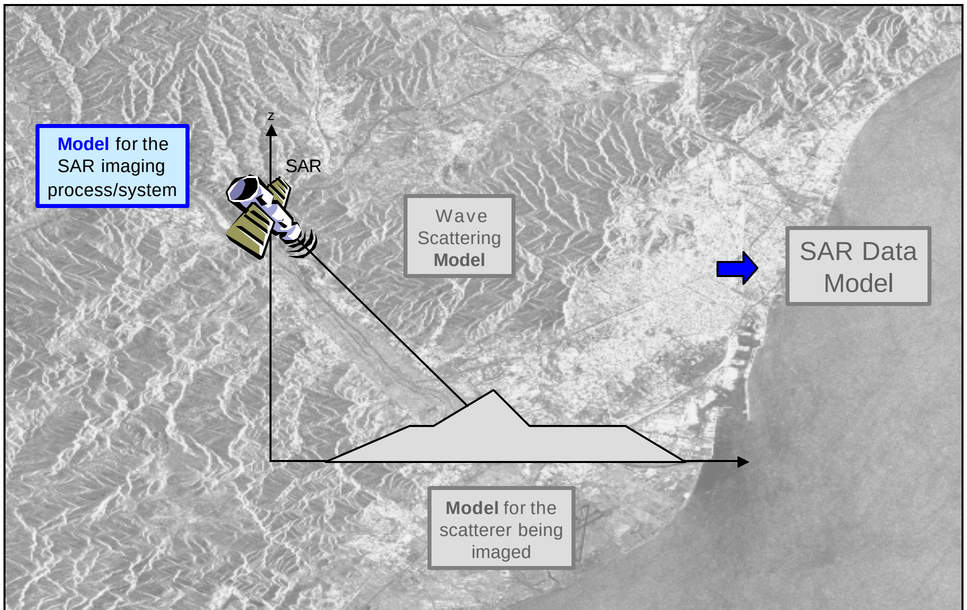
→ The objective of this tutorial

Data Models

The analysis and understanding of data acquired by a SAR system needs from the following considerations

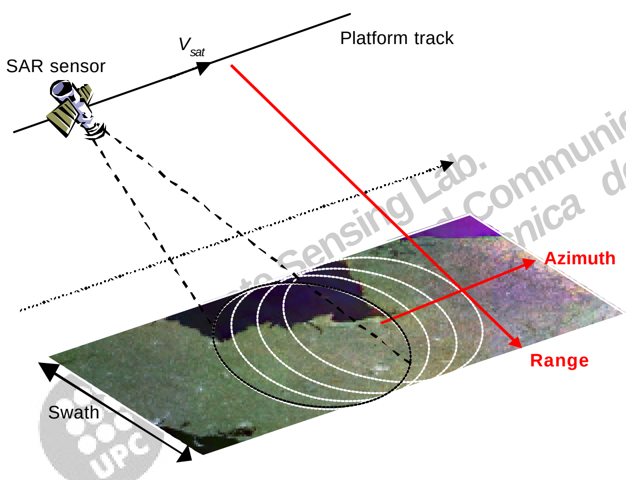






Synthetic Aperture Radar Principles

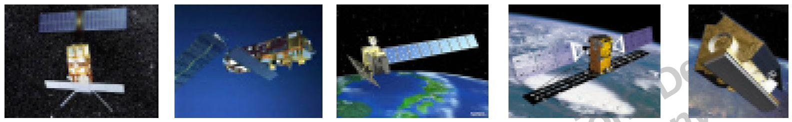
Synthetic Aperture Radar Principles



- Side looking geometry
- Two-dimensional imaging system: Range vs. Azimuth
- Different imaging modes. Compromise between resolution and swath coverage
  - Stripmap
  - Scansar
  - Spolight
- SAR images present a complex nature

Moving Platforms

- Satellite: Orbital systems

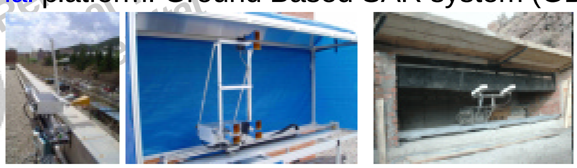


ERS-1/2 ESA (EU) · ENVISAT / ASAR ESA (EU) · ALOS / PALSAR NASDA / JAROS (J) · RADARSAT 2 CSA - MDA (CA) · TERRASAR BMBF / DLR / ASTRIUM

- Airplane: Airborne systems

AIRSAR NASA / JPL (USA) · ESAR DLR (D) · PISAR NASDA / CRL (J) · RAMSES ONERA (F) · SAR580 Environment Canada (CA)

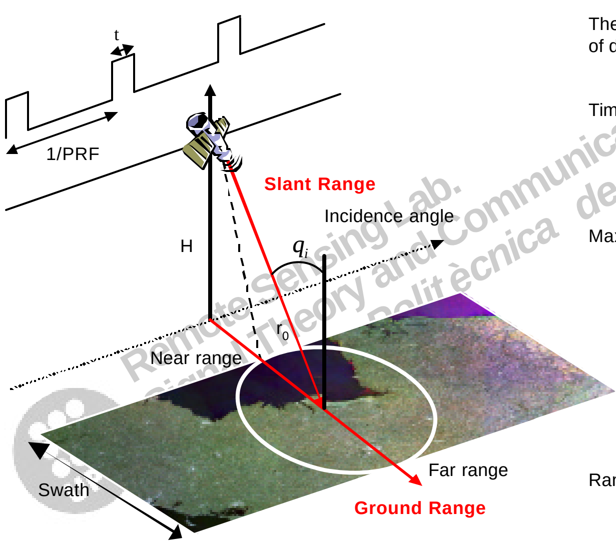
- Terrestrial platform: Ground Based SAR system (GB-SAR)



UPC GB-SAR UPC (SP) · GBInSAR Lisa LisaLab (I)

Range Analysis

In range a SAR system operates as a conventional radar



The SAR system transmits pulses of duration τ at PRF frequency

Time delay

$$t_d = \frac{2r_0}{c}$$

Maximum swath

$$SW_{max} \approx \frac{c}{2\,PRF\,\sin(\theta_i)}$$

$$\theta_i \in \left[20^0..60^0\right]$$

Range resolution

$$\delta_r = \frac{c\tau}{2} = \frac{c}{2B}$$

Synthetic Aperture Radar Principles

Range Analysis

- High spatial resolution, i.e., short transmitted pulses, with sufficient SNR imposes the use of high energy pulses

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- Pulse compression techniques based on modulated long pulses
  - Large radiated energy
  - Range resolution of short pulses

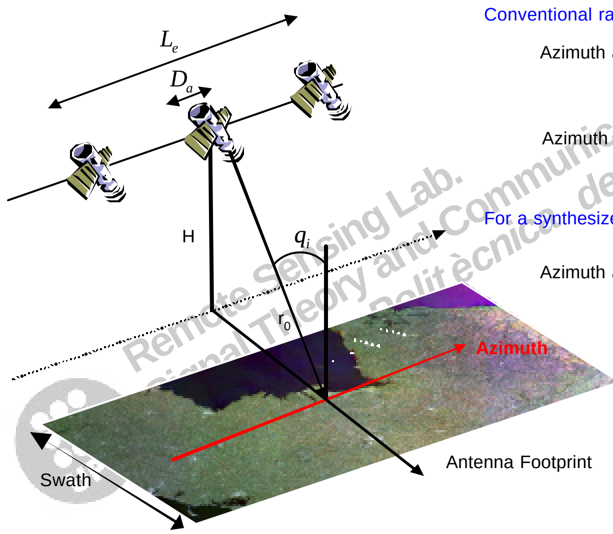
- Implementation based on frequency or phase modulation of the pulse with a bandwidth B_{pulse}
  - Chirp pulses
  - In reception, the pulse is processed in a matched filter compressing the long pulse to a duration $1/B_{pulse}$

Range resolution $$d_r = \frac{ct}{2} = \frac{c}{2B_{pulse}}$$

Synthetic Aperture Radar Principles

Azimuth Analysis

Difference between SAR system and conventional radars



Conventional radar

Azimuth angular spread $q_a \propto \frac{l}{D_a}$

Azimuth resolution $d_a = r_0 q_a = r_0 \frac{l}{D_a}$

For a synthesized aperture

Azimuth angular spread $q_{sa} \propto \frac{l}{2L_e}$

Two-way

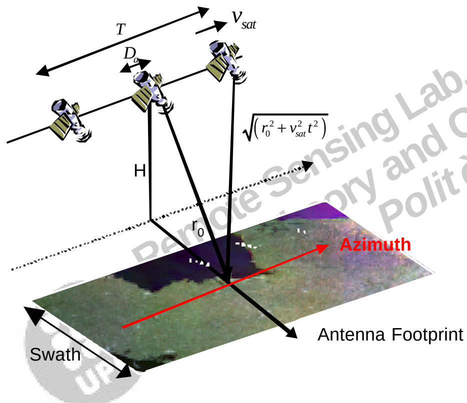
Azimuth resolution $d_a = r_0 \frac{l}{2L_e}$

$L_e < \frac{l R}{D_a}$ $$d_a > \frac{D_a}{2}$$

Synthetic Aperture Radar Principles

Azimuth Analysis

- Azimuth processing is based on the fact that a given target is observed all the time that it is within the antenna footprint. The different observation points are labelled through the Doppler frequency



Synthetic Aperture Radar Principles

Azimuth Analysis

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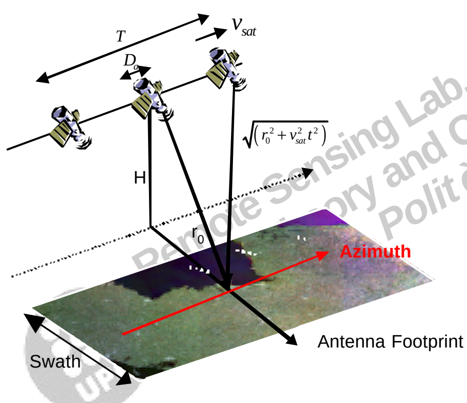


Doppler frequency definition $f_{dop}=\frac{1}{2\pi}\frac{d\phi}{dt}$

Instant phase $\phi=\phi_0+2\frac{2\pi}{\lambda}\sqrt{(r_0^2+v_{sat}^2t^2)}$

Doppler frequency $f_{dop}=2\frac{v_{sat}^2t}{\lambda r_0}$ $\quad B_{dop}=2\frac{v_{sat}^2T}{\lambda r_0}$

Doppler bandwidth $T\approx R\frac{\lambda}{L_e}\frac{1}{v_{sat}}$ $\quad B_{dop}\approx 2\frac{v_{sat}}{D_a}$

Azimuth resolution $\delta_a\approx\frac{D_a}{2}$

SAR Impulse Response

SAR data processing, i.e., SAR image formation process comprises

- Data acquisition process
  - Raw data generation. Data recorded by the SAR system
- Image formation process
  - Raw data compression
    - Generation of the synthesized aperture
    - Collecting/Focusing all the contributions of a given target
  - Non-separable/Non-homogeneous bi-dimensional problem

Consider all the process as a linear system

Phase information

$s_s(x_0, r_0)$ → $h(x,r)$ → $S(x,r)$

Complex scattering amplitude

$s_s(x_0, r_0) = \sqrt{s_0} e^{j\varphi} \delta(x - x_0, r - r_0)$

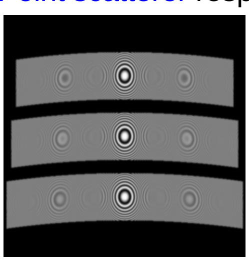
SAR impulse response or Point Spread Function (PSF)

Complex SAR image

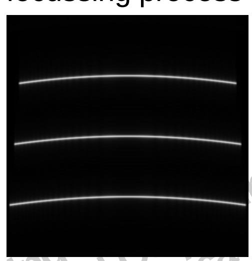
Finite bandwidth impulse response $\delta_a \times \delta_r = \frac{D_a}{2} \times \frac{c}{2B_{pulse}}$

SAR Impulse Response

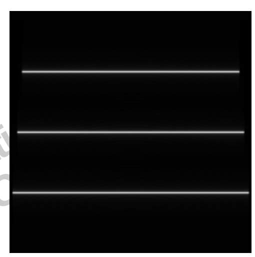
Point scatterer response focussing process



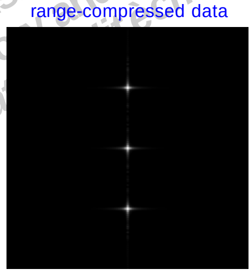
Real part of the raw data



Amplitude of range-compressed data



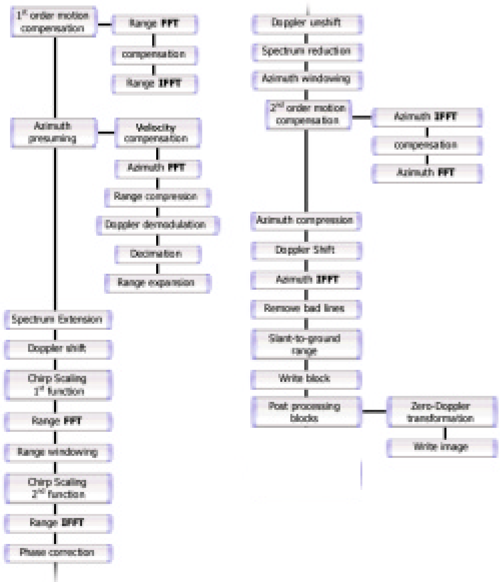
Amplitude of range-compressed data after RCMC



Amplitude of the final compressed image

Synthetic Aperture Radar Principles

SAR Processing Chain



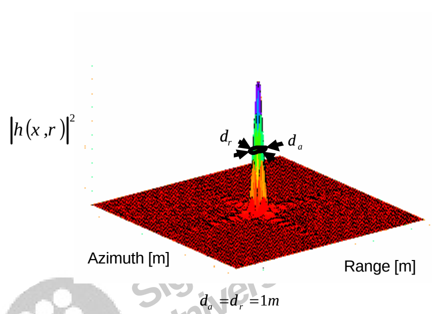
Example of a real processing chain in charge of the image formation process

Synthetic Aperture Radar Principles

SAR Impulse Response

SAR impulse response function

$$h(x,r) \propto \mathrm{sinc}\left(\frac{p\,x}{d_a}\right)\mathrm{sinc}\left(\frac{p\,r}{d_r}\right)$$



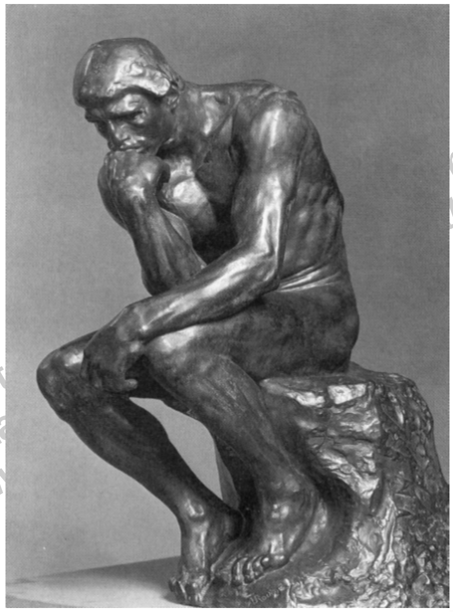
Point scatterer

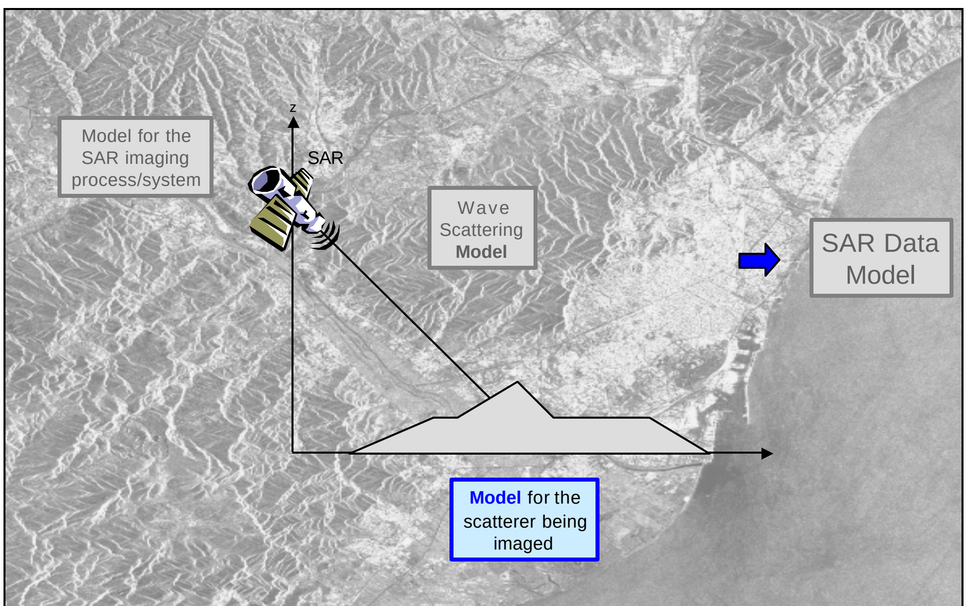
How it appears in the SAR image $S(x,r)$

Distributed scatterer

Idea of resolution cell $d_a \times d_r$

- The resolution cell is not the pixel of the SAR image. The pixel properties depend on how the SAR impulse response is sampled
  - Over sampling induces image spatial correlation





Scatterer Models

Scatterers Heterogeneity

|Shh| |Shv| |Svv|

SAR images reflect the Nature's heterogeneity

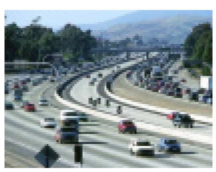
L-band (1.3 GHz) fully PolSAR data
E-SAR system. Oberpfaffenhofen test area (D)

Optical Image
Oberpfaffenhofen test area (D)

Scattering from Point Scatterers

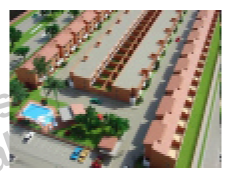
Examples of point targets imaged by SAR systems

Power lines



Vehicles

Railways



Houses

Types of microwave scattering
- Point scattering
- Complex scattering

Man-made media present a strong point scattering behaviour

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Scattered field dominated by canonical scattering mechanisms

Scatterer Models

Point Scatterers Description

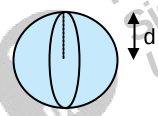
These scatterers make reference to canonical bodies as plates, cylinders, etc...the properties of which can be determined. Each body is finite in extend, and because in the far field zone, the scattered field appears to originated at a point, the body is described as a point target. [Ulaby'90]

$$\sigma_{i,j} = 4\pi \left|S_{ij}\right|^2 \quad i,j = h,v$$

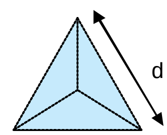
Radar Cross Section or RCS of a body. Scattered power as a function of the incident power. Depends on the imaging geometry

$$s_s(x_0, r_0) = \sqrt{\sigma}\, e^{j\varphi} \delta(x - x_0, r - r_0)$$

Object description (Deterministic description)



$$\sigma_{max}(m^2) = \pi d^2$$



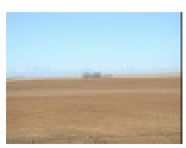
$$\sigma_{max}(m^2) = \frac{4\pi d^4}{3\lambda^2}$$

Scatterer Models

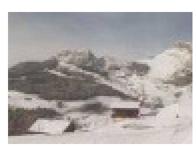
Scattering from Natural/Distributed Targets

Examples of natural targets imaged by SAR systems

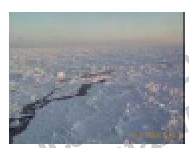
Rocks



Rough surface



Snow



Sea ice

Vegetation cover

Types of microwave scattering
- Surface scattering
- Volume scattering

Geophysical media present complicate structures and/or compositions

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Exact knowledge of the scattered field very difficult

Scatterer Models

Distributed Scatterers Description

Radar scattering from terrain involves complicated interactions because the scattering elements have complicated geometries and are randomly distributed in space. (...) Hence, we usually focus our attention on the development of generic models that can help to understand the nature of wave propagation and scattering in random media (...) allowing to interpret radar observations and extract useful information. [Ulaby'90]

$u(\vec{r})$

Object scattering function. (Random function - microscopic structure) NOT ACCESIBLE

Distributed scatterers have complex geometries and are randomly distributed

$$\langle u(\vec{r}) \cdot u(\vec{r}')^* \rangle = s^0 \cdot d(\vec{r} - \vec{r}')$$

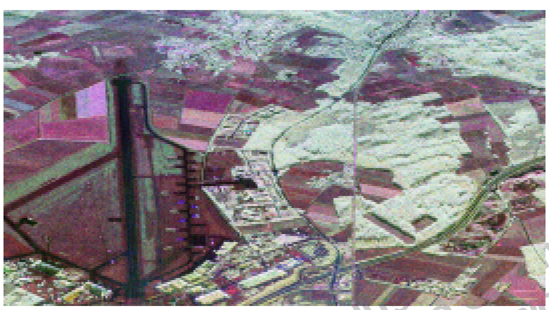
Object description. (2nd order descriptor - *macroscopic structure*)

$$s^0 = E\left\{\frac{s}{A}\right\}$$

Average scattering coefficient or Differential backscattering coefficient. Does not depend on the area of the cell of resolution. This normalisation is necessary as a distributed target can occupy more than one cell of resolution.

Scatterer Models

Texture in Distributed Scatterers



|Shh| |Shv| |Svv|

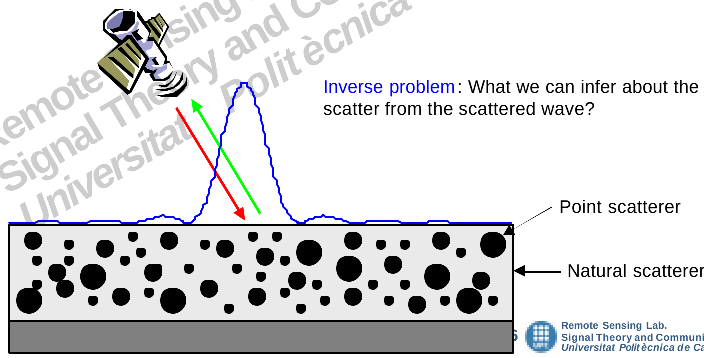
L-band (1.3 GHz) fully PolSAR data
E-SAR system. Oberpfaffenhofen test area (D)

- Homogeneous areas: Zones characterized for presenting a constant behaviour, i.e., a stationary mean value
- Non-Homogeneous areas: Zones presenting the same natural characteristics (forest, grass, etc…) but characterized by non-stationary properties
  - Data texture
- Data texture is subjected to the notion of information scale

Description of Distributed Targets

Real scatterers present very complex structure

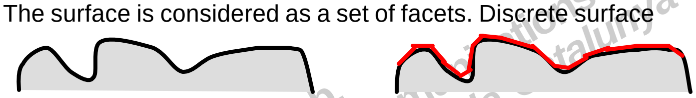
- Consider real scatterers as a collection of point or individual scatterers, randomly located
- The internal structure, i.e., microscopic structure, of the scatterer covered by the resolution cell can non be resolved
  - Partially solved by high dimensional SAR systems (PolInSAR)
- Scattered field results from the interaction of the incident wave with the individual scatterers



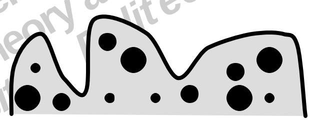
Description of Distributed Targets

Types of complex scatterers

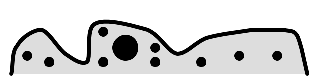
- Surface scatterer
  - The scattered wave is produced on the surface of the scatterer (Conducting or homogeneous media)
  - The surface is considered as a set of facets. Discrete surface

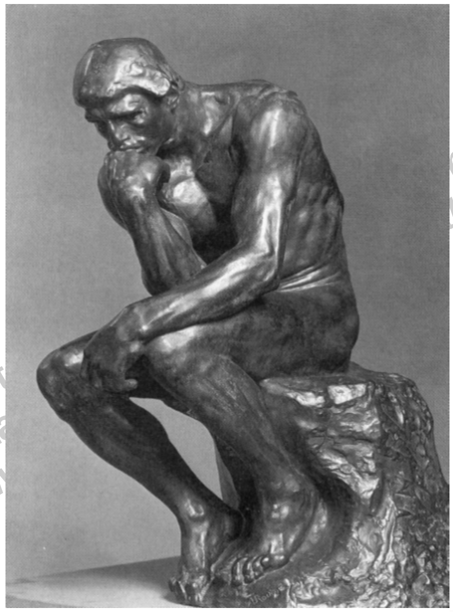


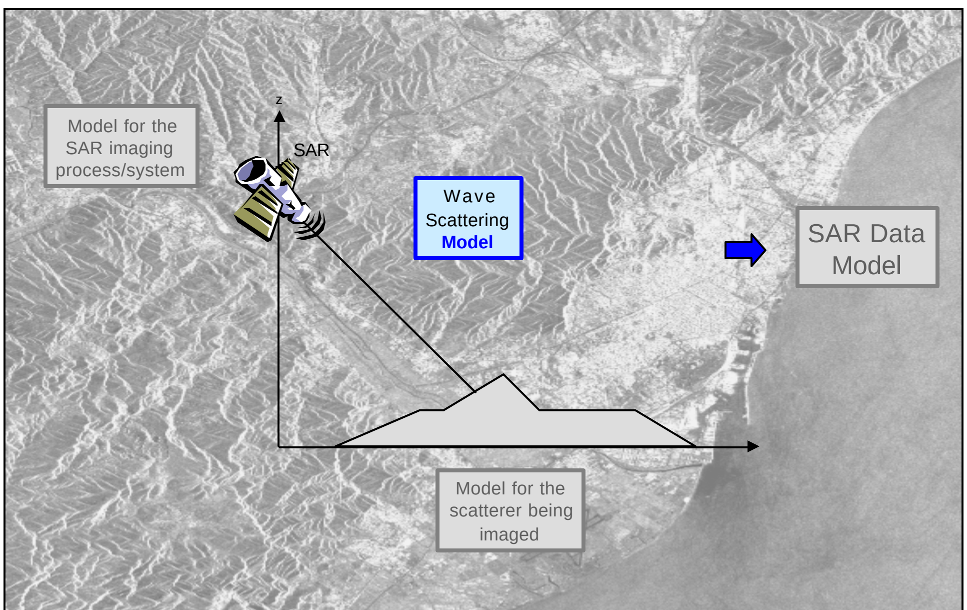
- Volume scatterer
  - The scattered wave is produced within the scatterer (Inhomogeneous media)



- Point scatterer
  - The scattered wave is mainly produced by a dominant point scatterer







Wave Scattering Models, Interaction with Matter

Scattering by Point Scatterers

The response of a point scatterer, i.e., how it appears in a SAR image, can be considered as deterministic

$$s_s(x_0, r_0) = \sqrt{\sigma}\, e^{j\theta} \delta(x - x_0, r - r_0) \qquad h(x, r) \propto \mathrm{sinc}\left(\frac{\pi x}{\delta_a}\right) \mathrm{sinc}\left(\frac{\pi r}{\delta_r}\right)$$

$$S(x, r) = s_s(x_0, r_0) ** h(x, r)$$

Convolution in the Range-Azimuth space

$$S(x, r) \propto s_s(x_0, r_0) \exp\left(j2\frac{2\pi}{\lambda}(r - r_0)\right) \mathrm{sinc}\left(\frac{\pi(x - x_0)}{\delta_a}\right) \mathrm{sinc}\left(\frac{\pi(r - r_0)}{\delta_r}\right)$$

Complex SAR image

- Given the SAR image, it is possible to determine the properties of the scatterer from the image itself
- The pixel contains all the necessary information to characterize the scatterer

Scattering by Distributed Scatterers

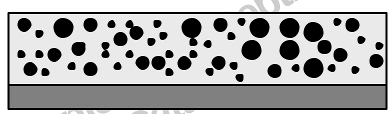
The complexity of a distributed scatterer is translated to the scattering process in the scatterer

Given a set of resolution cells/pixels, the internal structure (microscopic structure) changes from pixel to pixel, then, the scattering process changes from pixel to pixel

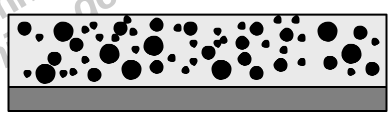
- Scattering processes can not be characterized by the value of single pixels as its values depend on the internal random arrangement

- Scattering processes must be characterized by global parameters common to all the pixels

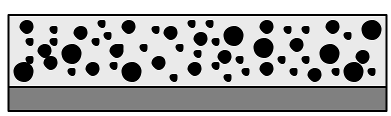
Characterization based on Statistical parameters



Pixel 1



Pixel 2

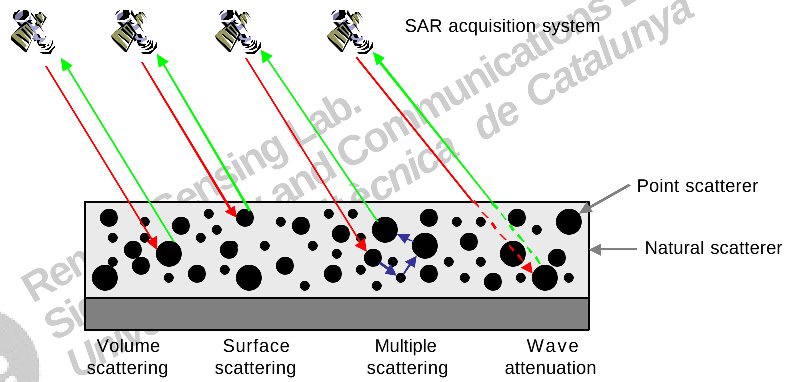


Pixel 3

Born Scattering Approximation

Scattering based on the Born approximation or single scattering approximation

- The scattering is supposed to be the linear coherent addition of the individual scattered waves from a set of discrete or point scatterers



- The model does not consider attenuation or multiple scattering

SAR Imaging Process

Real three dimensional scenes are translated to a two dimensional SAR image

- SAR system impulse response

$$h(x,r) \propto \text{sinc}\left(\frac{p\,x}{d_a}\right)\text{sinc}\left(\frac{p\,r}{d_r}\right)$$

- Scatterer model
  - Complex reflectivity function. Object scattering function
  - Random nature
  - Describes the reflectivity of each point scatterer

$$u(\vec{r}) = u(x,y,z)$$

- The complex reflectivity function must be transformed into a two-dimensional function. Introduction of image distortions

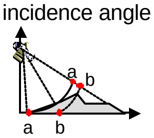
$$u(x,r) = \int u(x, y_0 + r\sin q, z_0 - r\cos q)\, r\,dq$$

Wave Scattering Models

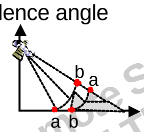
SAR Imaging Distortions

The side-looking geometry and the two-dimensional nature of the SAR images introduce image distortions

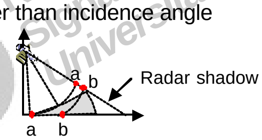
- Foreshortening. Slope of local terrain is less that incidence angle

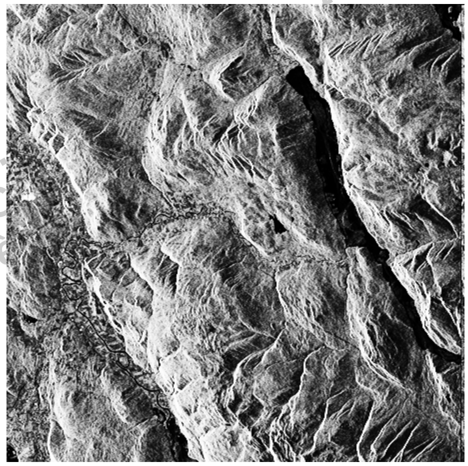


- Layover. Slope of local terrain is higher that incidence angle



- Shadowing. Magnitude of negative slopes is greater than incidence angle





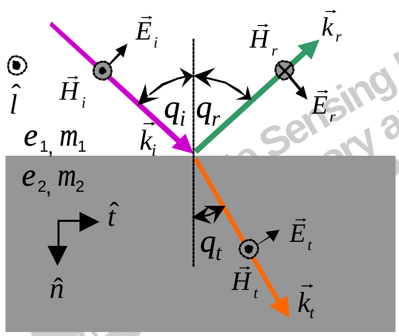
ERS 1 Image

Wave Scattering Models

Surface Scattering

Reflected waves, coming from surface scattering, appear on the surface plane dividing two semi-infinite homogeneous media

- Oblique incidence in lossless media where E-field is parallel to the plane of incidence



Incident electric field

$$\vec{E}_i = \left[\hat{t}\cos\theta_i - \hat{n}\sin\theta_i\right] E_0\, e^{-j\beta_1[t\sin\theta_i + n\cos\theta_i]}$$

Reflected electric field

$$\vec{E}_r = \left[\hat{t}\cos\theta_r + \hat{n}\sin\theta_r\right] E_0\Gamma_{\parallel}^{b}\, e^{-j\beta_1[t\sin\theta_r - n\cos\theta_r]}$$

Transmitted electric field

$$\vec{E}_t = \left[\hat{t}\cos\theta_t - \hat{n}\sin\theta_t\right] E_0 T_{\parallel}^{b}\, e^{-j\beta_2[t\sin\theta_t + n\cos\theta_t]}$$

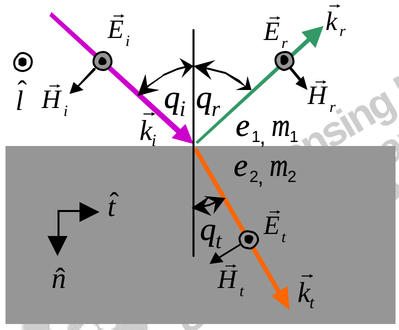
Transmission and reflection coefficients

$$\Gamma_{\parallel}^{b} = [\eta_2\cos\theta_t - \eta_1\cos\theta_i]/[\eta_2\cos\theta_t + \eta_1\cos\theta_i]$$

$$T_{\parallel}^{b} = 2\eta_2\cos\theta_i/[\eta_2\cos\theta_t + \eta_1\cos\theta_i]$$

Surface Scattering

- Oblique incidence in lossless media where E-field is perpendicular to the plane of incidence



Incident electric field

$$\vec{E}_i = \hat{l}\, E_0\, e^{-jb_1[t\sin q + n\cos q_i]}$$

Reflected electric field

$$\vec{E}_r = \hat{l}\, E_0 \Gamma_{\perp}^{b}\, e^{-jb_1[t\sin q_r - n\cos q_r]}$$

Transmitted electric field

$$\vec{E}_t = \hat{l}\, E_0 \mathrm{T}_{\perp}^{b}\, e^{-jb_2[t\sin q_t + n\cos q_t]}$$

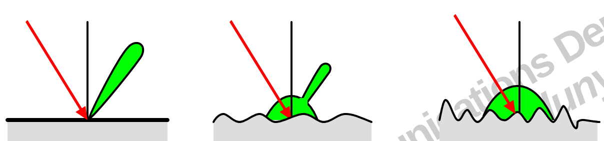
Transmission and reflection coefficients

$$\Gamma_{\perp}^{b} = [h_2\cos q_i - h_1\cos q_t]/[h_2\cos q_i + h_1\cos q_t]$$
$$\mathrm{T}_{\perp}^{b} = 2h_2\cos q_i/[h_2\cos q_i + h_1\cos q_t]$$

Surface Scattering

RCS (s^0) of surfaces depends on

- Roughness



$s^0 = -\infty\ dB$ $s^0 \leq 0\ dB$ $s^0 \approx 0\ dB$

  - Roughness ➚ ⇨ s^0 ➚

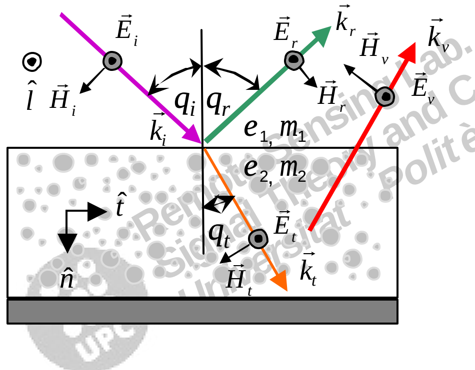
- Dielectric permittivity, related with the water content
  - Humidity ➚ ⇨ s^0 ➚

- Roughness and humidity are coupled parameters in SAR measurements

Wave Scattering Models

Volume Scattering

Volume scattering appears in non-homogeneous media. The capacity to produce volume scattering depends on the penetration depth



The penetration depth d_p corresponds to the distance in which the energy is attenuated a factor equal to 1/e

Propagation constant

$$k = \frac{2\pi}{\lambda} = k_0\sqrt{\varepsilon_r} = \alpha + j\beta \qquad \beta \geq 0$$

Transmitted field

$$\vec{E}_t = \vec{E}_0 e^{j\alpha z} e^{\beta z}$$

Penetration depth (field attenuation $1/\sqrt{e}$)

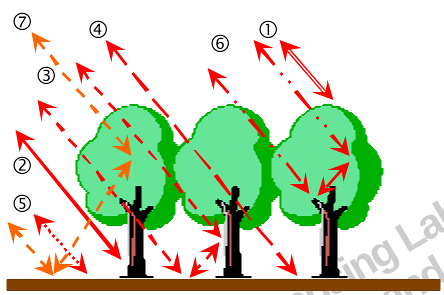
$$d_p = \frac{1}{2\beta} = \frac{1}{2k_0\Im\left\{\sqrt{\varepsilon_r}\right\}} \propto \lambda$$

Wavelength ↗ ⇨ d_p ↗

Frequency ↗ ⇨ d_p ↙

Wave Scattering Models

Volume Scattering Mechanisms



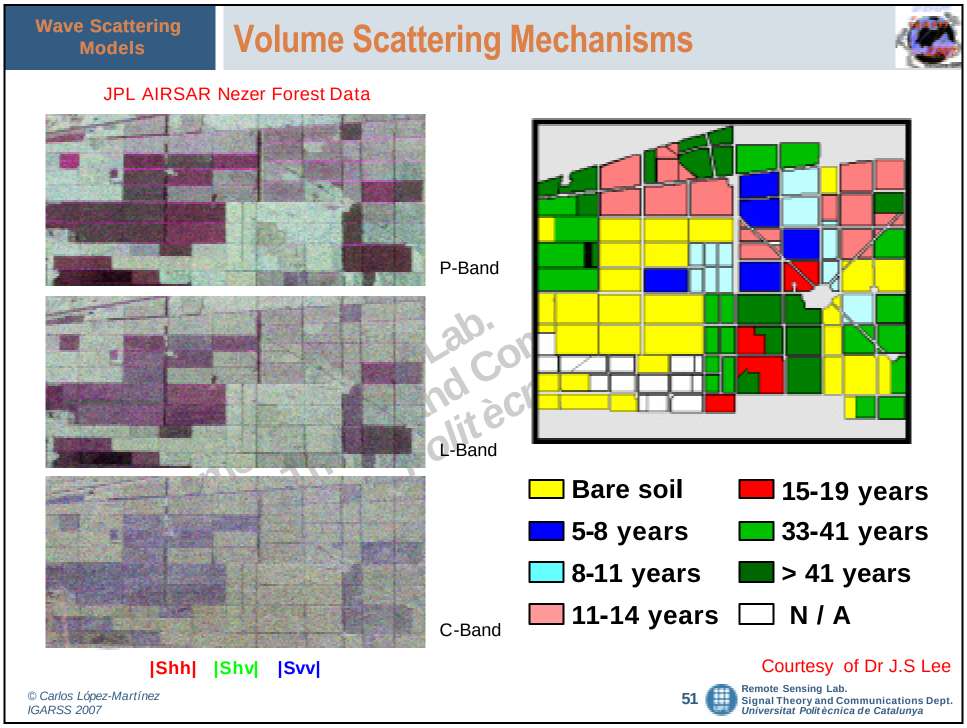
Main scattering mechanism in forest

1. Canopy scattering
2. Trunk scattering
3. Trunk-soil interaction
4. Attenuated soil scattering
5. Direct soil scattering
6. Trunk-branches scattering
7. Branches -soil interaction

Volume scattering composed by complex scattering mechanisms. Born approximation is no longer valid

Biomass ↗ ⇨ σ^0 ↗

Frequency ↗ ⇨ Penetration ↙


Wave Scattering Models
Volume Scattering Mechanisms
JPL AIRSAR Nezer Forest Data
P-Band
L-Band
C-Band
|Shh| |Shv| |Svv|
Bare soil
5-8 years
8-11 years
11-14 years
15-19 years
33-41 years
> 41 years
N / A
Courtesy of Dr J.S Lee
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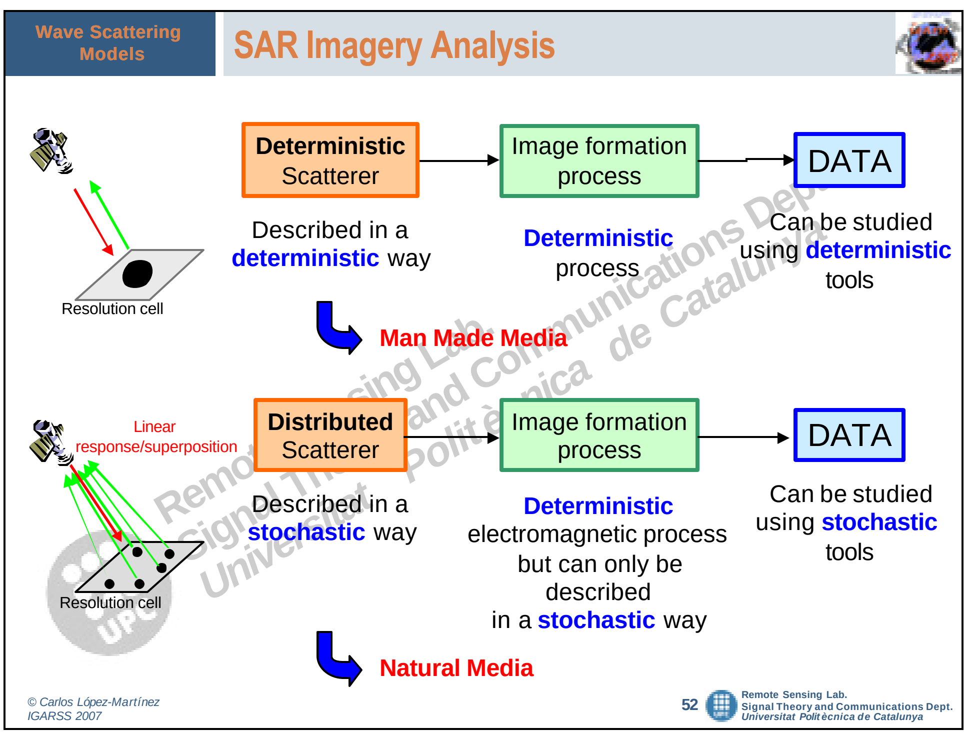
Wave Scattering Models
SAR Imagery Analysis
Deterministic Scatterer
Image formation process
DATA
Resolution cell
Described in a deterministic way
Deterministic process
Can be studied using deterministic tools
Man Made Media
Linear response/superposition
Distributed Scatterer
Image formation process
DATA
Resolution cell
Described in a stochastic way
Deterministic electromagnetic process but can only be described in a stochastic way
Can be studied using stochastic tools
Natural Media
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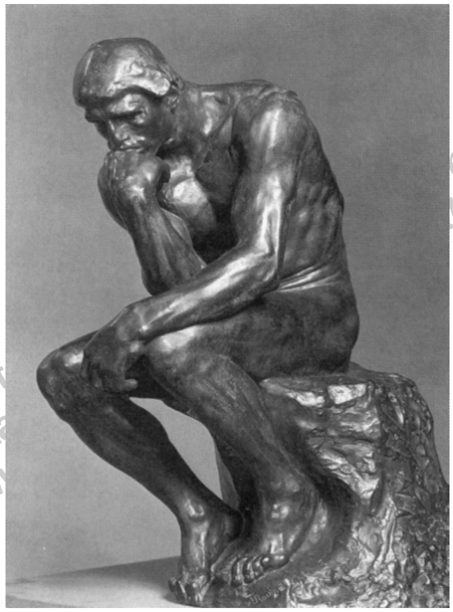

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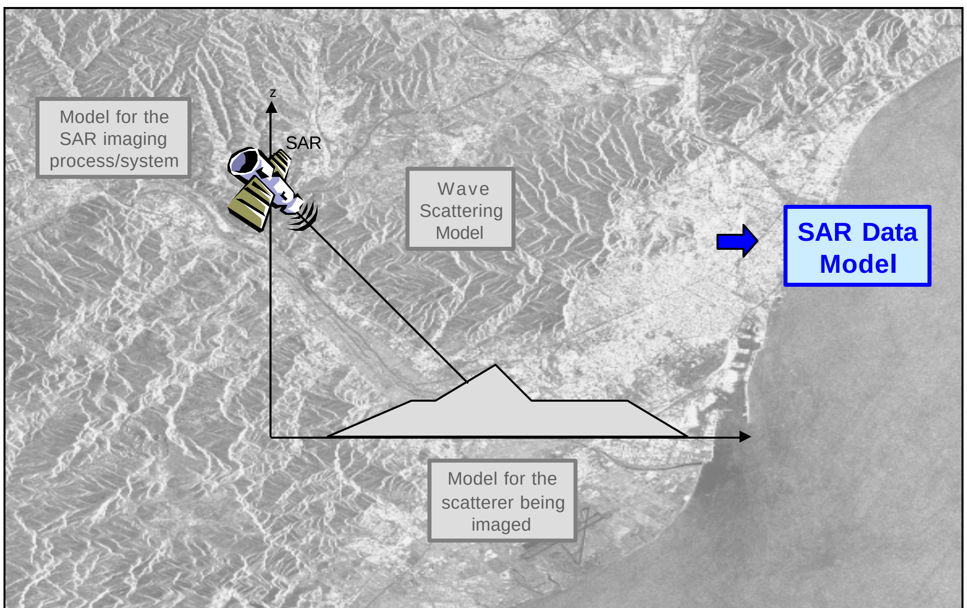
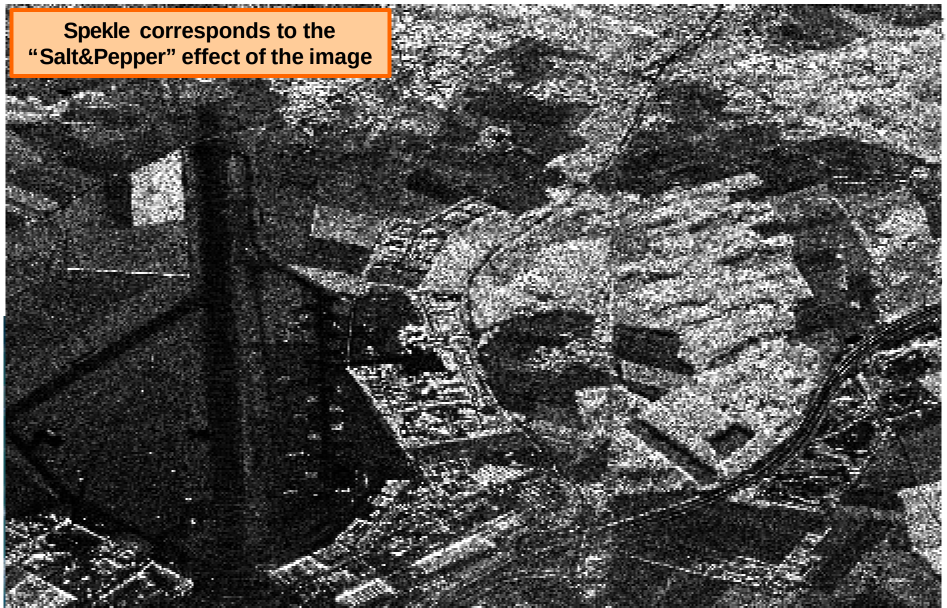


SAR Data Models and Speckle Noise

Speckle Noise

Spekle corresponds to the "Salt&Pepper" effect of the image



|Svv|

Speckle Noise

On the basis of the discrete scatterer description

$$S(x,r)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} u(x',r')h(x-x',r-r')\,\mathrm{d}x'\mathrm{d}r' \;\Rightarrow\; S(x,r)=\frac{1}{\sqrt{L}}\sum_{k=1}^{L}\sqrt{s_k}\,e^{j\theta_k}h(x-x_k,r-r_k)$$

Normalizing factor

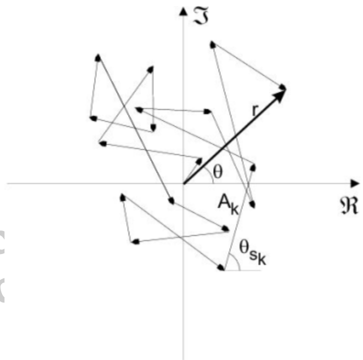
L: Number of point scatterers embraced by the resolution cell

- L as a deterministic quantity
  - L = 1: or a dominating point scatterer: Deterministic scattering
    - Rice/Rician model
  - L >1: Partially developed speckle
    - Not solved model. Even numerical solution difficult
  - L >>1: Fully developed speckle
    - Gaussian model
- L as a stochastic quantity
  - L characterized by a pdf: Image texture
    - K-distribution model

Fully Developed Speckle Noise

- SAR image formation process

$$S(x,r)=\frac{1}{\sqrt{L}}\sum_{k=1}^{L}\sqrt{s_k}\,e^{j\theta_k}h(x-x_k,r-r_k)$$



Random Walk Process

- Complex SAR data for L>>1

$$S(r(x,r),q(x,r))=\Re\{S\}+j\Im(S)$$
$$=r(x,r)\exp(jq(x,r))$$

- Real part

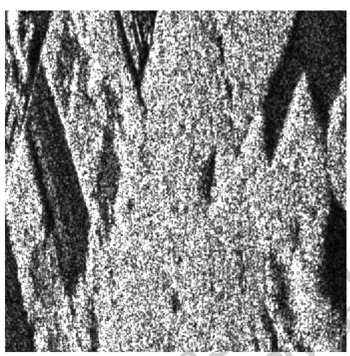
$$\Re\{S\}=\frac{1}{\sqrt{L}}\sum_{k=1}^{L}A_k\cos(q_{s_k})$$

- Imaginary part

$$\Im\{S\}=\frac{1}{\sqrt{L}}\sum_{k=1}^{L}A_k\sin(q_{s_k})$$

$$r(x,r)\exp(jq(x,r))=\frac{1}{\sqrt{L}}\sum_{k=1}^{L}A_k\exp(jq_{s_k})$$

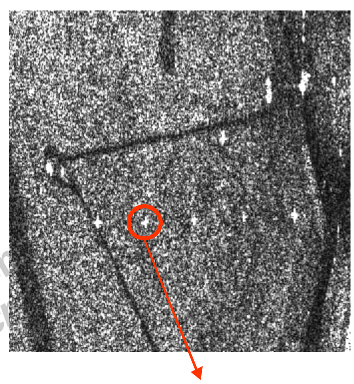
Fully Developed Speckle Noise



Fully Developed speckle

Bright points: Points where the interference is constructive

Dark points: Points where the interference is destructive



Corner reflector
Dominant scatter
No speckle

Speckle is the interference or fading pattern

S_{hh} amplitude
E-SAR L-band system

Fully Developed Speckle Noise

- Completely developed Speckle (large L and no dominant scatterer)
  - Hypotheses
    - The amplitude A_k and the phase q_{s_k} of the kth scattered wave are statistically independent of each other and from the amplitudes and phases of all other elementary waves (Uncorrelated point scatterers)
    - The phases of the elementary contributions q_{s_k} are equally likely to lie anywhere in the primary interval *[-p, p)*
- Central Limit Theorem $S = \mathcal{N}_{C^2}\left(0, s^2/2\right)$ { Gaussian Model / Gaussian Hypothesis
  - Real Part

$$p_{\Re\{S\}}\left(\Re\{S\}\right) = \frac{1}{\sqrt{2p s^2}} \exp\left(-\frac{1}{2}\left(\frac{\Re\{S\}}{s}\right)^2\right) \quad \Re\{S\} \in (-\infty, \infty) \quad \text{Gaussian pdf}$$

  - Imaginary Part

$$p_{\Im\{S\}}\left(\Im\{S\}\right) = \frac{1}{\sqrt{2p s^2}} \exp\left(-\frac{1}{2}\left(\frac{\Im\{S\}}{s}\right)^2\right) \quad \Im\{S\} \in (-\infty, \infty) \quad \text{Gaussian pdf}$$

  - Real and imaginary parts are uncorrelated $E\left\{\Re\{S\}\,\Im\{S\}\right\} = 0$

Fully Developed Speckle Noise

- Amplitude: Rayleigh pdf

$$p_r(r) = \frac{r}{s^2} \exp\left(-\frac{1}{2}\left(\frac{r}{s}\right)^2\right) \quad r \in [0, \infty)$$

$$E\{r\} = \sqrt{\frac{p}{2}}\, s \qquad E\{r^2\} = 2s^2 \qquad s_r^2 = E\{r^2\} - E^2\{r\} = \left(2 - \frac{p}{2}\right) s^2$$

- Intensity ($I = r^2$): Exponential pdf

$$p_I(I) = \frac{1}{2s^2} \exp\left(-\frac{I}{2s^2}\right) \quad I \in [0, \infty)$$

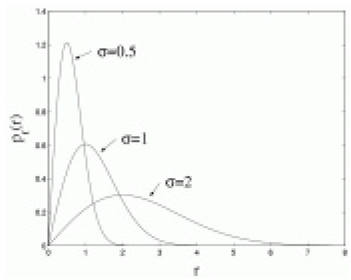
$$E\{I\} = 2s^2 \equiv s \qquad E\{I^2\} = 2\left(2s^2\right)^2 \qquad s_I^2 = E\{I^2\} - E^2\{I\} = \left(2s^2\right)^2$$

- Phase: Uniform pdf. Contains NO information

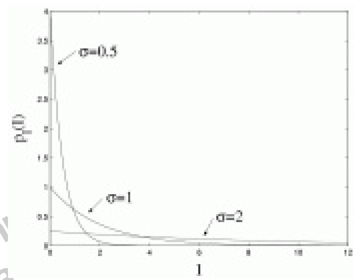
$$p_q(q) = \frac{1}{2p} \quad q \in [-p, p)$$

- Amplitude and phase are uncorrelated

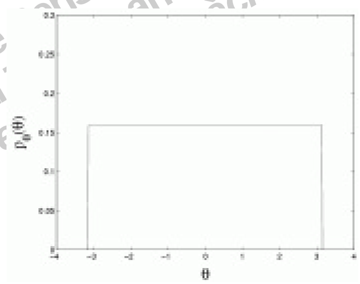
Fully Developed Speckle Noise



Amplitude: Rayleigh pdf



Intensity (I=r^2): Exponential pdf



Phase: Uniform pdf

Fully Developed Speckle Noise

Important considerations

- Speckle is a deterministic electromagnetic effect, but due to the complexity of the image formation process, it must be analysed statistically
- Considering completely developed speckle, a SAR image pixel does not give information about the target. Only statistical moments can describe the target or the process

Information

What does it mean information in the presence of Speckle?

- Phase contains no information
- Intensity exponentially distributed

$$p_I(I)=\frac{1}{2s^2}\exp\left(-\frac{I}{2s^2}\right) \quad I\in[0,\infty) \quad \Rightarrow \quad E\{I\}=2s^2, \quad s_I=2s^2$$

Exponential pdf — First and second order moments

- Intensity, under the previous hypotheses, is completely determined by the exponential pdf
  - Pdf completely determined by the pdf shape
  - Pdf shape parameterized by s ⇒ INFORMATION ⇒ RCS s^0
- Not useful information is considered as NOISE

Fully Developed Speckle Noise Model

Objectives of a Noise Model

- To embed the data distribution into a noise model, that is, a function that allows identifying of the useful information to be retrieved, the noise sources, and how these terms interact
- Optimize the information extraction process, i.e., the noise filtering process

SAR image intensity noise model

SAR image intensity ($I=r^2$) $p_I(I)=\frac{1}{2s^2}\exp\left(-\frac{I}{2s^2}\right) \quad I\in[0,\infty)$ $\quad E\{I\}=2s^2, \quad s_I=2s^2$

$I=2s^2n \quad p_n(n)=\exp(-n) \quad n\in[0,\infty)$ $\quad E\{I\}=1, \quad s_I=1$

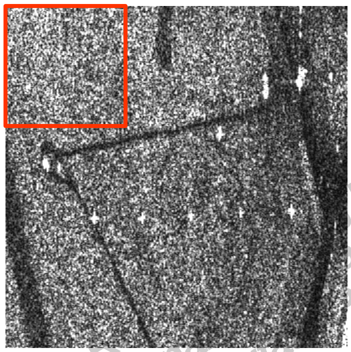
One dimensional speckle noise model (Model over the SAR image intensity - 2nd moment) ⇒ $I(x,r)=s(x,r)n(x,r)$

Multiplicative Speckle Noise Model

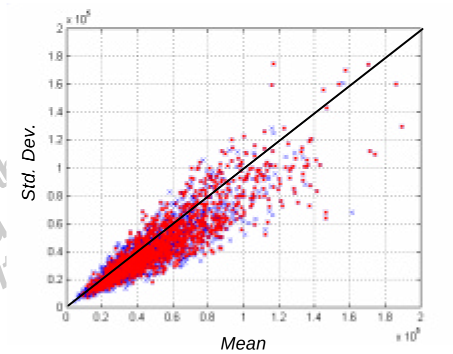
Fully Developed Speckle Noise Model

Moments calculated over local 7x7 local windows

Statistics area



Grass area



Blue: $|S_{hh}|^2$
Red: $|S_{vv}|^2$

S_{hh} amplitude
E-SAR L-band system

Fully Developed Speckle Noise Model

Analysis of the Coefficient of Variation CV

$$CV = \frac{std}{mean}$$

$$E\{I\} = 2s^2 \qquad s_I = 2s^2 \qquad CV = \frac{std}{mean} = \frac{2s^2}{2s^2} = 1$$

For the exponential PDF *CV*=1

- An increase of the power transmitted by the SAR system does not produce an increase of the Signal to Noise Ratio (SNR)

Analysis of the Equivalent Number of Looks (ENL)

$$ENL = CV^{-1} = \frac{mean}{std}$$

SAR Image Texture

Statistical Product Model

- Intensity is decomposed into a three term product

$$I(x,r) = s\ (x,r)\,T(x,r)\,n(x,r)$$

s : Mean value
T : Texture random variable
n : Fading random variable (speckle)

Three scale model

- Coarsest scale : Mean reflectivity, constant value
- Finest scale : Speckle, noise
- Intermediate scale : Texture, spatially correlated fluctuations

As observed, the definition of the three terms is subjected to the notion of scale, or in other words, to where limits between them are placed

- Analysis based in time/frequency tools

SAR Image Texture

How to describe texture in SAR images

- One-point statistics: Mean and Variance
  - K-distribution model

- Two-point statistics: Autocovariance, Autocorrelation function (ACF)
  - Modelization of the autocovariance and autocorrelation functions

One-Point Statistics Texture

• Texture can be considered as a fluctuating mean value

$$I(x,r) = s(x,r)T(x,r)n(x,r)$$

$$p_I(I) = \frac{1}{2s^2}\exp\left(-\frac{I}{2s^2}\right) \quad I \in [0,\infty) \quad \Rightarrow \quad p_I(I) = \frac{1}{s}\exp\left(-\frac{I}{s}\right) \quad I \in [0,\infty)$$

Simplification

$$P(I) = \int_0^\infty P(I|s)P(s)\,ds = \frac{L^L I^{L-1}}{\Gamma(L)}\int_0^\infty \frac{ds}{s^L}\exp\left[-\frac{LI}{s}\right]P(s)$$

Gaussian PDF — Fluctuating RCS

• Model results from considering the number of scatterers L within the resolution cell as a random quantity

SAR Data Models and Speckle Noise

One-Point Statistics Texture

RCS model ⇒ Gamma pdf

$$P(s) = \left(\frac{\nu}{\langle s\rangle}\right)^{\nu}\frac{s^{\nu-1}}{\Gamma(\nu)}\exp\left[-\frac{\nu s}{\langle s\rangle}\right]$$

ν : Order parameter

$\langle s\rangle$: Mean RCS $s(x,r)$

Number of scatterers controlled by a bird, death and migration process, the population would be negative binomial

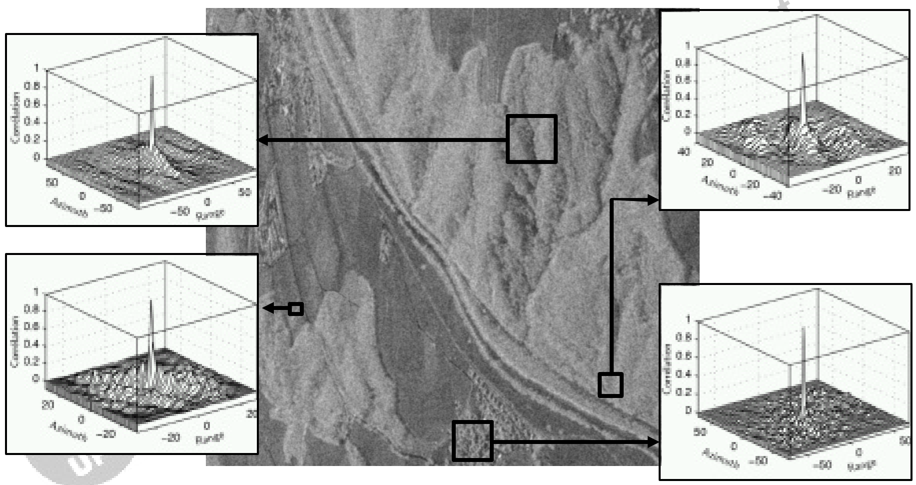
$$P(I) = \frac{2}{\Gamma(L)\Gamma(\nu)}\left(\frac{L\nu}{\langle I\rangle}\right)^{(L+\nu)/2} I^{(L+\nu-2)/2}K_{\nu-L}\left[2\left(\frac{\nu LI}{\langle I\rangle}\right)^{1/2}\right]$$

Intensity distributed as K-distribution

Two-Point Statistics Texture

Texture can be considered as a fluctuating autocovariance function

Trauntstein, ESAR, DLR, L-Band



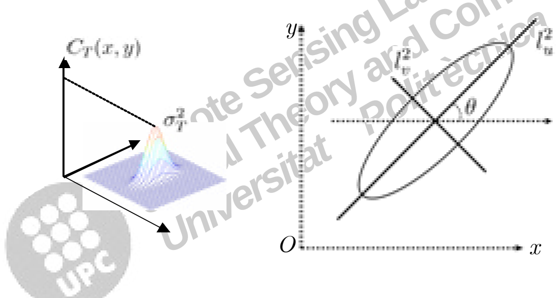
Nonstationary texture
Anisotropic texture

Two-Point Statistics Texture

Modelling of the local autocovariance function considering Anisotropic Gaussian Kernels AGK

- Consider that locally, the autocovariance function can be approximated by an orientated two-dimensional Gaussian function

$$C_T(\mathbf{d}) = \sigma_T^2 \exp\left(-\mathbf{d}^T \Sigma^{-1} \mathbf{d}\right) \quad \mathbf{d} = [x, y]^T$$



$$\Sigma = \mathbf{R}_\theta^T \Lambda\, \mathbf{R}_\theta$$

Correlation lengths

$$\Lambda = \begin{bmatrix} l_u^2 & 0 \\ 0 & l_v^2 \end{bmatrix}$$

Orientation Angle

$$\mathbf{R}_\theta = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

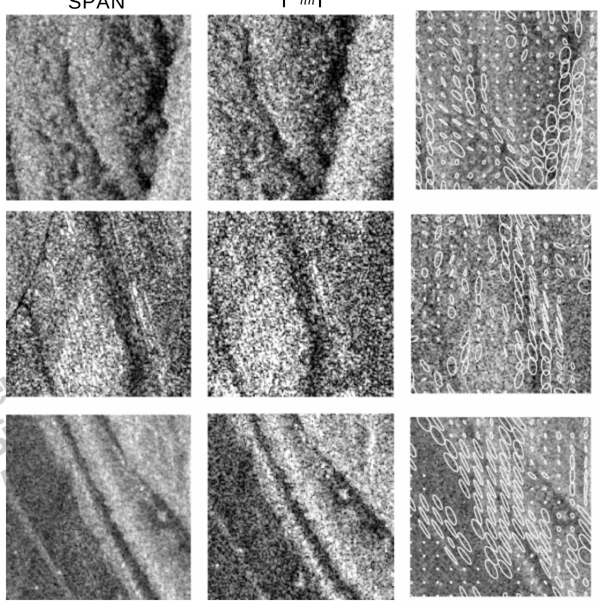
Two-Point Statistics Texture

SPAN | $|S_{hh}|$

Zone 1

Zone 2

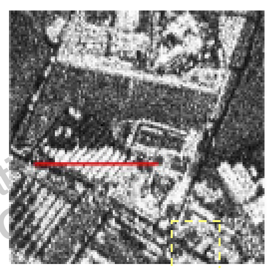
Zone 3



Speckle Noise Model

Observation

- The Gaussian statistical model is unable to accommodate larger tails, i.e., a higher probability of larger SAR images amplitudes



⬇

Gaussian statistics must be extended

⬇

Consider a family of distributions in which the Gaussian distribution is a member

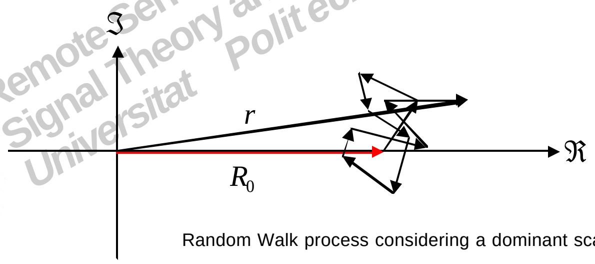
Rice Model

- SAR image formation process

$$S(x,r)=\frac{1}{\sqrt{L}}\sum_{k=1}^{L}\sqrt{\sigma_k}e^{j\theta_k}h(x-x_k,r-r_k)$$

- Now consider that within the resolution cell there is a dominant point scatterer

$$S(x,r)=R_0+\frac{1}{\sqrt{L}}\sum_{k=1}^{L}\sqrt{\sigma_k}e^{j\theta_k}h(x-x_k,r-r_k)$$



Random Walk process considering a dominant scatterer

Rice Model

Under the same assumptions for fully developed speckle, but considering the contribution of the dominant point scatterer

- Real and Imaginary Parts

$$p_{\Re\{S\},\Im\{S\}}\left(\Re\{S\},\Im\{S\}\right)=\frac{1}{2\pi\sigma^2}\exp\left(-\frac{\left(\Re\{S\}-R_0\right)^2+\Im^2\{S\}}{2\sigma^2}\right)$$

- Amplitude: Rician pdf

$$p_r(r)=\frac{r}{\sigma^2}\exp\left(-\frac{r^2+R_0^2}{2\sigma}\right)I_0\left(\frac{rR_0}{\sigma^2}\right)$$ $I_0(x)$ Bessel function of first kind, order zero

$$E\{r\}=\frac{1}{2}\sqrt{\frac{\pi}{2\sigma^2}}\exp\left(-\frac{R_0^2}{4\sigma^2}\right)\left[\left(R_0^2+2\sigma^2\right)I_0\left(\frac{R_0^2}{4\sigma^2}\right)+R_0^2I_1\left(\frac{R_0^2}{4\sigma^2}\right)\right]$$

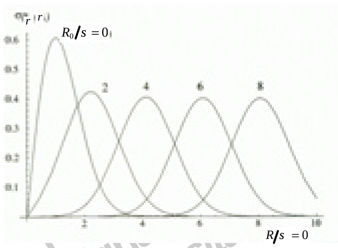
$$E\{r^2\}=R_0^2-2\sigma^2$$

Rice Model

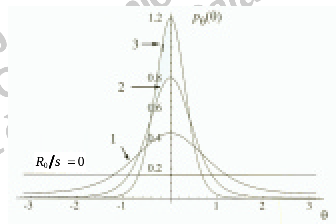
- Phase

$$p_\theta(\theta) = \frac{e^{-\frac{R_0^2}{2\sigma^2}}}{2\pi} + \sqrt{\frac{1}{2\pi}}\frac{R_0}{\sigma}e^{-\frac{R_0^2}{2\sigma^2}\sin^2\theta}\,\frac{1+\operatorname{erf}\left(\frac{R_0\cos\theta}{\sqrt{2}\sigma}\right)}{2}\cos\theta$$

- Examples of pdfs

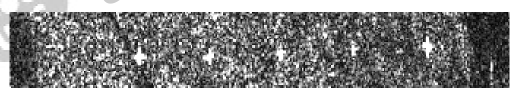


Amplitude pdf



Phase pdf

- SAR image example



Corners reflectors

S_{hh} amplitude
E-SAR L-band system

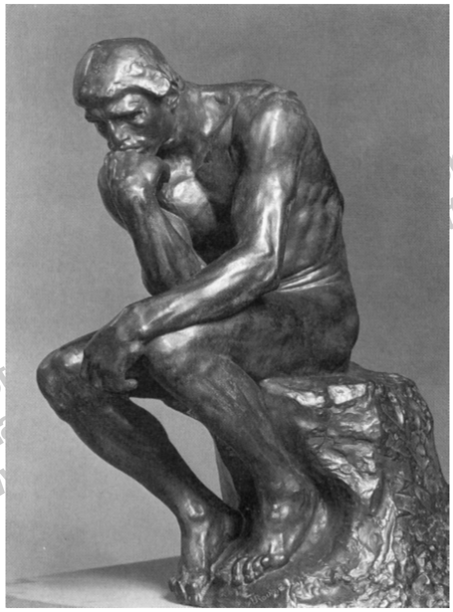
Models for Extremely Heterogeneous Areas

In extremely heterogeneous areas the Gaussian distribution is unable to predict the data distribution

- The solution is to consider more complex distributions with a larger number of parameters
- Difficulty to estimate these parameters with a reduced number of samples
- These models tend to model the pair Texture/Spekle and not only Speckle. No differences are established between point and distributed scatterers

$$I(x,r) = s(x,r)T(x,r)n(x,r)$$

- Extremely heterogeneous areas correspond mainly to urban areas



Multidimensional SAR Systems

Multidimensional SAR Systems

- SAR Interferometry (InSAR): $m=2$. Topographic information
- Differential SAR interferometry (DInSAR): $m=3$. Topographic changes information
- SAR Polarimetry (PolSAR): $m=3,4$. Geometric characterization and classification of the scatterers being imaged
- Polarimetric SAR interferometry (PolInSAR): $m=6,8$. Study and characterization of volumetric structures
- SAR Tomography/Multibaseline: $m>2$. Vertical profiling
- Multitemporal SAR: $m>2$. Change detection and temporal analysis
- Multifrequency SAR: $m>2$. Characterization of the scatterers being imaged

Multidimensional SAR Systems

Important aspects to consider in Multidimensional SAR Imagery

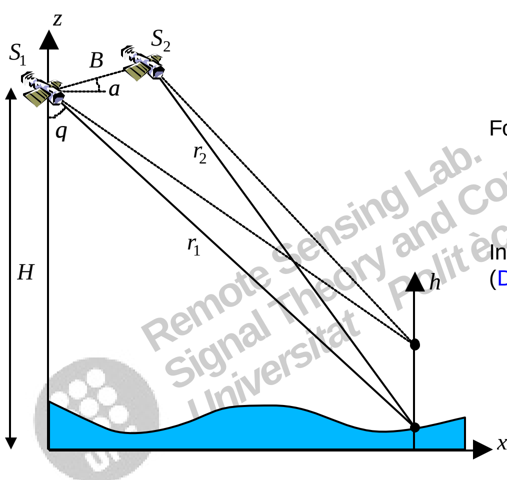
- Physics
  - Depending on the configuration of the multidimensional SAR system, information is sensitive to one or several properties of the target being imaged
  - Data processing, and specially, data estimation can not be done without taking into account the physics behind de scattering process
  - The most clear example is the number of channels m. Represents a clear limitation for multidimensional SAR imagery
- Mathematical representation. Statistics
  - A mathematical description is necessary to systemize data description and understanding

⬇

Electromagnetic Signal Processing

SAR Interferometry - InSAR

Spatial diversity makes SAR data sensitive to terrain topography



Coherent information

$$S_1 = r_1 e^{jf_1} = r_1 e^{j\left(-\frac{4p}{l} r_1 + f_{obj1}\right)}$$

$$S_2 = r_2 e^{jf_2} = r_2 e^{j\left(-\frac{4p}{l} r_2 + f_{obj2}\right)}$$

For small baselines

$$r_1 \approx r_2$$

$$f_{obj1} \approx f_{obj2}$$

Interferometric phase difference (Derministic phase component)

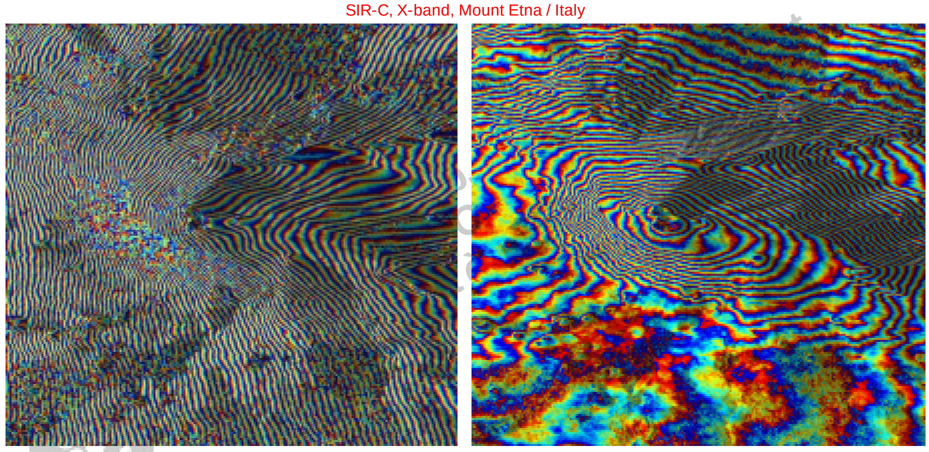
$$\Delta f_{12} = Arg\left(S_1 S_2^*\right) = \frac{4p}{l} \Delta r_{12}$$

Δr_{12} can be estimated with a sub-wavelength accuracy

High accuracy estimation of h

SAR Interferometry - InSAR

SIR-C, X-band, Mount Etna / Italy



$\Delta f_{12} = \Delta f_{topo} + \Delta f_{fe}$

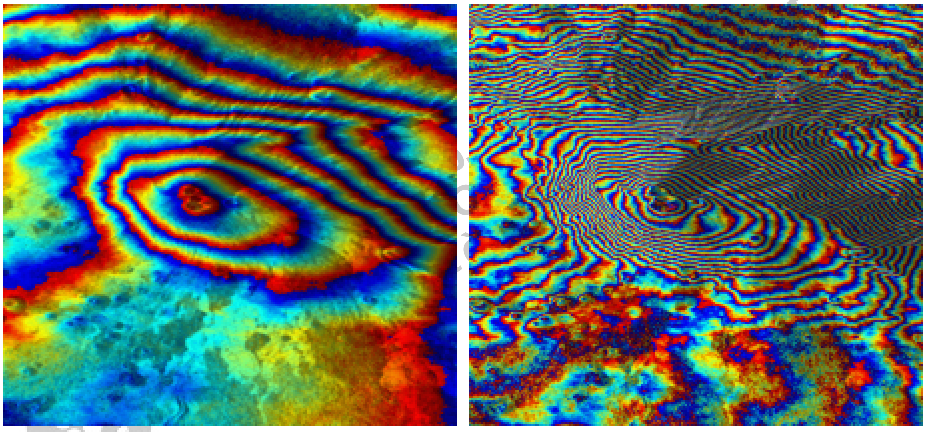
60m baseline

$\Delta f_{12} - \Delta f_{fe} = \Delta f_{topo}$

Multidimensional SAR Systems

SAR Interferometry - InSAR

SIR-C, X-band, Mount Etna / Italy



Simulated 12m baseline — 60m baseline

$$\Delta f_{topo} = \frac{4p}{l}\frac{B_{\perp}}{r_1 \sin q}\Delta h$$

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Multidimensional SAR Systems

SAR Interferometry - InSAR

Interferometric coherence

- Joint interferometric representation

$$\mathbf{k} = \begin{bmatrix} S_1 \\ S_2 \end{bmatrix} \quad \text{is} \quad S = \mathcal{N}_{C^2}(0, \mathbf{C_k})$$

with $\mathbf{C_k} = E\{\mathbf{k}\mathbf{k}^H\} = \begin{bmatrix} \bar{I}_1 & r\sqrt{\bar{I}_1\bar{I}_2} \\ r^*\sqrt{\bar{I}_1\bar{I}_2} & \bar{I}_2 \end{bmatrix}$ and $\bar{I}_i = E\{S_i S_i^H\}$

- Interferometric coherence

$$r = \frac{E\{S_1 S_2^H\}}{\sqrt{\bar{I}_1\bar{I}_2}} = |r| e^{jf_0}$$

$|r|$ indicator of the interferometric information quality

$|r| = 1$ interferometric assumptions are fulfilled

$|r| = 0$ interferometric images are totally uncorrelated

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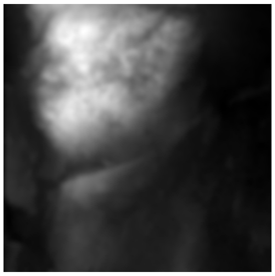
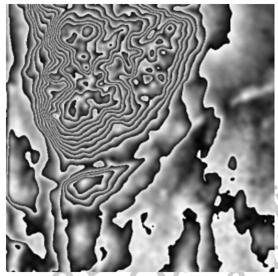
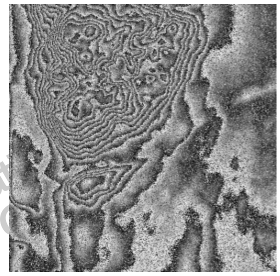
SAR Interferometry - InSAR



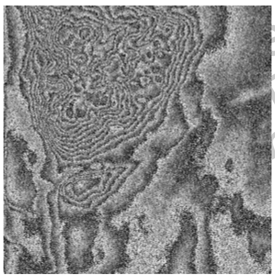
Absolute «True » Phase



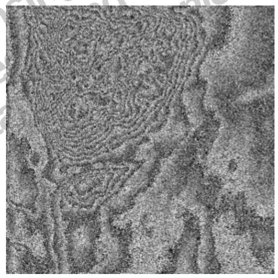
Coherence=1.0



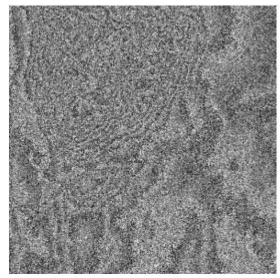
Coherence=0.8



Coherence=0.6



Coherence=0.4



Coherence=0.2

SAR Polarimetry - PolSAR

Polarimetry represents a cornerstone for the scatterers analysis

- Polarimetric data allows a better characterization of the scatterer being imaged
- Polarimetric data is basically sensitive to the geometry and the electrical properties of the scatterer being imaged
- Polarimetric synthesis allows, from the response of the scatterer to a particular polarization basis, the response to any polarization basis

SAR Polarimetry

- Extend the advantages of SAR systems, mainly, the high spatial resolution, to polarimetric data
- Considerations
  - Wave polarimetry
  - Wave scattering
  - Target Decomposition Theorems

Wave Polarimetry

Polarimetry is the most clear example of the Electromagnetic Waves' vectorial nature

- Electric field description

Real electric field vector $\vec{E}(z,t)=\Re\left(\underline{E}(z)e^{jwt}\right)$ Harmonic time dependence

Complex electric field vector $\underline{E}(z)$

Helmotz Propagation Equation

$$\nabla^2\underline{E}(z)+\underline{k}^2\underline{E}(z)=0$$

General Solution

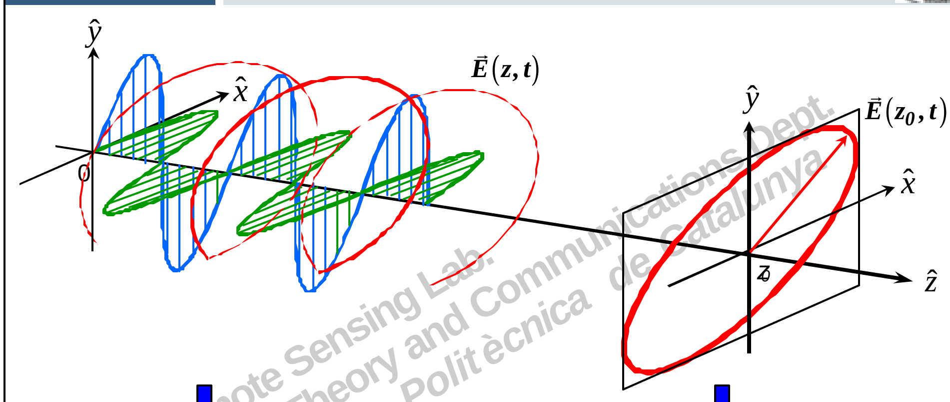
$$\underline{E}(z)=\underline{E}e^{-jkz}$$

With: $\underline{E}=\begin{bmatrix}E_x\\E_y\\E_z\end{bmatrix}=\begin{bmatrix}E_{ox}e^{jd_x}\\E_{oy}e^{jd_y}\\E_{oz}e^{jd_z}\end{bmatrix}$

Sinusoidal Plane Wave

$$\nabla\cdot\vec{E}(z,t)=0\Rightarrow\frac{\partial E_z}{\partial z}=0$$

Wave Polarimetry



Real electric field vector

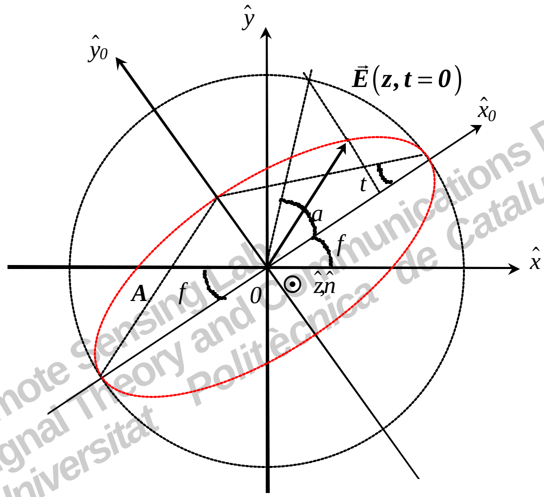
$$\vec{E}(z,t)=\begin{cases}E_x=E_{0x}\cos(wt-kz-d_x)\\E_y=E_{0y}\cos(wt-kz-d_y)\\E_z=0\end{cases}$$

Polarization ellipse

$$\left(\frac{E_x}{E_{0x}}\right)^2-2\frac{E_xE_y}{E_{0x}E_{0y}}\cos(d)+\left(\frac{E_y}{E_{0y}}\right)^2=\sin^2(d)$$

With: $d=d_y-d_x$

Wave Polarimetry



A : Wave Amplitude

a : Absolute Phase

f : Orientation Angle $-\frac{p}{2} \le f \le \frac{p}{2}$

t : Ellipticity Angle $0 \le t \le \frac{p}{4}$

Wave Polarimetry

Representation of the wave polarization state with the minimum amount of information

Real electric field vector

$$\vec{E}(z,t) = \begin{cases} E_x = E_{0x}\cos(wt - kz - d_x) \\ E_y = E_{0y}\cos(wt - kz - d_y) \\ E_z = 0 \end{cases}$$

$$\vec{E}(z,t) = \Re\left(\underline{E} e^{j(wt-kz)}\right)$$

Phasor = Jones Vector

$$\underline{E} = \begin{bmatrix} E_x = E_{ox} e^{jd_x} \\ E_y = E_{oy} e^{jd_y} \end{bmatrix}$$

Jones vector does not contain neither time nor propagation information

Polarization ellipse geometrical parameters

Absolute Phase $a = d_x$

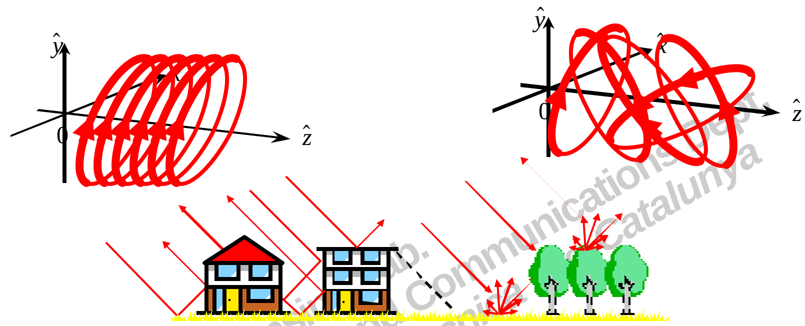
Orientation Angle $\tan 2f = 2\frac{E_{0x}E_{0y}}{E_{0x}^2 - E_{0y}^2}\cos d$

Ellipticity Angle $\sin 2t = 2\frac{E_{0x}E_{0y}}{E_{0x}^2 + E_{0y}^2}\sin d$

Wave Amplitude $A = \sqrt{E_{0x}^2 + E_{0y}^2}$

Polarization handedness Sign(t)

Wave Polarimetry



Deterministic Scattering

Completely Polarized Wave

Random Scattering

Partially Polarized Wave

Polarisation Ellipse varies in time/space
Amplitude, Phase: Random processes

Statistical description needed

Wave Polarimetry

Random electromagnetic fields ➡ $E\{E_x\}=0$, $E\{E_y\}=0$

Wave polarization descriptors based on first order moments (field) not valid to describe partially polarized waves

☒ Jones vector varies in time/space $\underline{E}(t)=\begin{bmatrix} E_{ox}(t)e^{j\delta_x(t)} \\ E_{oy}(t)e^{j\delta_y(t)} \end{bmatrix}$

⬇

Wave polarization descriptors based on higher order moments

☑ Stokes Vector

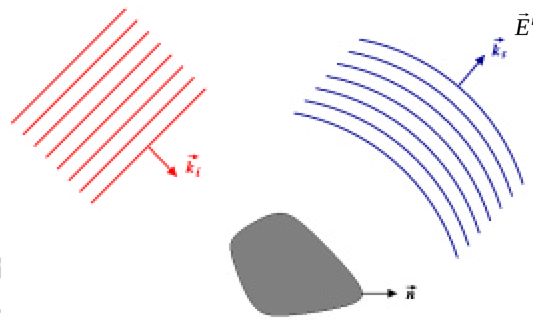
☑ Wave Covariance Matrix

Polarimetric Wave Scattering

What is the objective to employ wave polarimetry?

Incident Wave

$\vec{E}^i(\vec{r}) = \vec{E}_0^i e^{(j k_i r)}$



Far Field Approximation

$\vec{E}^i(\vec{r}) = \vec{E}_0^s e^{(j k_s r)}$

$|\vec{r}| >> |\vec{r}'|$

and

$|\vec{r}| >> l$

The scatterer under study reacts differently to incident waves different polarization states. Hence, the relation between the incident and the scattered waves permits to analyze and characterize the scatterer.

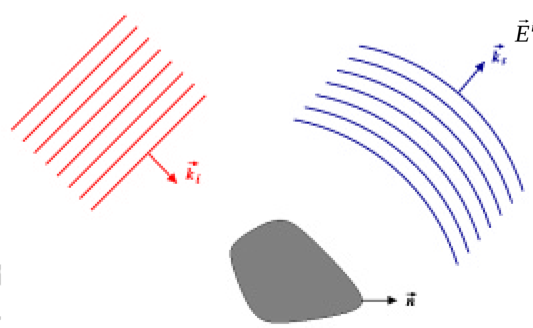
SAR Polarimetry
PolSAR

Polarimetric Wave Scattering

What is the objective to employ wave polarimetry?

Incident Wave

$\vec{E}^i(\vec{r}) = \vec{E}_0^i e^{(j k_i r)}$



Far Field Approximation

$\vec{E}^i(\vec{r}) = \vec{E}_0^s e^{(j k_s r)}$

$|\vec{r}| >> |\vec{r}'|$

and

$|\vec{r}| >> l$

$$\begin{bmatrix} E_\perp^s(\vec{r}) \\ E_{//}^s(\vec{r}) \end{bmatrix} = \frac{e^{jkr}}{r} \begin{bmatrix} S_{\perp\perp}(\vec{r}) & S_{\perp //}(\vec{r}) \\ S_{//\perp}(\vec{r}) & S_{////}(\vec{r}) \end{bmatrix} \begin{bmatrix} E_\perp^i(\vec{r}) \\ E_{//}^i(\vec{r}) \end{bmatrix}$$

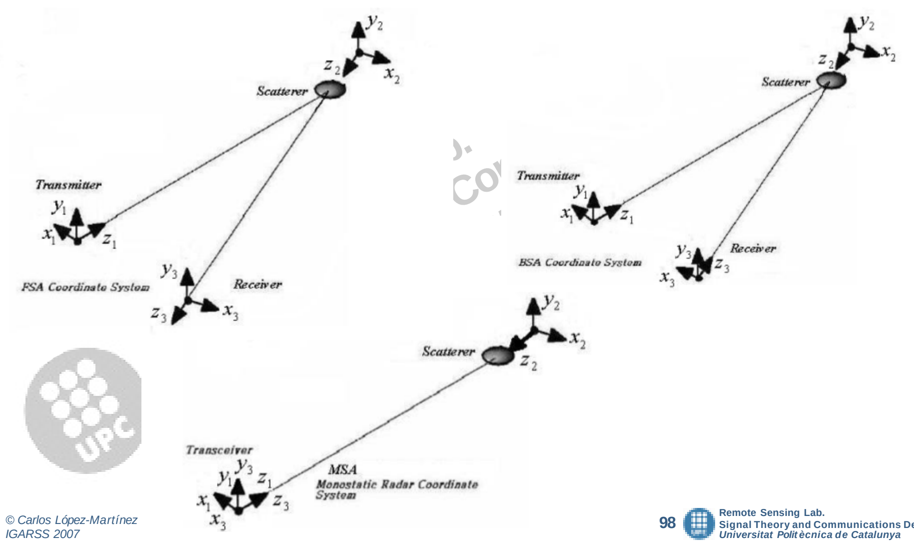
Scattered Jones Vector

2x2 Complex Scattering matrix

Incident Jones Vector

Polarimetric Wave Scattering

It is necessary to establish a scattering coordinate framework since the scattering problem deals with (complex vectorial) quantities



Polarimetric Wave Scattering

Bistatic Case: Scattering or Jones matrix

$$\begin{bmatrix} E_X^s \\ E_Y^s \end{bmatrix} = \frac{e^{jkr}}{r} \begin{bmatrix} S_{XX} & S_{XY} \\ S_{YX} & S_{YY} \end{bmatrix} \begin{bmatrix} E_X^i \\ E_Y^i \end{bmatrix}$$

- X and Y are a pair of orthogonal polarization states
- Defined in the local coordinates system
- [S] is independent of the polarisation of the incident wave
- [S] is dependent on the frequency, the geometrical and electrical properties of the scatterer
- Total scattered power

$$Span(\mathbf{S}) = Trace\left(\mathbf{SS}^H\right) = |S_{XX}|^2 + |S_{XY}|^2 + |S_{YX}|^2 + |S_{YY}|^2$$

Polarimetric Wave Scattering

$$\mathbf{S}=\frac{e^{jkr}}{r}\begin{bmatrix} S_{XX} & S_{XY} \\ S_{YX} & S_{YY} \end{bmatrix}=\frac{e^{jkr}}{r}\begin{bmatrix} |S_{XX}|e^{j\phi_{XX}} & |S_{XY}|e^{j\phi_{XY}} \\ |S_{YX}|e^{j\phi_{YX}} & |S_{YY}|e^{j\phi_{YY}} \end{bmatrix}$$

Absolute Scattering Matrix

$$\mathbf{S}=\frac{e^{jkr}e^{j\phi_{XX}}}{r}\begin{bmatrix} |S_{XX}| & |S_{XY}|e^{j(\phi_{XY}-\phi_{XX})} \\ |S_{YX}|e^{j(\phi_{YX}-\phi_{XX})} & |S_{YY}|e^{j(\phi_{YY}-\phi_{XX})} \end{bmatrix}$$

Absolute phase factor

Relative Scattering Matrix
Seven Parameters: 4 amplitudes and 3 phase

SCATTERER POLARIMETRIC DIMENSION = 7

Polarimetric Wave Scattering

Monostatic Case: Backscattering or Sinclair matrix

- In the case of Backscattering from reciprocal scatterers

RECIPROCITY THEOREM $\quad S_{XY}^{BSA}=S_{YX}^{BSA} \Leftrightarrow S_{XY}^{FSA}=-S_{YX}^{FSA}$

$$\begin{bmatrix} E_X^s \\ E_Y^s \end{bmatrix}=\frac{e^{jkr}}{r}\begin{bmatrix} S_{XX} & S_{XY} \\ S_{XY} & S_{YY} \end{bmatrix}\begin{bmatrix} E_X^i \\ E_Y^i \end{bmatrix}$$

(BSA CONVENTION)

- Total scattered power

$$Span(\mathbf{S})=Trace\left(\mathbf{S}\mathbf{S}^{T*}\right)=|S_{XX}|^2+2|S_{XY}|^2+|S_{YY}|^2$$

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Polarimetric Wave Scattering

$$\mathbf{S} = \frac{e^{jkr}}{r}\begin{bmatrix} S_{XX} & S_{XY} \\ S_{XY} & S_{YY} \end{bmatrix} = \frac{e^{jkr}}{r}\begin{bmatrix} |S_{XX}|e^{j\phi_{XX}} & |S_{XY}|e^{j\phi_{XY}} \\ |S_{XY}|e^{j\phi_{XY}} & |S_{YY}|e^{j\phi_{YY}} \end{bmatrix}$$

Absolute Scattering Matrix

$$\mathbf{S} = \frac{e^{jkr}e^{j\phi_{XX}}}{r}\begin{bmatrix} |S_{XX}| & |S_{XY}|e^{j(\phi_{XY}-\phi_{XX})} \\ |S_{XY}|e^{j(\phi_{XY}-\phi_{XX})} & |S_{YY}|e^{j(\phi_{YY}-\phi_{XX})} \end{bmatrix}$$

Absolute phase factor

Relative Scattering Matrix
Seven Parameters: 3 amplitudes and 2 phase

SCATTERER POLARIMETRIC DIMENSION = 5

Polarimetric Wave Scattering

Vectorial formulation of the scattering problem

$$\mathbf{S} = \begin{bmatrix} S_{XX} & S_{XY} \\ S_{YX} & S_{YY} \end{bmatrix} \quad \Rightarrow \quad \mathbf{k} = V(\mathbf{S}) = \frac{1}{2}Trace(\mathbf{S}\ ?) = \begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \end{bmatrix} \in \mathbf{C}_4$$

Vectorization process

Scattering vector

With: $V(\mathbf{S})$ Matrix Vectorization Operator
? Set of orthogonal 2x2 matrices

Frobenius norm of: $\mathbf{S}$

$$\|\mathbf{S}\|^2 = \mathbf{S}^H \cdot \mathbf{S} = |S_1|^2 + |S_2|^2 + |S_3|^2 + |S_4|^2$$
$$= Span(\mathbf{S}) = |S_{XX}|^2 + |S_{YX}|^2 + |S_{XY}|^2 + |S_{YY}|^2$$

Polarimetric Wave Scattering

Pauli scattering vector $\mathbf{k}_p = V(\mathbf{S}) = \frac{1}{2} Trace(\mathbf{S}\ ?_P)$

Set of 2x2 complex matrices from the Pauli matrices group

$$?_P = \left\{ \sqrt{2}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}, \sqrt{2}\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}, \sqrt{2}\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}, \sqrt{2}\begin{bmatrix}0 & -j\\ j & 0\end{bmatrix} \right\}$$

⬇

Bistatic case $\mathbf{k}_p = \frac{1}{\sqrt{2}}\begin{bmatrix} S_{XX}+S_{YY} \\ S_{XX}-S_{YY} \\ S_{XY}+S_{YX} \\ j(S_{XY}-S_{YX}) \end{bmatrix}$ Monostatic case $\mathbf{k}_p = \frac{1}{\sqrt{2}}\begin{bmatrix} S_{XX}+S_{YY} \\ S_{XX}-S_{YY} \\ 2S_{XY} \end{bmatrix}$

Advantage: Closer related to physical properties of the scatterer

Note: Also known as $\mathbf{k}_{4P}$ or $\mathbf{k}_{3P}$

Polarimetric Wave Scattering

Lexicographic scattering vector $\mathbf{k} = V(\mathbf{S}) = \frac{1}{2} Trace(\mathbf{S}\ ?_L)$

Set of 2x2 complex matrices from the Lexicographic matrices group

$$?_L = \left\{ 2\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix}, 2\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}, 2\begin{bmatrix}0 & 0\\ 1 & 0\end{bmatrix}, 2\begin{bmatrix}0 & 0\\ 0 & 1\end{bmatrix} \right\}$$

⬇

Bistatic case $\mathbf{k} = \begin{bmatrix} S_{XX} \\ S_{XY} \\ S_{YX} \\ S_{YY} \end{bmatrix}$ Monostatic case $\mathbf{k} = \begin{bmatrix} S_{XX} \\ \sqrt{2}S_{XY} \\ S_{YY} \end{bmatrix}$

Advantage: Directly related to the system measurables

Note: Also known as $\mathbf{k}_{4L}$ or $\mathbf{k}_{3L}$

Polarimetric Wave Scattering

Bistatic Pauli scattering vector $\mathbf{k}_p = \frac{1}{\sqrt{2}}\left[S_{XX}+S_{YY} \quad S_{XX}-S_{YY} \quad S_{XY}+S_{YX} \quad j(S_{XY}-S_{YX})\right]^T$

Coherency matrix ➡ $\mathbf{T}=\mathbf{k}_p\mathbf{k}_p^{*} = \begin{bmatrix} 2A_0 & C-jD & H+jG & L-jK \\ C+jD & B_0+B & E+jF & M-jN \\ H-jG & E-jF & B_0-B & J+jI \\ L+jK & M+jN & J-jI & 2A \end{bmatrix}$

Monostatic Pauli scattering vector $\mathbf{k} = \frac{1}{\sqrt{2}}\left[S_{XX}+S_{YY} \quad S_{XX}-S_{YY} \quad 2S_{XY}\right]^T$

Coherency matrix ➡ $\mathbf{T}=\mathbf{k}_p\mathbf{k}_p^{*} = \begin{bmatrix} 2A_0 & C-jD & H+jG \\ C+jD & B_0+B & E+jF \\ H-jG & E-jF & B_0-B \end{bmatrix}$

Hermitian positive semi definite matrix – Rank 1

Polarimetric Wave Scattering

Bistatic Lexicographic scattering vector $\mathbf{k} = \left[S_{XX} \quad S_{XY} \quad S_{YX} \quad S_{YY}\right]^T$

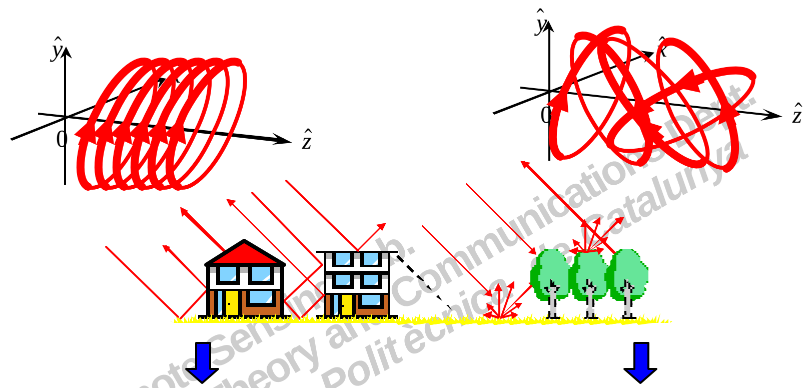
Covariance matrix ➡ $\mathbf{C} = \mathbf{k}\mathbf{k}^H = \begin{bmatrix} S_{XX}S_{XX}^* & S_{XX}S_{XY}^* & S_{XX}S_{YX}^* & S_{XX}S_{YY}^* \\ S_{XY}S_{XX}^* & S_{XY}S_{XY}^* & S_{XY}S_{YX}^* & S_{XY}S_{YY}^* \\ S_{YX}S_{XX}^* & S_{YX}S_{XY}^* & S_{YX}S_{YX}^* & S_{YX}S_{YY}^* \\ S_{YY}S_{XX}^* & S_{YY}S_{XY}^* & S_{YY}S_{YX}^* & S_{YY}S_{YY}^* \end{bmatrix}$

Monostatic Lexicographic scattering vector $\mathbf{k} = \left[S_{XX} \quad \sqrt{2}S_{XY} \quad S_{YY}\right]^T$

Covariance matrix ➡ $\mathbf{C} = \mathbf{k}\mathbf{k}^H = \begin{bmatrix} S_{XX}S_{XX}^* & \sqrt{2}S_{XX}S_{XY}^* & S_{XX}S_{YY}^* \\ \sqrt{2}S_{XY}S_{XX}^* & 2S_{XY}S_{XY}^* & \sqrt{2}S_{XY}S_{YY}^* \\ S_{YY}S_{XX}^* & \sqrt{2}S_{YY}S_{XY}^* & S_{YY}S_{YY}^* \end{bmatrix}$

Hermitian positive semi definite matrix – Rank 1

Polarimetric Wave Scattering



Deterministic Scattering

Completely Polarized Scattering

Random Scattering

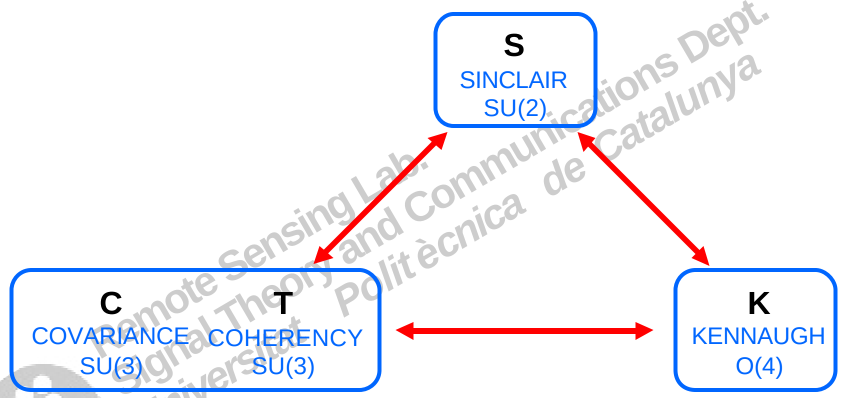
Partially Polarized Scattering

Can not be described by $\mathbf{S}, \mathbf{k}_p, \mathbf{k}$

Necessity of statistical descriptors

Polarimetric Wave Scattering

Relations for deterministic scattering



All polarimetric descriptors contain the same information !!!

All polarimetric descriptors the same number of independent parameters !!!
(Bistatic 7 – Monostatic 5)

Polarimetric Wave Scattering

Relations for random scattering

Independent parameters !!!
(Bistatic 7 – Monostatic 5)
Absolute phase is not considered

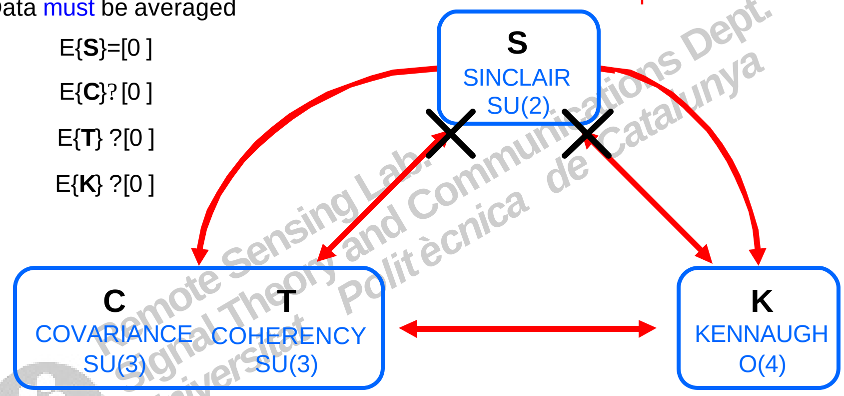
Data must be averaged

E{**S**}=[0]

E{**C**}? [0]

E{**T**} ?[0]

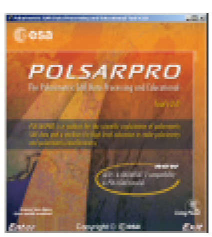
E{**K**} ?[0]



Independent parameters !!!
(Bistatic 16 – Monostatic 9)

[S] is not a statistical descriptor, it is a zero-mean multidimensional random value !!!

Software: PolSARPro v3.0



PolSARpro v3.0 Software:

Tool specifically designed to handle Polarimetric data.

Educational Software offering a tool for self-education in the field of Polarimetric SAR and Interferometric Polarimetric SAR data processing and analysis.

Developed to be accessible to a wide range of users, from novices to experts in the field of Polarimetry.

Courtesy of Dr. E: Pottier

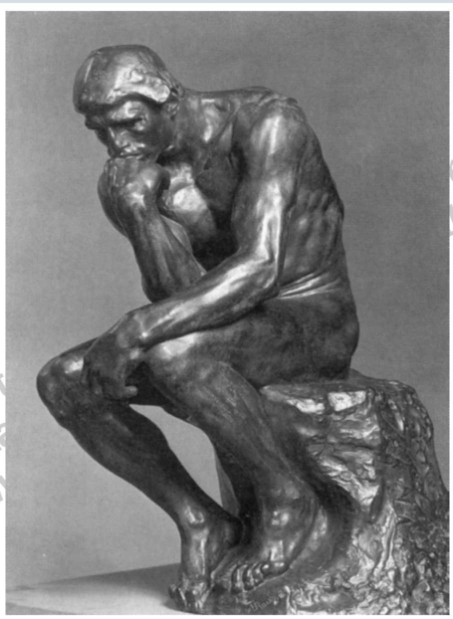
Software: PolSARPro v3.0

Visit regularly the Web Site
http://earth.esa.int/polsarpro

The Web Site provides

- **Details of the project**
- **Access to the tutorial and software**
- **Information about status of the development**
- **Demonstration Sample Datasets**
- **Recently obtained results**

Courtesy of Dr. E: Pottier



Polarimetric SAR Interferometry

Polarimetric SAR Interferometry

Polarimetry and Interferometry are complementary information

|HH+VV| |HH-VV| |HV|

DLR E-SAR L Band
POL-InSAR (1.5m x 3m)
Baseline 5m

$|g_{VV}|$

0 1

- Polarimetry provides sensitivity to scattering mechanisms
- Interferometry provides sensitivity to height information

Location of scattering mechanism in height

Polarimetric SAR Interferometry

PolInSAR data mathematical representation

- (6×6) coherency matrix $\langle \mathbf{T}_6 \rangle$

$$\mathbf{k} = \begin{bmatrix} \mathbf{k}_1 \\ \mathbf{k}_2 \end{bmatrix} \Rightarrow \langle \mathbf{T}_6 \rangle = \langle \mathbf{k}\mathbf{k}^\dagger \rangle = \begin{bmatrix} \langle \mathbf{T}_1 \rangle & \langle \mathbf{O}_{12} \rangle \\ \langle \mathbf{O}_{12} \rangle^\dagger & \langle \mathbf{T}_2 \rangle \end{bmatrix}$$

$\langle \mathbf{T}_1 \rangle$ and $\langle \mathbf{T}_2 \rangle$ separate image coherency matrices

$\langle \mathbf{O}_{12} \rangle$ correlation matrix

- Coherence set in the radar polarization basis

$$(r_1, r_2, r_3) \text{ with } r_i = \frac{\langle k_{1i} \; k_{2i}^* \rangle}{\sqrt{\langle k_{1i} \; k_{1i}^* \rangle \langle k_{2i} \; k_{2i}^* \rangle}} \text{ for } i = 1,2,3$$

Polarimetric SAR Interferometry

PolInSAR data mathematical representation

- Coherence representation using projection vectors

$$k_{1i} = \mathbf{w}_i^{*T}\mathbf{k}_1, \; k_{2i} = \mathbf{w}_i^{*T}\mathbf{k}_2 \qquad r_i = \frac{\langle k_{1i} \; k_{2i}^* \rangle}{\sqrt{\langle k_{1i} \; k_{1i}^* \rangle \langle k_{2i} \; k_{2i}^* \rangle}}$$

$$r_i = \frac{\langle \mathbf{w}_i^{*T}\mathbf{k}_1 \; \mathbf{k}_2^{*T}\mathbf{w}_i \rangle}{\sqrt{\langle \mathbf{w}_i^{*T}\mathbf{k}_1 \; \mathbf{k}_1^{*T}\mathbf{w}_i \rangle \langle \mathbf{w}_i^{*T}\mathbf{k}_2 \; \mathbf{k}_2^{*T}\mathbf{w}_i \rangle}} = \frac{\mathbf{w}_i^{*T} \langle \mathbf{O}_{12} \rangle \mathbf{w}_i}{\sqrt{\mathbf{w}_i^{*T} \langle \mathbf{T}_{11} \rangle \mathbf{w}_i \mathbf{w}_i^{*T} \langle \mathbf{T}_{22} \rangle \mathbf{w}_i}}$$

- Coherence set in an arbitrary polarization basis

$$\mathbf{k}_1^{'} = \mathbf{U}_{SU3}\mathbf{k}_1, \; \mathbf{k}_2^{'} = \mathbf{U}_{SU3}\mathbf{k}_2 \qquad r(\mathbf{w}) = \frac{\mathbf{w}^{*T} \langle \mathbf{O}_{12} \rangle \mathbf{w}}{\sqrt{\mathbf{w}^{*T} \langle \mathbf{T}_{11} \rangle \mathbf{w} \; \mathbf{w}^{*T} \langle \mathbf{T}_{22} \rangle \mathbf{w}}}$$

- Coherence set in different emission/reception polarization basis

$$r(\mathbf{w}_1, \mathbf{w}_2) = \frac{\mathbf{w}_1^{*T} \langle \mathbf{O}_{12} \rangle \mathbf{w}_2}{\sqrt{\mathbf{w}_1^{*T} \langle \mathbf{T}_{11} \rangle \mathbf{w}_1 \mathbf{w}_2^{*T} \langle \mathbf{T}_{22} \rangle \mathbf{w}_2}}$$

Polarimetric SAR Interferometry

Optimal coherence set

Interferometric coherence $r(\mathbf{w}_1,\mathbf{w}_2)=\dfrac{\mathbf{w}_1^{*T}\langle\mathbf{O}_{12}\rangle\mathbf{w}_2}{\sqrt{\mathbf{w}_1^{*T}\langle\mathbf{T}_{11}\rangle\mathbf{w}_1\mathbf{w}_2^{*T}\langle\mathbf{T}_{22}\rangle\mathbf{w}_2}}$

Which polarization combination leads to the maximum possible interferometric coherence?

$$L=\mathbf{w}_1^{*T}\langle\mathbf{O}_{12}\rangle\mathbf{w}_2+l_1\left(\mathbf{w}_1^{*T}\langle\mathbf{T}_{11}\rangle\mathbf{w}_1-C_1\right)+l_2\left(\mathbf{w}_2^{*T}\langle\mathbf{T}_{22}\rangle\mathbf{w}_2-C_2\right)$$

$$\max_{\mathbf{w}_1,\mathbf{w}_2}\left(LL^*\right)$$

$$\frac{\partial L}{\partial\mathbf{w}_1^{*T}}=\langle\mathbf{O}_{12}\rangle\mathbf{w}_2+l_1\langle\mathbf{T}_{11}\rangle\mathbf{w}_1=0$$

$$\frac{\partial L^*}{\partial\mathbf{w}_2^{*T}}=\langle\mathbf{O}_{12}\rangle^{*T}\mathbf{w}_1+l_2^*\langle\mathbf{T}_{22}\rangle\mathbf{w}_2=0$$

⇒

$$\langle\mathbf{T}_{11}\rangle^{-1}\langle\mathbf{O}_{12}\rangle\langle\mathbf{T}_{22}\rangle^{-1}\langle\mathbf{O}_{12}\rangle^{*T}\mathbf{w}_1=n\ \mathbf{w}_1$$

$$\langle\mathbf{T}_{22}\rangle^{-1}\langle\mathbf{O}_{12}\rangle^{*T}\langle\mathbf{T}_{11}\rangle^{-1}\langle\mathbf{O}_{12}\rangle\mathbf{w}_2=n\ \mathbf{w}_2$$

These matrices are not hermitian, but $n=l_1l_2^*$ is real

Polarimetric SAR Interferometry

Optimal coherence set

Coherence optimisation

$$\langle\mathbf{T}_{11}\rangle^{-1}\langle\mathbf{O}_{12}\rangle\langle\mathbf{T}_{22}\rangle^{-1}\langle\mathbf{O}_{12}\rangle^{*T}\mathbf{w}_1=n\ \mathbf{w}_1$$

$$\langle\mathbf{T}_{22}\rangle^{-1}\langle\mathbf{O}_{12}\rangle^{*T}\langle\mathbf{T}_{11}\rangle^{-1}\langle\mathbf{O}_{12}\rangle\mathbf{w}_2=n\ \mathbf{w}_2$$

⇓

Three real eigenvalues $n_1\geq n_2\geq n_3$

Three pairs of eigenvectors $(\mathbf{w}_{11},\mathbf{w}_{12}),(\mathbf{w}_{21},\mathbf{w}_{22}),(\mathbf{w}_{31},\mathbf{w}_{32})$

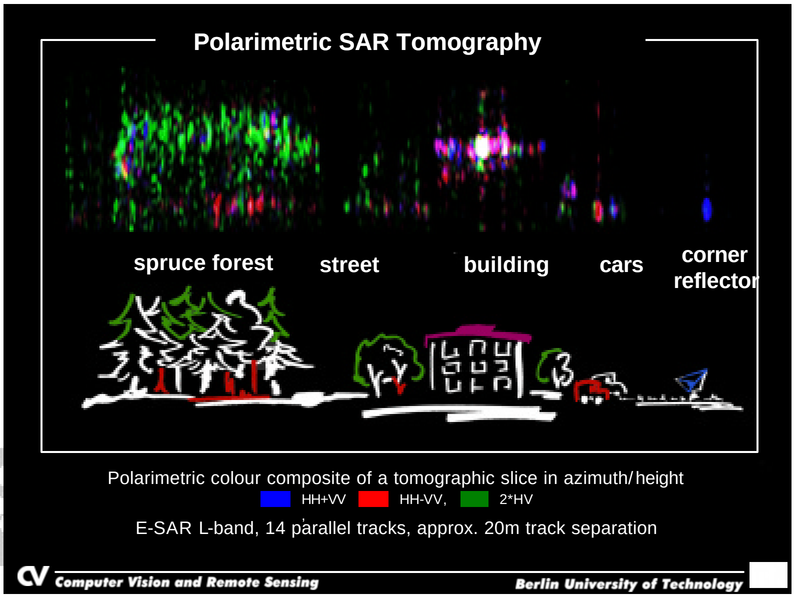
- Optimum coherence values $r_i=\sqrt{n_i}$
- Optimum scattering mechanisms $(\mathbf{w}_{i1},\mathbf{w}_{i2})$

Image formation $m_1=\mathbf{w}_1^{*T}\mathbf{k}_1 \quad m_2=\mathbf{w}_2^{*T}\mathbf{k}_2$

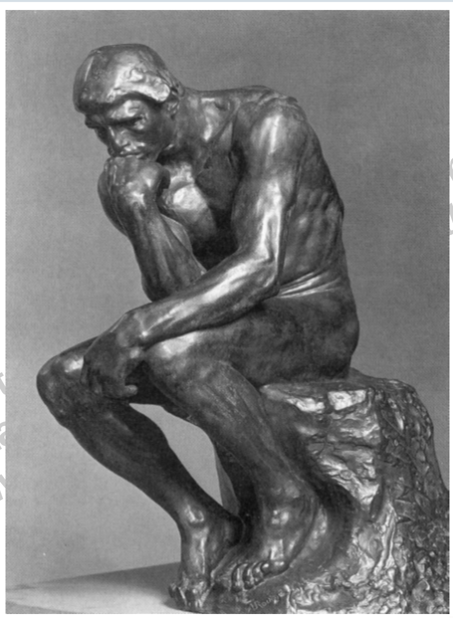
Eigen phase normalisation $f_e=\arg(\mathbf{w}_1^{*T}\mathbf{w}_2)=0$

Interferogram $m_1m_2^*=\left(\mathbf{w}_1^{*T}\mathbf{k}_1\right)\left(\mathbf{w}_2^{*T}\mathbf{k}_2\right)^{*T}=\mathbf{w}_1^{*T}\langle\mathbf{O}_{12}\rangle\mathbf{w}_2$

Polarimetric SAR Tomography



Courtesy of Dr. A. Reigber



Multidimensional SAR Data Models

Multidimensional SAR Data Models

Multidimensional SAR systems present a wide spectra of configurations

- SAR Interferometry
- Differential SAR interferometry
- SAR Polarimetry
- Polarimetric SAR Interferometry
- SAR Tomography/ Multibaseline
- Multitemporal SAR
- Multifrequency SAR

How to deal with all these types of configurations?

- Statistics: Speckle noise imposes the use of statistical descriptors
  - Speckle affects the SAR images
  - Speckle affects the correlation structure
- Physics: The interpretation of the data must be done according to the physics behind the imaging process

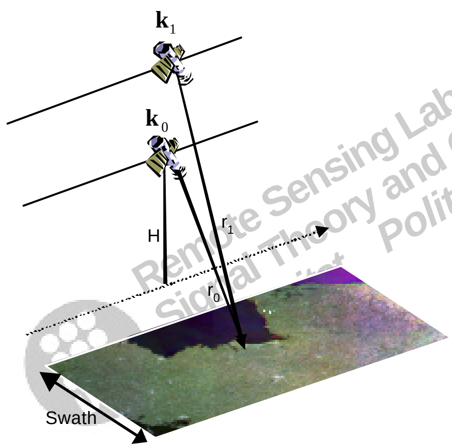
Electromagnetic Signal Processing

Multidimensional SAR Data Models

Mathematical Representation

The multidimensional SAR system acquires *m* complex SAR images

Target vector $\mathbf{k} = [S_1, S_2, \ldots, S_m]^T$



The properties of the target vector follow from the properties of a single SAR image

- **k** is deterministic for point scatterers. It contains all the necessary information to characterize the scatterer
- **k** is a multidimensional random variable for distributed scatterers due to speckle. A single sample does not characterize the scatterer

SAR images characterized through second order moments

- Second order moments in multidimensional SAR data are matrix quantities

Multidimensional SAR Data Models

Mathematical Representation

PDF for non-correlated SAR images (Distributed scatterers)

- Zero-mean multidimensional complex (also circular) Gaussian pdf

$$p_{\mathbf{k}}(\mathbf{k}) = \prod_{k=1}^{m} \frac{1}{\pi\sigma^2} \exp\left(-\frac{S_k S_k^H}{\sigma^2}\right) = \frac{1}{\pi^m \sigma^{2m}} \exp\left(-\sum_{k=1}^{m} \frac{S_k S_k^H}{\sigma^2}\right) = \frac{1}{\pi^m \sigma^{2m}} \exp\left(-\frac{1}{\sigma^2} \mathrm{tr}\left(\mathbf{k}\mathbf{k}^H\right)\right)$$

Independent SAR images with the same power $S_k = \mathcal{N}_{C^2}\left(0, \sigma^2/2\right)$

- First order moment

$$E\{\mathbf{k}\} = \mathbf{0}$$

- Second order moment: Covariance matrix

$$\mathbf{C} = E\{\mathbf{k}\mathbf{k}^H\} = \sigma^2 \mathbf{I}_{m\times m}$$

Multidimensional Gaussian pdf Properties

Characterization of random variables

- Probability Density Function (pdf)
- Moment-generating function
- Statistical moments (mean, power, kurtosis, skewness...)

Zero-mean multidimensional complex Gaussian pdf

$$p_{\mathbf{k}}(\mathbf{k}) = \frac{1}{\pi^{m}|\mathbf{C}|}\exp\left(-\mathbf{k}^{H}\mathbf{C}^{-1}\mathbf{k}\right)$$

- First order moment $E\{\mathbf{k}\} = \mathbf{0}$
- Second order moment: Covariance matrix

$$\mathbf{C} = E\{\mathbf{k}\mathbf{k}^{H}\} = \begin{bmatrix} E\{S_1 S_1^H\} & E\{S_1 S_2^H\} & \cdots & E\{S_1 S_m^H\} \\ E\{S_2 S_1^H\} & E\{S_2 S_2^H\} & \cdots & E\{S_2 S_m^H\} \\ \vdots & \vdots & \ddots & \vdots \\ E\{S_m S_1^H\} & E\{S_m S_2^H\} & \cdots & E\{S_m S_m^H\} \end{bmatrix}$$

$E\{S_k S_l^*\} \neq 0 \quad k,l \in \{1,\ldots,m\},\ k \neq l$

Correlated SAR images

Multidimensional Gaussian pdf Properties

A zero-mean multidimensional complex Gaussian pdf is completely characterized by the second order moments, i.e., the covariance matrix

- Moment theorem for complex Gaussian processes, given Q correlated SAR images
  - For $k \neq l$, where m_k and n_l are integers from $\{1,2,\ldots,Q\}$

    $$E\left\{S_{m_1}S_{m_2}\cdots S_{m_k}S_{n_1}^{*}S_{n_2}^{*}\cdots S_{n_l}^{*}\right\} = 0$$

  - For $k = l$, where p is a permutation of the set of integers $\{1,2,\ldots,Q\}$

    $$E\left\{S_{m_1}S_{m_2}\cdots S_{m_k}S_{n_1}^{*}S_{n_2}^{*}\cdots S_{n_l}^{*}\right\} = \sum_{p} E\left\{S_{m_{p(1)}}S_{n_1}^{*}\right\}E\left\{S_{m_{p(2)}}S_{n_2}^{*}\right\}\cdots E\left\{S_{m_{p(l)}}S_{n_l}^{*}\right\}$$

- Considering the covariance matrix
  - Higher order moments are function of the covariance matrix

Remote Sensing Lab. Signal Theory and Communications Dept. Universitat Politècnica de Catalunya

Mathematical Representation

The covariance matrix contains the correlation structure of the set of *m* SAR images

$$\mathbf{C} = E\{\mathbf{k}\mathbf{k}^H\} = \begin{bmatrix} E\{S_1 S_1^H\} & E\{S_1 S_2^H\} & \cdots & E\{S_1 S_m^H\} \\ E\{S_2 S_1^H\} & E\{S_2 S_2^H\} & \cdots & E\{S_2 S_m^H\} \\ \vdots & \vdots & \ddots & \vdots \\ E\{S_m S_1^H\} & E\{S_m S_2^H\} & \cdots & E\{S_m S_m^H\} \end{bmatrix}$$

Information

- Diagonal elements: Power information

$$E\{S_k S_k^H\} = E\{|S_k|^2\} \quad k \in \{1,2,\ldots,m\}$$

- Off diagonal elements: Correlation information

$$E\{S_k S_l^H\} \quad k,l \in \{1,2,\ldots,m\}, k \neq l$$

Mathematical Representation

PDF for correlated SAR images (Distributed scatterers)

- Zero-mean multidimensional complex Gaussian pdf

$$p_{\mathbf{k}}(\mathbf{k}) = \frac{1}{p^m |\mathbf{C}|} \exp\left(-\mathbf{k}^H \mathbf{C}^{-1} \mathbf{k}\right)$$

- First order moment

$$E\{\mathbf{k}\} = \mathbf{0}$$

- Second order moment: Covariance matrix

$$\mathbf{C} = E\{\mathbf{k}\mathbf{k}^H\} = \begin{bmatrix} E\{S_1 S_1^H\} & E\{S_1 S_2^H\} & \cdots & E\{S_1 S_m^H\} \\ E\{S_2 S_1^H\} & E\{S_2 S_2^H\} & \cdots & E\{S_2 S_m^H\} \\ \vdots & \vdots & \ddots & \vdots \\ E\{S_m S_1^H\} & E\{S_m S_2^H\} & \cdots & E\{S_m S_m^H\} \end{bmatrix}$$

All the information characterizing the set of *m* SAR Images is contained in the covariance matrix

Multidimensional SAR Data Models

Complex Correlation Coefficient

How to consider the correlation information

- Off-diagonal covariance matrix elements

$$E\{S_k S_l^H\} \quad k,l \in \{1,2,\ldots,m\},\ k \neq l$$

  - Absolute correlation information

- Complex correlation coefficient

$$r_{k,l} = \frac{E\{S_k S_l^*\}}{\sqrt{E\{|S_k|^2\} \cdot E\{|S_l|^2\}}} = |r_{k,l}| e^{jq_{k,l}} \qquad 0 \leq |r_{k,l}| \leq 1 \text{ Coherence} \qquad -p \leq q_{k,l} \leq p$$

  - Normalized correlation information

- The complex correlation information represents the most important observable for multidimensional SAR data. Its physical interpretation depends on the multidimensional SAR system configuration

Complex Correlation Coefficient

Information Content

- SAR Interferometry
  - Phase $q_{k,l}$ contains topographic information
  - Coherence $|r_{k,l}|$ is sensitive to different properties of the imaged area
    - Study and retrieval of stem volume over forested areas
    - Study of dry and wet snow covered areas
    - Characterization of glaciers, valleys, and fjord ice
- SAR Polarimetry
  - Off-diagonal information related with the geometry and the electrical properties of the target being imaged
- Polarimetric SAR Interferometry
  - Complex correlation coefficient related with the vegetation height and the vegetation structural properties

Multidimensional SAR Data Models

Mathematical Representation

Multidimensional SAR data descriptors for distributed scatterers

First order moment | Second order moment

$$\mathbf{S}=\begin{bmatrix} S_{hh} & S_{hv} \\ S_{hv}^{H} & S_{vv} \end{bmatrix}$$

Scattering matrix

$$\mathbf{kk}^{H}=\begin{bmatrix} S_{hh}S_{hh}^{H} & \sqrt{2}S_{hh}S_{hv}^{H} & S_{hh}S_{vv}^{H} \\ \sqrt{2}S_{hv}S_{hh}^{H} & 2S_{hv}S_{hv}^{H} & \sqrt{2}S_{hv}S_{vv}^{H} \\ S_{vv}S_{hh}^{H} & \sqrt{2}S_{vv}S_{hv}^{H} & S_{vv}S_{vv}^{H} \end{bmatrix}$$

Covariance matrix

Description for PolSAR data

Point scatterers characterization

Description for generalized multidimensional SAR data

$$\mathbf{k}=\left[S_1,S_2,\ldots,S_m\right]^T$$

Target vector

$$\mathbf{kk}^{H}=\begin{bmatrix} S_1S_1^H & S_1S_2^H & \cdots & S_1S_m^H \\ S_2S_1^H & S_2S_2^H & \cdots & S_2S_m^H \\ \vdots & \vdots & \ddots & \vdots \\ S_mS_1^H & S_mS_2^H & \cdots & S_mS_m^H \end{bmatrix}$$

$$E\{\mathbf{k}\}=\mathbf{0}$$

Characterizes completely the data distribution

Distributed scatterers characterization

$$E\{\mathbf{kk}^H\}=\mathbf{C}$$

Covariance matrix

$$p_{\mathbf{k}}(\mathbf{k})=\frac{1}{p^m|\mathbf{C}|}\exp\left(-\mathbf{k}^H\mathbf{C}^{-1}\mathbf{k}\right)$$

Multidimensional SAR Data Models

Information Estimation/Filtering

For multidimensional SAR data, under the hypothesis of Gaussian scattering, all the information is contained in the covariance matrix

$$\mathbf{C}=E\{\mathbf{kk}^H\}=\begin{bmatrix} E\{S_1S_1^H\} & E\{S_1S_2^H\} & \cdots & E\{S_1S_m^H\} \\ E\{S_2S_1^H\} & E\{S_2S_2^H\} & \cdots & E\{S_2S_m^H\} \\ \vdots & \vdots & \ddots & \vdots \\ E\{S_mS_1^H\} & E\{S_mS_2^H\} & \cdots & E\{S_mS_m^H\} \end{bmatrix}$$

This matrix must be estimated from the available information

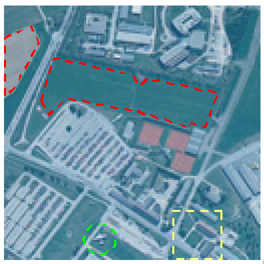
- The scattering vector for each pixel/sample of the SAR data

$$\mathbf{k}_i=\left[S_{1_i},S_{2_i},\ldots,S_{m_i}\right]^T$$

- The estimation process reduces to estimate the ensemble average (expectation operator) $E\{\}$
- The estimation process also receives the name of data filtering process. In case of homogeneous areas.

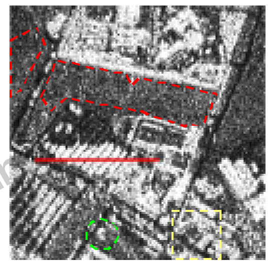
Information Estimation/Filtering

Considerations about speckle noise reduction



Optical image DLR OP

SAR images reflex the Nature's complexity



SAR image DLR OP

Homogeneous areas	Image details	Heterogeneous areas
Maintain useful information (s)	Maintain spatial details (Shape and value)	Maintain both
RADIOMETRIC RESOLUTION	SPATIAL RESOLUTION	LOCAL ANALYSIS

Image data: S_{hh} amplitude. E-SAR L-band system

Information Estimation

Multidimensional SAR data information estimation, i.e., data filtering, based on two main hypotheses

- **Ergodicity in mean**: The different time/space averages of each process converge to the same limit, i.e., the ensemble average $E\{\}$
  - The statistics in the realizations domain can be calculated in the time/spatial domain
  - Necessary to assume ergodicity since there are not multiple data realizations over the same area
  - Applied to the processes $E\left\{|S_k|^2\right\}$, $E\left\{|S_l|^2\right\}$ and $E\left\{S_k S_l^H\right\}$ $\quad k,l \in \{1,2,\ldots,m\}$
- **Wide-sense stationary**: Given a spatial domain all the samples in this spatial domain belong to the same statistical distribution
  - SAR images can not be considered as wide-sense stationary processes since they are a reflex of the data heterogeneity
  - SAR images can be considered locally wide-sense stationary
  - Applied to the processes $E\left\{|S_k|^2\right\}$, $E\left\{|S_l|^2\right\}$ and $E\left\{S_k S_l^H\right\}$ $\quad k,l \in \{1,2,\ldots,m\}$
- **Homogeneity**: Refers to non-textured data
  - Gaussian distributed data

Sample Covariance Matrix

Covariance matrix estimation by means of a MultiLook (BoxCar)

- Maximum likelihood estimator: Sample covariance matrix

$$\mathbf{Z}_n = \frac{1}{n}\sum_{k=1}^{n}\mathbf{k}\mathbf{k}^H = \begin{bmatrix} \frac{1}{n}\sum_{k=1}^{n} S_1(k)\,S_1^*(k) & \frac{1}{n}\sum_{k=1}^{n} S_1(k)S_2^*(k) & \cdots & \frac{1}{n}\sum_{k=1}^{n} S_1(k)\,S_m^*(k) \\ \frac{1}{n}\sum_{k=1}^{n} S_2(k)\,S_1^*(k) & \frac{1}{n}\sum_{k=1}^{n} S_2(k)\,S_2^*(k) & \cdots & \frac{1}{n}\sum_{k=1}^{n} S_2(k)\,S_m^*(k) \\ \vdots & \vdots & \ddots & \vdots \\ \frac{1}{n}\sum_{k=1}^{n} S_m(k)\,S_1^*(k) & \frac{1}{n}\sum_{k=1}^{n} S_m(k)\,S_2^*(k) & \cdots & \frac{1}{n}\sum_{k=1}^{n} S_m(k)S_m^*(k) \end{bmatrix}$$

  - n represents the total number of samples employed to estimate the covariance matrix, taken a region (square, rectangular, adapted…)
  - $\mathbf{Z}_n$ as estimator of $\mathbf{C}$
    - Does not consider signal morphology/heterogeneity
    - Loss of spatial resolution

The sample covariance matrix Z_n is itself a multidimensional random variable

Sample Covariance Matrix Distribution

The sample covariance matrix $\mathbf{Z}_n$ is characterized by the complex Wishart distribution $\mathbf{Z}_n \sim \mathcal{W}(n,\mathbf{C})$

$$p_{\mathbf{Z}_n}(\mathbf{Z}_n) = \frac{n^{mn}|\mathbf{Z}_n|^{n-m}}{|\mathbf{C}|^n\,\tilde{\Gamma}_m(n)}\operatorname{etr}\left(-n\mathbf{C}^{-1}\mathbf{Z}_n\right) \qquad \tilde{\Gamma}_m(n) = p^{m(m-1)/2}\Pi_{i=1}^{m}\Gamma(n-i+1)$$

- Multidimensional data distribution
- Valid for $n \geq m$, otherwise $|\mathbf{Z}_n|^{n-m}$ is equal to zero and the Wishart pdf is undetermined
  - Equivalent to $Rank(\mathbf{Z}_n) = m$, i.e., the sample covariance matrix is a full rank matrix
  - The higher the data dimensionality m the higher the number of looks n for the Wishart pdf to be defined

Multidimensional SAR Data Description

$$S_k = \mathcal{N}_{C^2}\left(0, s^2/2\right)$$

Single SAR image

→

$$\mathbf{k} = \left[S_1, S_2, \ldots, S_m\right]^T$$

Multidimensional SAR dataset

→

$$p_{\mathbf{k}}\left(\mathbf{k}\right) = \frac{1}{p^m\left|\mathbf{C}\right|}\exp\left(-\mathbf{k}^H\mathbf{C}^{-1}\mathbf{k}\right)$$

Multidimensional complex Gaussian pdf

↓

$$\mathbf{Z}_n = \frac{1}{n}\sum_{i=1}^{n}\mathbf{k}_i\mathbf{k}_i^H$$

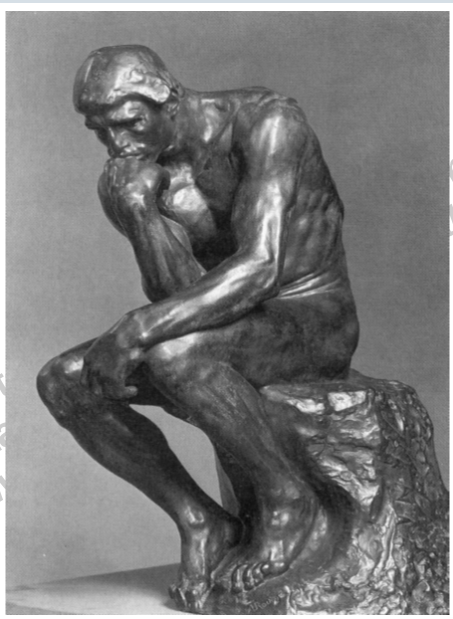
$$\mathbf{Z}_n \sim \mathcal{W}\left(n, \mathbf{C}\right)$$

Sample covariance matrix

↓

$$p_{\mathbf{Z}_n}\left(\mathbf{Z}_n\right) = \frac{n^{mn}\left|\mathbf{Z}_n\right|^{n-m}}{\left|\mathbf{C}\right|^n\tilde{\Gamma}_m\left(n\right)}\mathrm{etr}\left(-n\mathbf{C}^{-1}\mathbf{Z}_n\right)$$

Complex Wishart pdf



Multidimensional SAR Speckle Noise Models

Multidimensional Speckle Noise Model

$$\mathbf{kk}^H = \begin{bmatrix} S_1S_1^H & S_1S_2^H & \cdots & S_1S_m^H \\ S_2S_1^H & S_2S_2^H & \cdots & S_2S_m^H \\ \vdots & \vdots & \ddots & \vdots \\ S_mS_1^H & S_mS_2^H & \cdots & S_mS_m^H \end{bmatrix} = \begin{bmatrix} I_1 & S_1S_2^H & \cdots & S_1S_m^H \\ S_2S_1^H & I_2 & \cdots & S_2S_m^H \\ \vdots & \vdots & \ddots & \vdots \\ S_mS_1^H & S_mS_2^H & \cdots & I_m \end{bmatrix}$$

- Covariance matrix diagonal elements
  - Consist of the intensity of the SAR images
  - Speckle noise modelling/reduction as for one dimensional SAR images

$$I_k(x,r) = s_k(x,r)\,n_k(x,r) \qquad k \in \{1,2,\ldots,m\}$$

- Covariance matrix off-diagonal elements
  - Consist of the Hermitian products between pairs of SAR images

$$r_{k,l} = \frac{E\{S_k S_l^*\}}{\sqrt{E\{|S_k|^2\} \cdot E\{|S_l|^2\}}} = |r_{k,l}| e^{j\phi_{k,l}} \qquad k,l \in \{1,2,\ldots,m\}$$

  - A multiplicative speckle noise model can be also considered?

Multidimensional Speckle Noise Model

Multiplicative speckle noise model extension

- Model for a complex SAR image. Complex image model obtained from the intensity speckle model

$$S_k(x,r) = \sqrt{s_k(x,r)}\, e^{j\theta_k(x,r)} n_k(x,r) \qquad k \in \{1,2,\ldots,m\}$$

Construction of the covariance matrix terms

$$E\{S_k(x,r) S_l^H(x,r)\} = E\left\{\sqrt{s_k(x,r) s_l(x,r)}\, e^{j(\theta_k(x,r)-\theta_l(x,r))}\right\} E\{n_k(x,r) n_l^H(x,r)\} \quad k,l \in \{1,2,\ldots,m\}$$

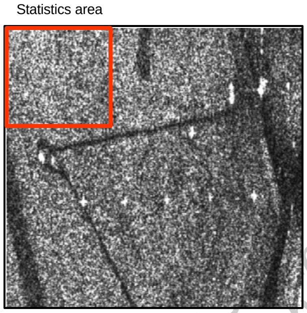
$$I_k = s_k n_k \qquad k \in \{1,2,\ldots,m\} \quad \overset{(?)}{\Rightarrow} \quad \hat{\mathbf{C}} = \begin{bmatrix} s_1^2 & \sqrt{s_1 s_2} e^{j(\theta_1-\theta_2)} & \cdots & \sqrt{s_1 s_m} e^{j(\theta_1-\theta_m)} \\ \sqrt{s_1 s_2} e^{j(\theta_2-\theta_1)} & s_2^2 & \cdots & \sqrt{s_2 s_m} e^{j(\theta_2-\theta_m)} \\ \vdots & \vdots & \ddots & \vdots \\ \sqrt{s_1 s_m} e^{j(\theta_m-\theta_1)} & \sqrt{s_2 s_m} e^{j(\theta_m-\theta_2)} & \cdots & s_m^2 \end{bmatrix} \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix}$$

Multidimensional extension

- Extension only possible for the extreme case $E\{\mathbf{n}\mathbf{n}^H\} = \mathbf{I}_{m\times m}$ as data's correlation is not considered

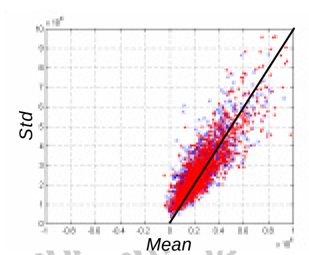
Multidimensional Speckle Noise Model

Statistics area



Grass area

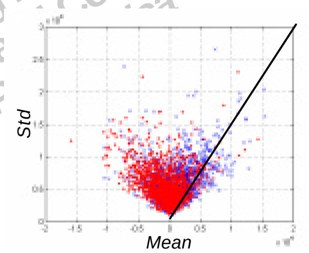
Statistics calculated over 7x7 pixel windows



Copolar

Blue: Real($S_{hh}S_{vv}$*)
Red: Imag($S_{hh}S_{vv}$*)

$|r_{hhvv}| = 0.77 e^{j\,0.807}$



Crosspolar

Blue: Real($S_{hh}S_{hv}$*)
Red: Imag($S_{hh}S_{hv}$*)

$|r_{hhhv}| = 0.118 e^{j\,0.638}$

S_{hh} amplitude
E-SAR L-band system

Multidimensional SAR Speckle Models — Multidimensional Speckle Noise Model

Objective of a multidimensional speckle noise model

Useful information ← C; Noise sources → $n_1, n_2, \ldots, n_p$

$$\mathbf{Z}_n = f\left(\mathbf{C}, n_1, n_2, \ldots, n_p\right)$$

- **Overcome** the limitations of the fully multiplicative speckle noise model. Noise model independent of the data dimensionality and valid for any correlation structure for the data
  - Observation: Any matrix entry consists of the Hermitian product of two complex SAR images

One-look sample covariance matrix $\mathbf{Z}_1 = \mathbf{k}\mathbf{k}^H = \begin{bmatrix} S_1S_1^H & S_1S_2^H & \cdots & S_1S_m^H \\ S_2S_1^H & S_2S_2^H & \cdots & S_2S_m^H \\ \vdots & \vdots & \ddots & \vdots \\ S_mS_1^H & S_mS_2^H & \cdots & S_mS_m^H \end{bmatrix}$

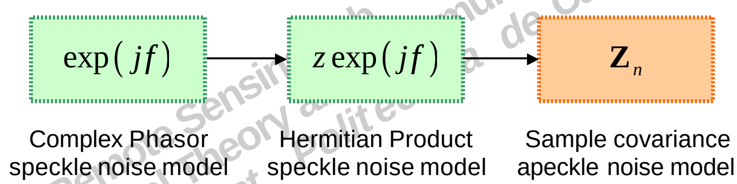
- Speckle noise model for the Hermitian product of a pair of SAR images

↓

Extension to model the sample covariance matrix independently of its dimensions

Multidimensional SAR Speckle Models — Hermitian Product Speckle Noise Model

Procedure to derive the multidimensional speckle noise model

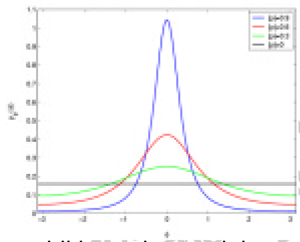


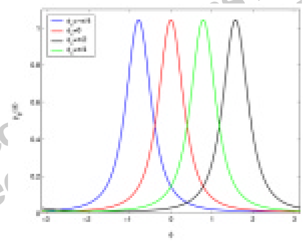
Complex Phasor Speckle Noise Model

$\exp(j\phi) \longrightarrow z\exp(j\phi) \longrightarrow [Z]$

- Phase distribution under the Gaussian Scattering Assumption

$$p_\phi(\phi) = \frac{\Gamma(n+1/2)(1-|r|^2)^n \beta}{2\sqrt{\pi}\,\Gamma(n)(1-\beta^2)^{n+1/2}} + \frac{(1-|r|^2)^n}{2\pi}\,{}_2F_1\left(n,1;\frac{1}{2};\beta^2\right) \qquad \beta = |r|\cos(\phi-\phi_x)$$





- Phase additive noise model

$$\phi = \phi_x + v \qquad [\phi_x - \pi, \phi_x + \pi)$$

$$E\{\phi\} = \phi_x$$

$$\mathrm{var}\{v\} = f(|\rho|)$$

- Useful signal separation from noise component under the complex phasor formulation

$$e^{j\phi} = \Re\{e^{j\phi}\} + j\Im\{e^{j\phi}\} = \cos(\phi) + j\sin(\phi) \qquad \begin{cases} \cos(\phi) = \cos(\phi_x)\cos(v) - \sin(\phi_x)\sin(v) \\ \sin(\phi) = \cos(\phi_x)\sin(v) + \sin(\phi_x)\cos(v) \end{cases}$$

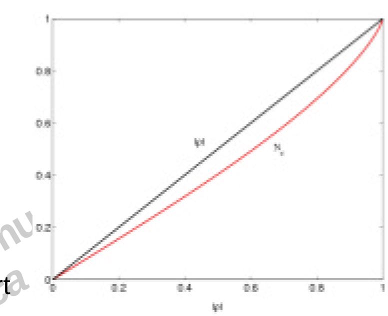
Complex Phasor Speckle Noise Model

- Interferometric phase noise real part

$$\cos(v) = N_c + v_1'$$

$$N_c = E\{\cos(v)\} = \frac{\pi}{4}|\rho|\;{}_2F_1\left(\frac{1}{2},\frac{1}{2};2;|\rho|^2\right)$$

$$E\{v_1'\} = 0 \qquad \sigma^2_{v_1'} = \frac{1}{2}(1-|\rho|^2)^{0.79}$$



- Interferometric phase noise imaginary part

$$\sin(v) = v_2'$$

$$E\{v_2'\} = 0 \qquad \sigma^2_{v_2'} = \frac{1}{2}(1-|\rho|^2)^{0.58}$$

- Information location within the complex phasor

$$\cos(\phi) = N_c\cos(\phi_x) + v_1'\cos(\phi_x) - v_2'\sin(\phi_x)$$

$$\sin(\phi) = N_c\sin(\phi_x) + v_1'\sin(\phi_x) + v_2'\cos(\phi_x)$$

N_c considered as signal component — Noise terms. Phase information is lost

Multidimensional SAR Speckle Models

Complex Phasor Speckle Noise Model

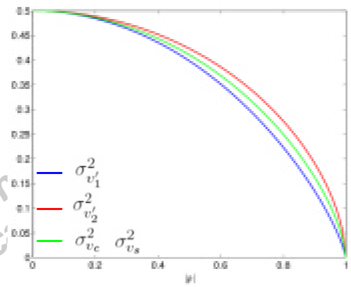
- Combination of the additive noise terms v'_1 and v'_2

$$
\begin{aligned}
v_c &= v'_1 \cos(\phi_x) - v'_2 \sin(\phi_x) \\
v_s &= v'_1 \sin(\phi_x) + v'_2 \cos(\phi_x)
\end{aligned}
$$

$$E\{v_c\} = E\{v_s\} = 0$$

$$
\begin{aligned}
\sigma^2_{v_c} &= \sigma^2_{v'_1} \cos^2(\phi_x) + \sigma^2_{v'_2} \sin^2(\phi_x) \\
\sigma^2_{v_s} &= \sigma^2_{v'_1} \sin^2(\phi_x) + \sigma^2_{v'_2} \cos^2(\phi_x)
\end{aligned}
$$

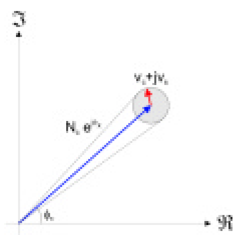
$$\sigma^2_{v_c} = \sigma^2_{v_s} = \frac{1}{2}(1 - |\rho|^2)^{0.685}$$



- Interferometric phasor noise model

$$e^{jf} = N_c e^{jf_x} + (v_c + jv_s)$$

Multiplicative component | Additive component



Multidimensional SAR Speckle Models

Hermitian Product Speckle Noise Model

$$\exp(j\phi) \longrightarrow z\exp(j\phi) \longrightarrow [Z]$$

Extended complex phasor

$$
\exp(j\phi) \left\{
\begin{aligned}
\cos(\phi) &= N_c \cos(\phi_x) + v'_1 \cos(\phi_x) - v'_2 \sin(\phi_x) \\
\sin(\phi) &= N_c \sin(\phi_x) + v'_1 \sin(\phi_x) + v'_2 \cos(\phi_x)
\end{aligned}
\right.
$$

- Complex Hermitian product speckle noise model as extension of the complex phasor speckle noise model

$$S_k S_l^* = |S_k S_l^*| e^{j(\theta_k - \theta_l)} = z e^{j\phi}$$

$$
\begin{aligned}
\Re\{z e^{j\phi}\} &= N_c\, z \cos(\phi_x) + z\, v'_1 \cos(\phi_x) - z\, v'_2 \sin(\phi_x) \\
\Im\{z e^{j\phi}\} &= N_c\, z \sin(\phi_x) + z\, v'_1 \sin(\phi_x) + z\, v'_2 \cos(\phi_x)
\end{aligned}
$$

$$z e^{jf} = \left[z N_c + (z v'_1 + j z v'_2) \right] \exp(j f_x)$$

- The phase ϕ_x is considered as the average phase difference in the case of homogeneous data
- Identification of noise sources in each additive element by considering homogeneous data

Multidimensional SAR Speckle Models

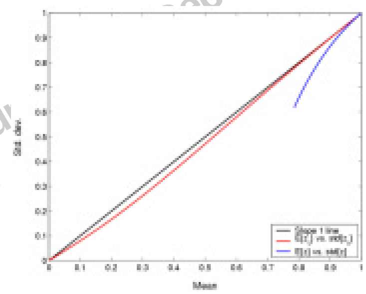
Hermitian Product Speckle Noise Model

$$ze^{j\phi} = [\boxed{zN_c} + (zv_1' + j\,zv_2')]\exp(j\phi_x)$$

- Random behaviour determined by the amplitude z

$$\Re\{ze^{j\phi}\}_1 = zN_c\cos(\phi_x)$$
$$\Im\{ze^{j\phi}\}_1 = zN_c\sin(\phi_x)$$

- N_c determines the final properties of this additive component as it depends also on the coherence $|\rho|$



$$E\left\{\Re\{ze^{j\phi}\}_1\right\} = \psi N_c \frac{\pi}{4}\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{2};1;|\rho|^2\right)\cos(\phi_x)$$

$$\mathrm{var}\left\{\Re\{ze^{j\phi}\}_1\right\} = \psi^2 N_c^2\left(1+|\rho|^2-\left(\frac{\pi}{4}\right)^2{}_2F_1^2\left(-\frac{1}{2},-\frac{1}{2};1;|\rho|^2\right)\right)\cos^2(\phi_x)$$

$$\mathrm{std}\left\{\Re\{ze^{j\phi}\}_1\right\} \simeq \mathrm{abs}\left(E\left\{\Re\{ze^{j\phi}\}_1\right\}\right)$$

$$\{ze^{j\phi}\}_1 = \psi N_c \overline{z}_n n_m e^{j\phi_x}$$

$$E\{n_m\} = 1$$
$$\mathrm{var}\{n_m\} = 1$$

Multiplicative noise behaviour

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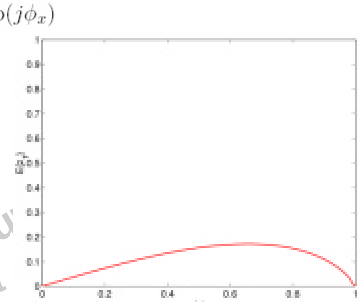
Multidimensional SAR Speckle Models

Hermitian Product Speckle Noise Model

$$ze^{j\phi} = [zN_c + (\boxed{zv_1'} + j\,zv_2')]\exp(j\phi_x)$$

- Random component determined by z and the phase noise term v_1'

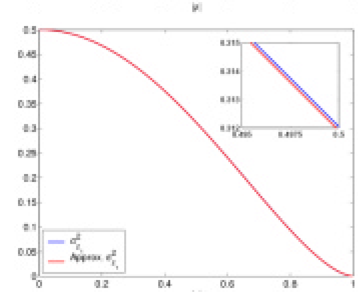


$$\Re\{ze^{j\phi}\}_2 = zv_1'\cos(\phi_x)$$
$$\Im\{ze^{j\phi}\}_2 = zv_1'\sin(\phi_x)$$

$$E\left\{\Re\{ze^{j\phi}\}_2\right\} = \psi\left(|\rho| - N_c\frac{\pi}{4}\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{2};1;|\rho|^2\right)\right)\cos(\phi_x)$$

$$\mathrm{var}\left\{\Re\{ze^{j\phi}\}_2\right\} \simeq \frac{1}{2}\psi^2(1-|\rho|^2)^{1.64}\cos^2(\phi_x)$$

- The mean and the variance depend differently on the coherence $|\rho|$



$$\{ze^{j\phi}\}_2 = \psi\left[(|\rho| - N_c\overline{z}_n) + n_{a1}\right]e^{j\phi_x}$$

$$E\{n_{a1}\} = 0$$
$$\mathrm{var}\{n_{a1}\} = \frac{1}{2}(1-|\rho|^2)^{1.64}$$

Additive noise behaviour

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Hermitian Product Speckle Noise Model

$$ze^{j\phi} = [zN_c + (zv_1' + j\,zv_2')]\exp(j\phi_x)$$

- Random component determined by z and the phase noise term v_2'

$$\Re\{ze^{j\phi}\}_3 = -zv_2'\sin(\phi_x)$$
$$\Im\{ze^{j\phi}\}_3 = zv_2'\cos(\phi_x)$$

$$E\{\Re\{ze^{j\phi}\}_3\} = 0$$

$$\mathrm{var}\{\Re\{ze^{j\phi}\}_3\} = \frac{1}{2}\psi^2(1-|\rho|^2)\sin^2(\phi_x)$$

$$\{ze^{j\phi}\}_3 = j\psi n_{a2}e^{j\phi_x}$$

$$E\{n_{a2}\} = 0$$
$$\mathrm{var}\{n_{a2}\} = \frac{1}{2}(1-|\rho|^2)$$

Additive noise behaviour

- Complex Hermitian product speckle noise model

$$S_kS_l^* = f\left(\psi|\rho|\exp(j\phi_x), [n_m, n_{ar}, n_{ai}]^T\right)$$

$$S_kS_l^* = ze^{j\phi} = \{ze^{j\phi}\}_1 + \{ze^{j\phi}\}_2 + \{ze^{j\phi}\}_3$$
$$= \psi N_c\bar{z}_n n_m e^{j\phi_x} + \psi\left[(|\rho| - N_c\bar{z}_n) + n_{a1}\right]e^{j\phi_x} + j\psi n_{a2}e^{j\phi_x}$$

$$\mathcal{C}_{n_m n_{a1}} \neq 0 \qquad \mathcal{C}_{n_m n_{a2}} = 0 \qquad \mathcal{C}_{n_{a1} n_{a2}} = 0$$

Hermitian Product Speckle Noise Model

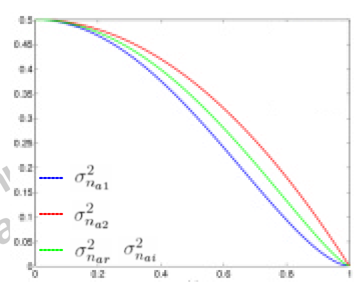
- Combination of the additive noise terms n_{ar} and n_{ai}

$$\psi n_{ar} = \psi n_{a1}\cos(\phi_x) - \psi n_{a2}\sin\phi_x$$
$$\psi n_{ai} = \psi n_{a1}\sin(\phi_x) + \psi n_{a2}\cos\phi_x$$

$$E\{n_{ar}\} = E\{n_{a1}\}\cos(\phi_x) - E\{n_{a2}\}\sin(\phi_x) = 0$$
$$E\{n_{ai}\} = E\{n_{a1}\}\sin(\phi_x) + E\{n_{a2}\}\cos(\phi_x) = 0$$

$$\sigma^2_{n_{ar}} = \sigma^2_{n_{a1}}\cos^2(\phi_x) + \sigma^2_{n_{a2}}\sin^2(\phi_x)$$
$$\sigma^2_{n_{ai}} = \sigma^2_{n_{a1}}\sin^2(\phi_x) + \sigma^2_{n_{a2}}\cos^2(\phi_x)$$

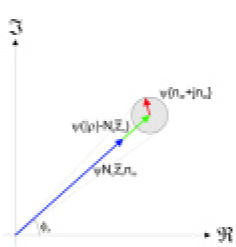
$$\sigma^2_{n_{ar}} = \sigma^2_{n_{ai}} = \frac{1}{2}(1-|\rho|^2)^{1.32}$$



- Complex Hermitian product linear speckle noise model

$$S_kS_l^* = \psi N_c\bar{z}_n n_m e^{j\phi_x} + \psi\left(|\rho| - N_c\bar{z}_n\right)e^{j\phi_x} + \psi\left(n_{ar} + jn_{ai}\right)$$

Multiplicative term — Additive term



Hermitian Product Speckle Noise Model

$$S_k S_l^* = \psi N_c \bar{z}_n n_m e^{j\phi_x} + \psi\left(|\rho| - N_c \bar{z}_n\right) e^{j\phi_x} + \psi\left(n_{ar} + j n_{ai}\right)$$

Multiplicative speckle component: n_m → High coherence areas

Stationary

$$\Re\{ze^{j\phi}\}_1 = z_c \cos(\phi_x) = \psi N_c \bar{z}_n n_m \cos(\phi_x)$$

$$\Im\{ze^{j\phi}\}_1 = z_c \sin(\phi_x) = \psi N_c \bar{z}_n n_m \sin(\phi_x)$$

$N_c \bar{z}_n \sigma_{n_m} = \sigma_{n_{ar}} = \sigma_{n_{ai}}$

$|\rho| = 0.675$

Additive speckle components: n_{ar}, n_{ai} → Low coherence areas

Non stationary

Final speckle noise behaviour: Combination of multiplicative and additive noise components, determined by ρ

Special cases

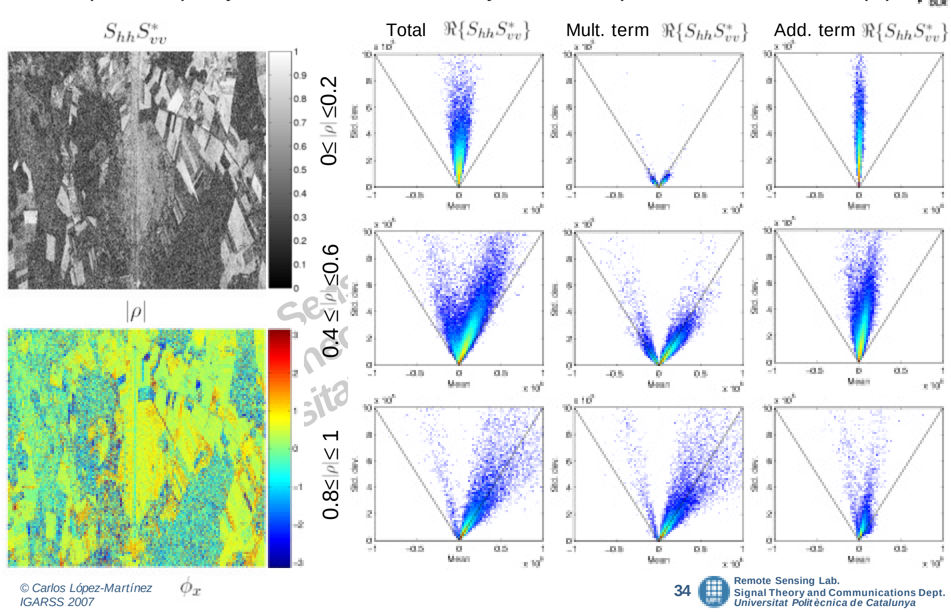
- Covariance matrix diagonal element

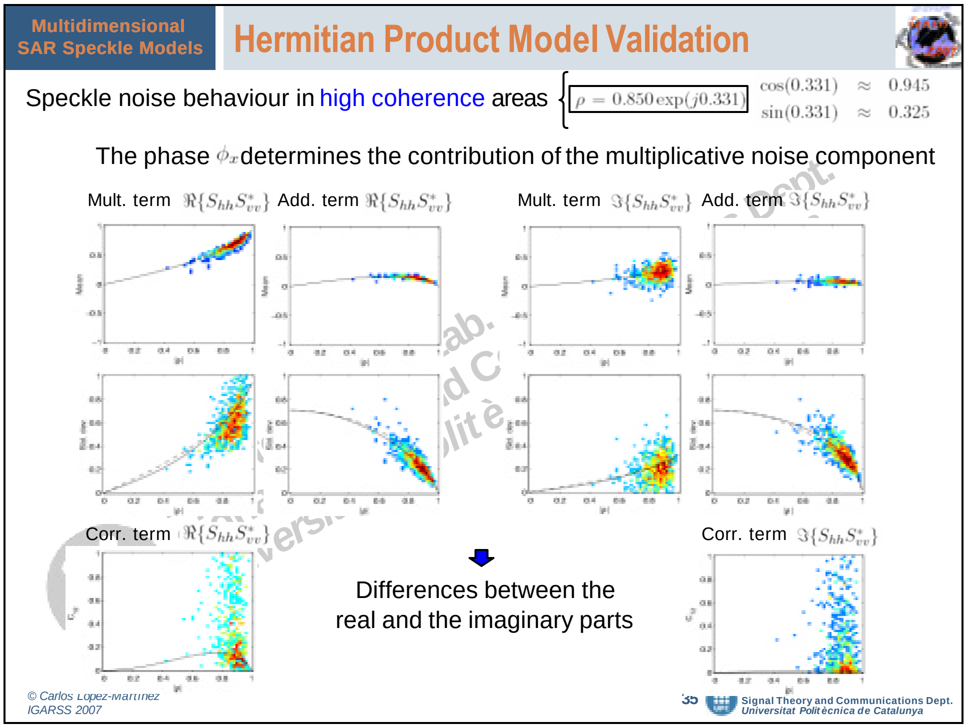
$\rho = 1\exp(j0)$ → $S_k S_k^* = \psi n_m$

- By construction, the complex Hermitian product phase difference is characterized by an additive noise model

Hermitian Product Model Validation

L-band (1.3 GHz) fully PolSAR data. E-SAR system. Oberpfaffenhofen test area (D)




Multidimensional SAR Speckle Models
Hermitian Product Model Validation
Speckle noise behaviour in high coherence areas
$\rho = 0.850\exp(j0.331)$
$\cos(0.331) \approx 0.945$
$\sin(0.331) \approx 0.325$
The phase ϕ_x determines the contribution of the multiplicative noise component
Mult. term $\Re\{S_{hh}S_{vv}^*\}$
Add. term $\Re\{S_{hh}S_{vv}^*\}$
Mult. term $\Im\{S_{hh}S_{vv}^*\}$
Add. term $\Im\{S_{hh}S_{vv}^*\}$
Corr. term $\Re\{S_{hh}S_{vv}^*\}$
Differences between the real and the imaginary parts
Corr. term $\Im\{S_{hh}S_{vv}^*\}$
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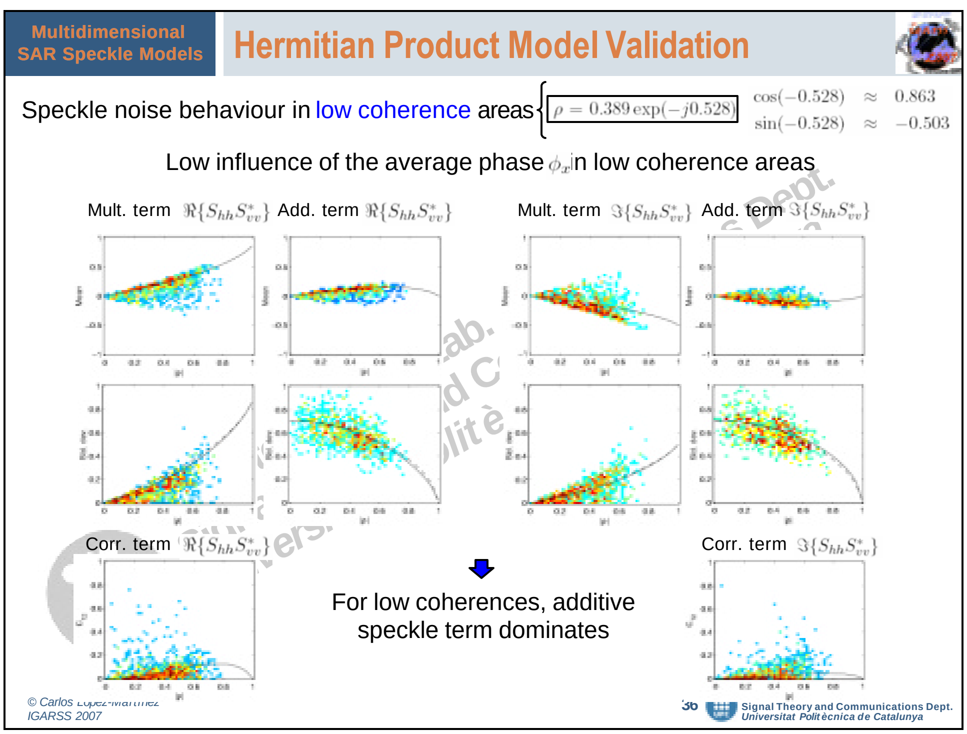
Multidimensional SAR Speckle Models
Hermitian Product Model Validation
Speckle noise behaviour in low coherence areas
$\rho = 0.389\exp(-j0.528)$
$\cos(-0.528) \approx 0.863$
$\sin(-0.528) \approx -0.503$
Low influence of the average phase ϕ_x in low coherence areas
Mult. term $\Re\{S_{hh}S_{vv}^*\}$
Add. term $\Re\{S_{hh}S_{vv}^*\}$
Mult. term $\Im\{S_{hh}S_{vv}^*\}$
Add. term $\Im\{S_{hh}S_{vv}^*\}$
Corr. term $\Re\{S_{hh}S_{vv}^*\}$
For low coherences, additive speckle term dominates
Corr. term $\Im\{S_{hh}S_{vv}^*\}$
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Multidimensional SAR Speckle Models

Sample Covariance Matrix Speckle Noise Model

Extension of the Hermitian Product Speckle Noise Model

Gaussian hypothesis

- $E\{\mathbf{Z}_n^p\} = E\{Q(\mathbf{Z}_n)\} \Rightarrow f(\mathbf{C})$
- $Q(\mathbf{Z}_n)$ Polynomial depending on the entries of $\mathbf{Z}_n$ invariant in the sense that it only depends on the eigenvalues of $\mathbf{Z}_n$

G. Letac and H. Massam, "All Invariant Moments of the Wishart Distribution", Scand. J. of Statistics, vol. 31, no. 2, pp. 295–318, June 2004

- Rearrangement of the Hermitian product speckle noise model

$$\langle S_k S_l^* \rangle_n = y n_m \exp(jf_x) + y\left(|r| - N_c \bar{z}_n\right)\exp(jf_x) + y\left(n_{ar} + jn_{ai}\right) \qquad k,l = 1,2,\ldots,m.$$

$$\langle S_k S_l^* \rangle_n = y|r|\exp(jf_x) + y\left(n_m - N_c \bar{z}_n\right)\exp(jf_x) + y\left(n_{ar} + jn_{ai}\right) \qquad k,l = 1,2,\ldots,m$$

- Matrix speckle noise model

$$\mathbf{Z}_n = \mathbf{C} + \mathbf{N}_m + \mathbf{N}_a$$

$$\begin{cases} \mathbf{N}_{m,ii} = n_{m,ii} \\ \mathbf{N}_{m,ij} = y_{ij}\left(n_{m,ij} - N_{c,ij}\bar{z}_{n,ij}\right)\exp(jf_{x,ij}) \end{cases}$$

$$\begin{cases} \mathbf{N}_{a,ii} = 0 \\ \mathbf{N}_{a,ij} = y_{ij}\left(n_{ar,ij} + jn_{ai,ij}\right) \end{cases}$$

- Coherent with previous results in the literature (Perturbation analysis)

$$\langle \mathbf{k}\mathbf{k}^* \rangle = \mathbf{C} + ?$$

H. Krim, P. Forster, J.G. Proakis, "Operator approach to performance analysis of root-MUSIC and root-min-norm" IEEE Trans. Signal Processing, vol.40, no. 7, pp. 1687–1696, July 1992

Multidimensional SAR Speckle Models

Multidimensional Multilook Speckle Model

A multidimensional SAR data speckle noise model for already multilooked data

- Sometimes, depending on the SAR sensor, only multilook data is available

$$\mathbf{Z}_n = \frac{1}{n}\sum_{k=1}^{n}\mathbf{k}\mathbf{k}^H = \begin{bmatrix} \frac{1}{n}\sum_{k=1}^{n} S_1(k)S_1^*(k) & \frac{1}{n}\sum_{k=1}^{n} S_1(k)S_2^*(k) & \cdots & \frac{1}{n}\sum_{k=1}^{n} S_1(k)S_m^*(k) \\ \frac{1}{n}\sum_{k=1}^{n} S_2(k)S_1^*(k) & \frac{1}{n}\sum_{k=1}^{n} S_2(k)S_2^*(k) & \cdots & \frac{1}{n}\sum_{k=1}^{n} S_2(k)S_m^*(k) \\ \vdots & \vdots & \ddots & \vdots \\ \frac{1}{n}\sum_{k=1}^{n} S_m(k)S_1^*(k) & \frac{1}{n}\sum_{k=1}^{n} S_m(k)S_2^*(k) & \cdots & \frac{1}{n}\sum_{k=1}^{n} S_m(k)S_m^*(k) \end{bmatrix}$$

Multidimensional Multilook Speckle Model

The Hermitian product phase difference pdf

$$\langle S_i S_j^* \rangle_n = z\exp(jf) \qquad i,j = 1,2,\dots,m \qquad p_f(f) = \frac{\Gamma(n+1/2)\left(1-|r|^2\right)^n b}{2\sqrt{p}\,\Gamma(n)\left(1-b^2\right)^{n+1/2}} + \frac{\left(1-|r|^2\right)^n}{2p}\,{}_2F_1\left(n,1;\frac{1}{2};b^2\right)$$

Phase difference model ➡ $f = f_x + v$ still valid

$$\Re\{\exp(jf)\} = N_c\cos(f_x) + v_1'\cos(f_x) - v_2'\sin(f_x)$$
$$\Im\{\exp(jf)\} = N_c\sin(f_x) + v_1'\sin(f_x) + v_2'\cos(f_x)$$

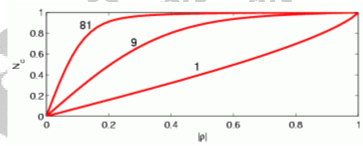
$$\exp(jf) = N_c\exp(jf_x) + (v_c' + jv_s')$$

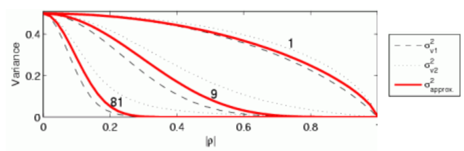
Amplitude information

$$N_c = \frac{\Gamma(n+1/2)\,\Gamma(3/2)}{\Gamma(n)}|r|\,{}_2F_1\left(\frac{3}{2}-n,\frac{1}{2};2;|r|^2\right)$$

Noise components

$$E\{v_c'\} = E\{v_s'\} = 0$$
$$s^2_{v_c'} = s^2_{v_s'} \simeq \frac{1}{2}\left(1-|r|^2\right)^{a\,n}$$
$$a = 0.685$$





Multidimensional Multilook Speckle Model

$$\langle S_i S_j^* \rangle_n = z\exp(jf) \qquad i,j = 1,2,\dots,m$$

$$\Re\{z\exp(jf)\} = zN_c\cos(f_x) + zv_1'\cos(f_x) - zv_2'\sin(f_x)$$
$$\Im\{z\exp(jf)\} = zN_c\sin(f_x) + zv_1'\sin(f_x) + zv_2'\cos(f_x)$$

- Single-look data

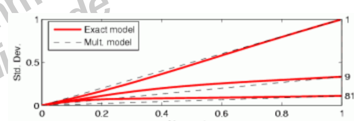
$$zN_c\exp(jf_x) = y\,\exp(jf_x)\,\overline{z}_n N_c n_m \qquad E\{n_m\} = 1 \qquad s^2_{n_m} = 1$$

- Multilook data

  Extension of the single-look model NOT possible

$$zN_c\exp(jf_x) = y\,\exp(jf_x)\,n_m$$

$$E\{n_m\} = N_c\overline{z}_n \quad \text{(Exact value)}$$
$$\mathrm{var}\{n_m\} = N_c^2\frac{\left(1+|r|^2\right)}{2n} \quad \text{(1st order approx.)}$$



- Asymptotic analysis: Mean values are exact. Std dev. are approximated

$$\lim_{n\to\infty} E\{zN_c\cos(f_x)\} = y\,|r|\cos(f_x) \qquad \lim_{n\to\infty} s^2_{zN_c\cos(f_x)} = 0$$
$$\lim_{n\to\infty} E\{zN_c\sin(f_x)\} = y\,|r|\sin(f_x) \qquad \lim_{n\to\infty} s^2_{zN_c\sin(f_x)} = 0$$

  - No loss of information
  - All information contained in the first additive term as $n \to \infty$

Multidimensional Multilook Speckle Model

$\langle S_i S_j^* \rangle_n = z\exp(j\phi) \qquad i,j=1,2,\dots,m$

$\Re\{z\exp(j\phi)\} = zN_c\cos(\phi_x) + zv_1'\cos(\phi_x) - zv_2'\sin(\phi_x)$

$\Im\{z\exp(j\phi)\} = zN_c\sin(\phi_x) + zv_1'\sin(\phi_x) + zv_2'\cos(\phi_x)$

- Single-look data

$zv_1'\exp(j\phi_x) = \psi\{(|\rho| - N_c\bar{z}_n) + n_{a1}\}\exp(j\phi_x)$

$E\{n_{a1}\} = 0$

$\sigma^2_{n_{a1}} \simeq \frac{1}{2}\left(1-|\rho|^2\right)^{1.64}$

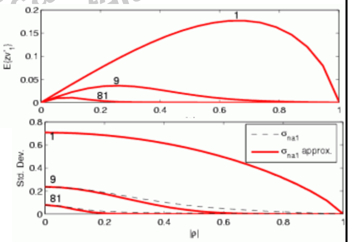
- Multilook data

Extension of the single-look model POSSIBLE

$zv_1'\exp(j\phi_x) = \psi\{(|\rho| - N_c\bar{z}_n) + n_{a1}\}\exp(j\phi_x)$

$E\{n_{a1}\} = 0$ (Exact value)

$\sigma^2_{n_{a1}} = \frac{1}{2n}\left(1-|\rho|^2\right)^{1.64n}$ (Approx.)



- Asymptotic analysis

$\lim_{n\to\infty} E\{zv_1'\cos(\phi_x)\} = 0 \qquad \lim_{n\to\infty}\mathrm{var}\{zv_1'\cos(\phi_x)\} = 0$

$\lim_{n\to\infty} E\{zv_1'\sin(\phi_x)\} = 0 \qquad \lim_{n\to\infty}\mathrm{var}\{zv_1'\sin(\phi_x)\} = 0$

  - No loss of information

Multidimensional Multilook Speckle Model

$\langle S_i S_j^* \rangle_n = z\exp(j\phi) \qquad i,j=1,2,\dots,m$

$\Re\{z\exp(j\phi)\} = zN_c\cos(\phi_x) + zv_1'\cos(\phi_x) - zv_2'\sin(\phi_x)$

$\Im\{z\exp(j\phi)\} = zN_c\sin(\phi_x) + zv_1'\sin(\phi_x) + zv_2'\cos(\phi_x)$

- Single-look data

$zv_2'\exp(j\phi_x) = \psi\, n_{a2}\exp(j\phi_x)$

$E\{n_{a2}\} = 0$

$\sigma^2_{n_{a2}} = \frac{1}{2n}\left(1-|\rho|^2\right)$

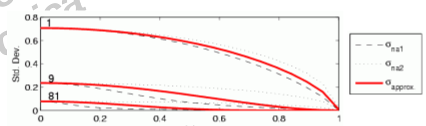
- Multilook data

Extension of the single-look model POSSIBLE

$zv_2'\exp(j\phi_x) = \psi\, n_{a2}\exp(j\phi_x)$

$E\{n_{a2}\} = 0$ (Exact value)

$\sigma^2_{n_{a2}} = \frac{1}{2n}\left(1-|\rho|^2\right)$ (Exact value)



- Unification of the additive noise terms

$n_{ar} = n_{a1}\cos(\phi_x) - n_{a2}\sin(\phi_x)$

$n_{ai} = n_{a1}\sin(\phi_x) + n_{a2}\cos(\phi_x)$

$E\{n_{ar}\} = E\{n_{ai}\} = 0 \qquad \sigma^2_{n_{ar}} = \sigma^2_{n_{ai}} \simeq \frac{1}{2n}\left(1-|\rho|^2\right)^{1.32\sqrt{n}}$ (Approx.)

Multidimensional SAR Speckle Models

Multidimensional Multilook Speckle Model

$$\langle S_i S_j^* \rangle_n = z \exp(jf) \qquad i,j = 1,2,\ldots,m$$

$$\langle S_i S_j^* \rangle_n = y\, n_m \exp(jf_x) + y\left(|r| - N_c \bar{z}_n\right)\exp(jf_x) + y\,(n_{ar} + jn_{ai}) \qquad i,j = 1,2,\ldots,m$$

- Multiplicative speckle noise component
  - Dominant for high coherences
  - Modulated by phase information

$$E\{n_m\} = N_c \bar{z}_n \qquad s_{n_m}^2 = N_c^2 \frac{\left(1+|r|^2\right)}{2n}$$

- Additive speckle noise component
  - Dominant for low coherences
  - Not affected by phase information

$$E\{n_{ar}\} = E\{n_{ai}\} = 0 \qquad s_{n_{ar}}^2 = s_{n_{ai}}^2 \simeq \frac{1}{2n}\left(1-|r|^2\right)^{1.32\sqrt{n}}$$

- Effect of the approximations
  - Mean value IS NOT approximated ➡ No loss of information

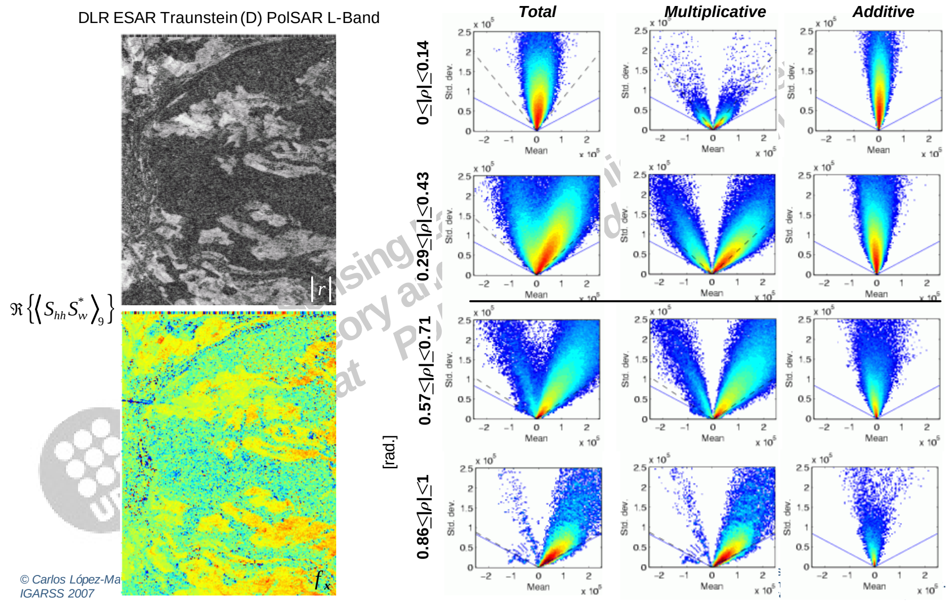
$$\lim_{n\to\infty}\left\{y\, n_m \exp(jf_x) + y\left(|r| - N_c \bar{z}_n\right)\exp(jf_x) + y\,(n_{ar} + jn_{ai})\right\} = y\,|r| \exp(jf_x)$$

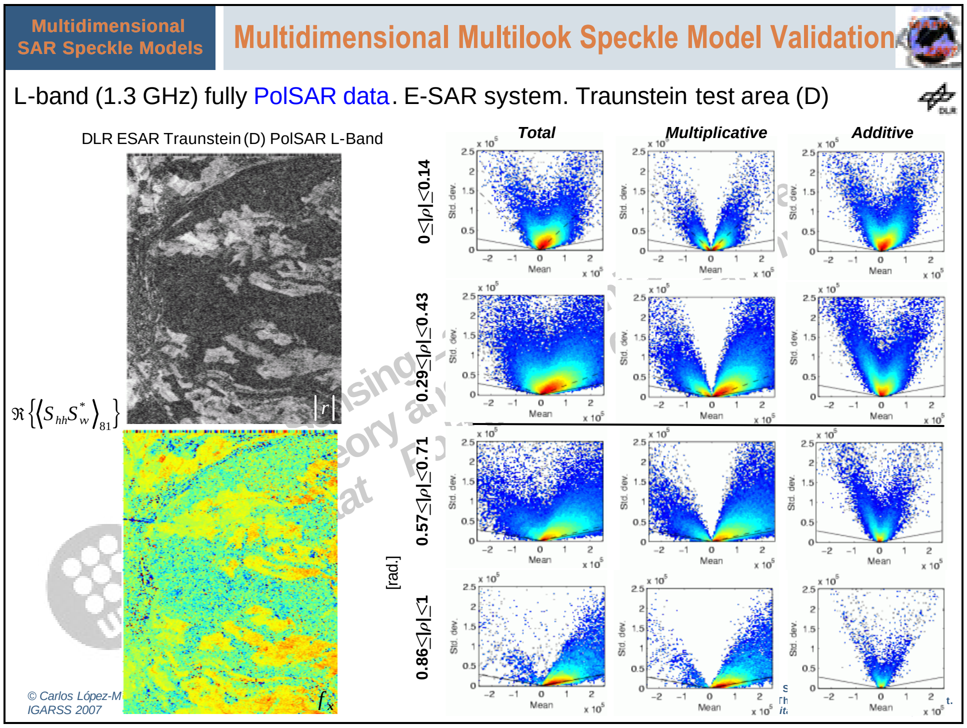
  - Std. Dev. ARE approximated

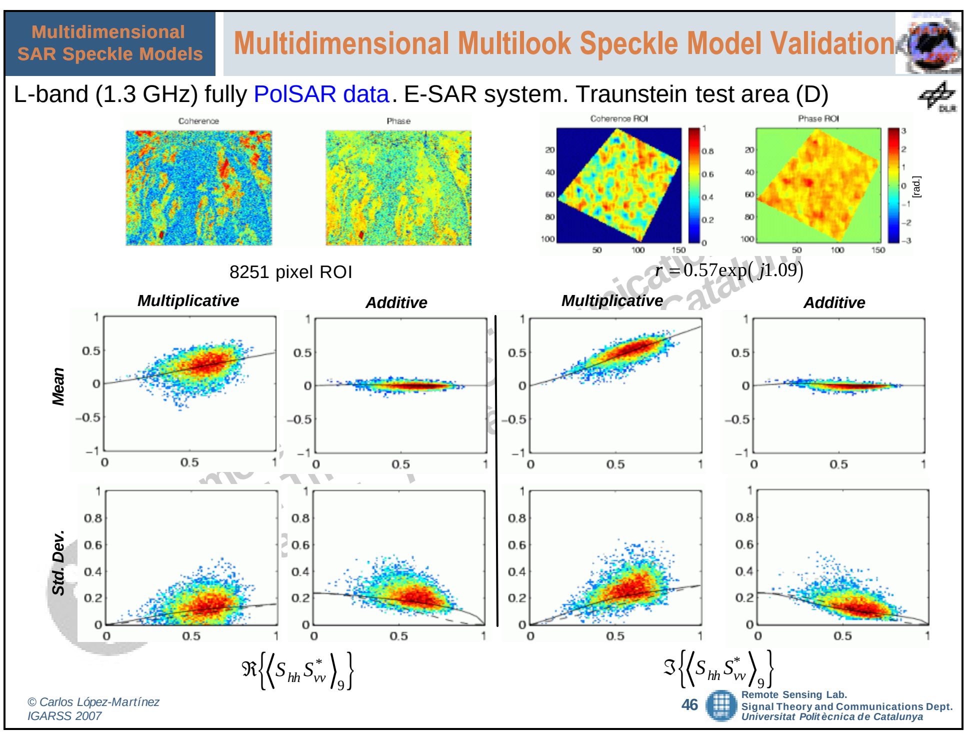
Multidimensional SAR Speckle Models

Multidimensional Multilook Speckle Model Validation

L-band (1.3 GHz) fully PolSAR data. E-SAR system. Traunstein test area (D)




Multidimensional SAR Speckle Models
Multidimensional Multilook Speckle Model Validation
L-band (1.3 GHz) fully PolSAR data. E-SAR system. Traunstein test area (D)
DLR ESAR Traunstein (D) PolSAR L-Band
$\Re\{\langle S_{hh}S_{vv}^*\rangle_{81}\}$
$|r|$
ϕ_x
[rad.]
Total
Multiplicative
Additive
$0\leq|\rho|\leq0.14$
$0.29\leq|\rho|\leq0.43$
$0.57\leq|\rho|\leq0.71$
$0.86\leq|\rho|\leq1$
Std. dev.
Mean
© Carlos López-M
IGARSS 2007


Multidimensional SAR Speckle Models
Multidimensional Multilook Speckle Model Validation
L-band (1.3 GHz) fully PolSAR data. E-SAR system. Traunstein test area (D)
Coherence
Phase
Coherence ROI
Phase ROI
[rad.]
8251 pixel ROI
$r = 0.57\exp(j1.09)$
Multiplicative
Additive
Multiplicative
Additive
Mean
Std. Dev.
$\Re\{\langle S_{hh}S_{vv}^*\rangle_9\}$
$\Im\{\langle S_{hh}S_{vv}^*\rangle_9\}$
© Carlos López-Martínez
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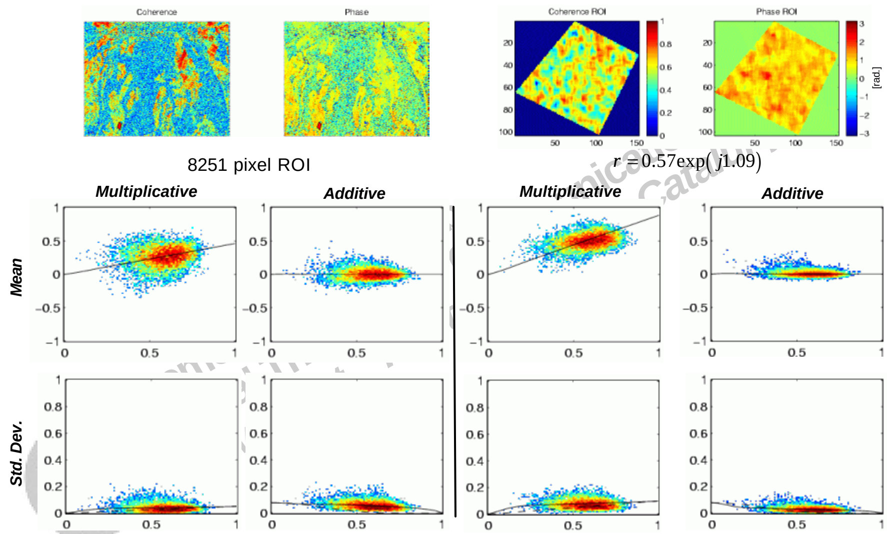
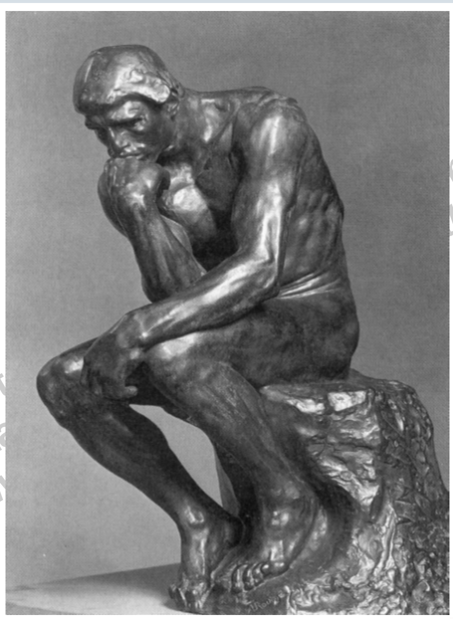
Remote Sensing Lab.
Signal Theory and Communications Dept.
Universitat Politècnica de Catalunya

Multidimensional Multilook Speckle Model Validation

L-band (1.3 GHz) fully PolSAR data. E-SAR system. Traunstein test area (D)



$\Re\{\langle S_{hh}S_{vv}^*\rangle_{81}\}$ $\Im\{\langle S_{hh}S_{vv}^*\rangle_{81}\}$



Coherence Modelling and Estimation

Coherence Estimation

SAR Single-Channel and Multi-Channel complex correlation

- Useful information contained in the Hermitian product (second order moment) of the pairs of SAR images

Complex correlation coefficient $r = \frac{E\{S_1 S_2^*\}}{\sqrt{E\{|S_1|^2\}E\{|S_2|^2\}}} = |r| e^{j\phi_x}$

$|r|$ → Coherence

- Coherence estimation through multilook techniques

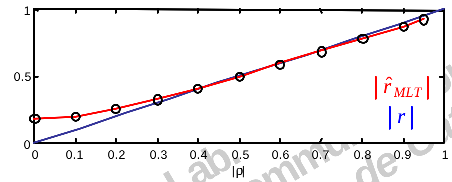
Multilook techniques $|\hat{r}_{MLT}| = \frac{\left|\sum_{m=1}^{M}\sum_{n=1}^{N} S_1(m,n) S_2^*(m,n)\right|}{\sqrt{\sum_{m=1}^{M}\sum_{n=1}^{N}|S_1(m,n)|^2 \sum_{m=1}^{M}\sum_{n=1}^{N}|S_2(m,n)|^2}}$

- Overestimation for low coherence values
- Bias due to systematic phase variations
- Origin?, Quantification?, Reduction/Elimination?

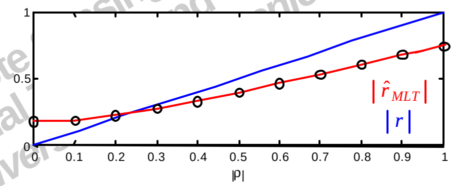
Topographic Effects

Effects of topography in coherence estimation

- Coherence estimated in a 5x5-pixel window



40-pixel fringes

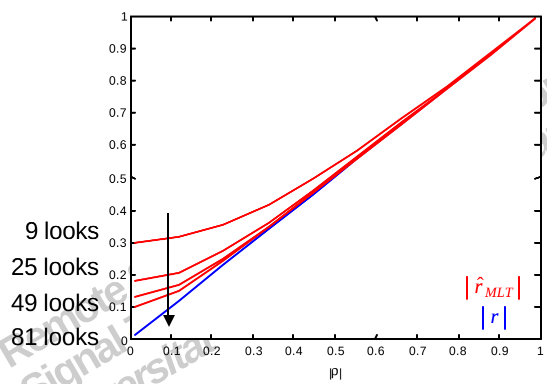


12-pixel fringes

Steep topography induces coherence bias

Speckle Bias

Effects of number of samples (looks) employed to estimate coherence



- Multilook coherence is as asymptotically non-biased coherence estimator.

Looks ↗ ⇨ Coherence bias ↙

Mathematical Modelling

Modelling of the Interferometric Coherence Parameter: Topographic Effects

Systematic phase variations corrupt coherence estimation since the hypothesis of data homogeneity is not valid

- Introduction of topographic effects in the speckle model of $S_1 S_2^*$

$$S_1 S_2^* = y\,\overline{z}_n n_m N_c e^{jf_x} + y\left(\left|r\right| - N_c \overline{z}_n\right) e^{jf_x} + y\left(n_{ar} + j n_{ai}\right)$$

Hypothesis: Topography model ➡ 2D separable slope

$$f_x(m,n) = \frac{2p}{S_x} m + \frac{2p}{S_y} n$$

Hypothesis: Multilook techniques to estimate coherence ➡ Simplicity of analysis

$$h(m,n) = \frac{1}{MN} \sum_{p=1}^{M} \sum_{q=1}^{N} d(m-p)\, d(n-q)$$

$$H(w_x, w_y) = \frac{1}{M} \frac{\sin\left(\frac{M}{2} w_x\right)}{\sin\left(\frac{w_x}{2}\right)} \frac{1}{N} \frac{\sin\left(\frac{N}{2} w_y\right)}{\sin\left(\frac{w_y}{2}\right)}$$

Mathematical Modelling

The availability of the speckle noise model for $S_1 S_2^*$ allows analyzing the effect of the multilook filter

$$x(m,n) = S_1 S_2^*(m,n) \longrightarrow \boxed{h(m,n)} \longrightarrow y(m,n) = \left(S_1 S_2^*\right)'(m,n)$$

- Stochastic Analysis

$$S_{xx}(w_x, w_y) \longrightarrow \boxed{H(w_x, w_y)} \longrightarrow S_{yy}(w_x, w_y)$$

$$S_{yy}(w_x, w_y) = \left|H(w_x, w_y)\right|^2 S_{xx}(w_x, w_y)$$

Hypothesis: Uncorrelated speckle components

$$r_{n_m n_m}(k,l) = 1 + d(k,l)$$

$$r_{n_{ar} n_{ar}}(k,l) = r_{n_{ai} n_{ai}}(k,l) = \frac{1}{2}\left(1 - \left|r\right|^2\right)^{1.32} d(k,l)$$

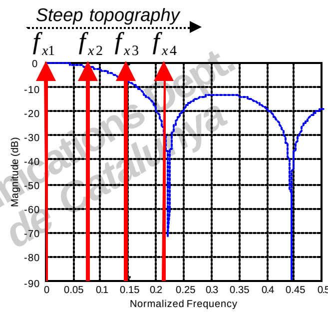
Mathematical Modelling

$$S_1S_2^* = y\,\overline{z}_n n_m N_c e^{jf_x} + y\left(|r| - N_c\overline{z}_n\right)e^{jf_x} + y\left(n_{ar} + jn_{ai}\right)$$

Spectral density function (*coherence information*) modulated by topography

$$S_{yy}(w_x, w_y) = \left|H(w_x, w_y)\right|^2 S_{xx}(w_x, w_y)$$

$$\Delta_{topo} = \left|\frac{1}{M}\frac{\sin\left(\frac{M}{2}w_x\right)}{\sin\left(\frac{w_x}{2}\right)}\frac{1}{N}\frac{\sin\left(\frac{N}{2}w_y\right)}{\sin\left(\frac{w_y}{2}\right)}\right|$$



$$\left\langle S_1S_2^*\right\rangle_{MN} = y\Delta_{topo}\overline{z}_n n'_m N_c e^{jf_x} + y\Delta_{topo}\left(|r| - N_c\overline{z}_n\right)e^{jf_x} + y\left(n'_{ar} + jn'_{ai}\right)$$

Quantification of the topographic bias (underestimates coherence)

Magnitude flat filters allow to estimate coherence independently from topography

Complex Coherence Speckle Noise Model

Modelling of the complex correlation coefficient

$$r = \frac{E\{S_1S_2^*\}}{\sqrt{E\{|S_1|^2\}E\{|S_2|^2\}}} \qquad r = \frac{S_1S_2^*}{\sqrt{|S_1|^2|S_2|^2}}$$

Separate noise models for S_1 and S_2 *do not take into account* the correlation between SAR images

- Hypothesis: Independent modelling of $S_1S_2^*$ and S_1 and S_2

$$S_1S_2^* = y\,\overline{z}_n n_m N_c e^{jf_x} + y\left(|r| - N_c\overline{z}_n\right)e^{jf_x} + y\left(n_{ar} + jn_{ai}\right)$$

$$|S_1|^2 = s_1 n_{m1}$$
$$|S_2|^2 = s_2 n_{m2} \Rightarrow |S_1|^2|S_2|^2 = y\sqrt{n_{m1}n_{m2}}$$

- Complex coherence noise model

$$r = \frac{S_1S_2^*}{\sqrt{|S_1|^2|S_2|^2}} = \frac{\overline{z}_n n_m N_c e^{jf_x} + (|r| - N_c\overline{z}_n)e^{jf_x} + (n_{ar} + jn_{ai})}{\sqrt{n_{m1}n_{m2}}}$$

- Multilook estimation

$$\hat{r}_{MLT} = \frac{\overline{z}_n n'_m N_c e^{jf_x} + (|r| - N_c\overline{z}_n)e^{jf_x} + (n'_{ar} + jn'_{ai})}{\sqrt{n'_{m1}n'_{m2}}}$$

$$E\{n'_{ar}\} = E\{n'_{ai}\} = 0$$
$$E\{n'_m\} = E\{n'_{m1}\} = E\{n'_{m2}\} = 1$$

Coherence Estimation

Complex Coherence Speckle Noise Model

- Denominator model

$$E\left\{\langle|S_1|^2\rangle_{MN}\langle|S_2|^2\rangle_{MN}\right\} = y\left(1+\frac{|r|^2}{MN}\right) \Rightarrow \sqrt{\langle|S_1|^2\rangle_{MN}\langle|S_2|^2\rangle_{MN}} \simeq y\sqrt{\left(1+\frac{1}{MN}\right)}$$

$$r_{MLT} = \frac{\Delta_{topo} n'_m \exp(jf_x) + \Delta_{topo}\left(|r| - N_c \bar{z}_n\right)\exp(jf_x) + (n'_{ar} + jn'_{ai})}{\sqrt{\left(1+\frac{1}{MN}\right)}}$$

- Complex correlation coefficient model

$$r_{MLT} \simeq |r|\Delta_{topo}\exp(jf_x) + \left(1+\frac{1}{MN}\right)^{-\frac{1}{2}}(n'_{ar} + jn'_{ai})$$

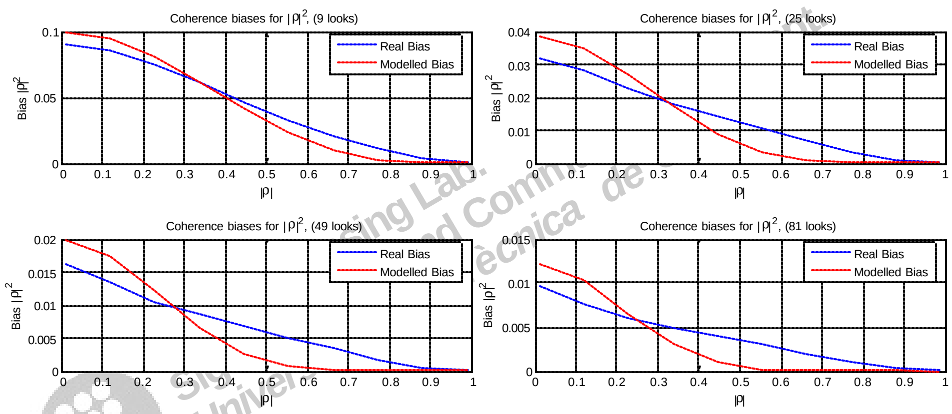
$$E\left\{|r_{MLT}|^2\right\} \simeq |r|^2\Delta^2_{topo} + \left(1+\frac{1}{MN}\right)^{-1}\frac{1}{MN}\left(1-|r|^2\right)^{1.32\sqrt{MN}}$$

$$\Delta^2_{speckle} = \left(1+\frac{1}{MN}\right)^{-1}\frac{1}{MN}\left(1-|r|^2\right)^{1.32\sqrt{MN}}$$

Coherence Estimation

Complex Coherence Speckle Noise Model

Simulated data considering no topography



Low coherence bias due to the additive speckle noise component

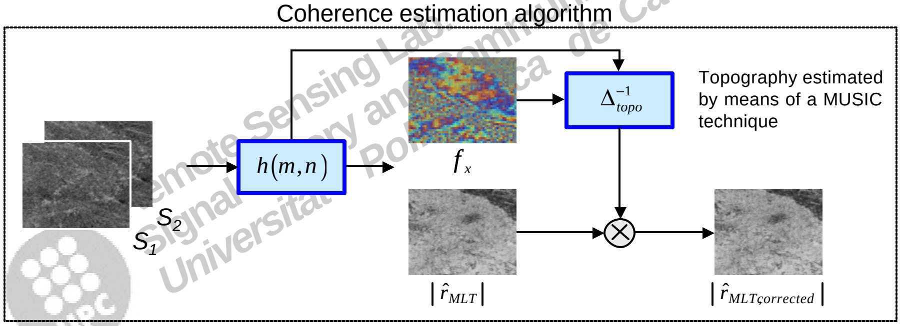
Non Biased Coherence Estimation

Topographic Bias Compensation

Inversion of the underestimation introduced by the topographic component

$$\Delta_{topo} = \left| \frac{1}{M} \frac{\sin\left(\frac{M}{2} w_x\right)}{\sin\left(\frac{w_x}{2}\right)} \frac{1}{N} \frac{\sin\left(\frac{N}{2} w_y\right)}{\sin\left(\frac{w_y}{2}\right)} \right|$$

Coherence estimation algorithm



Details

Topography must be estimated from data ➡ *Error source*

Non Biased Coherence Estimation

Filtering via Magnitude Flat Filters

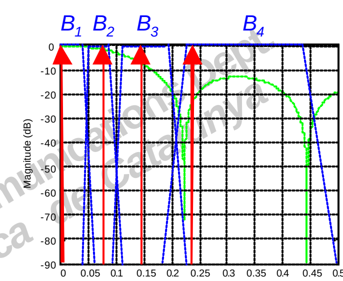
All band-pass filters can not be used as noise is not eliminated, i.e., signal is not correctly estimated

⬇

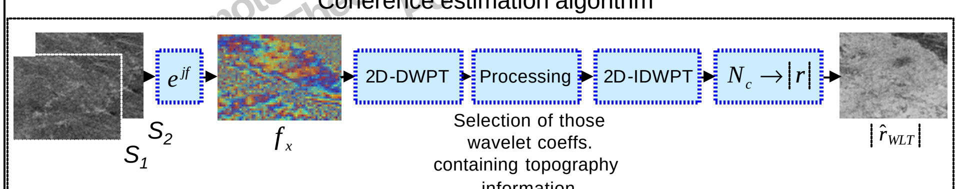
Multiscale filter banks

⬇

2D Discrete Wavelet Transform



Coherence estimation algorithm



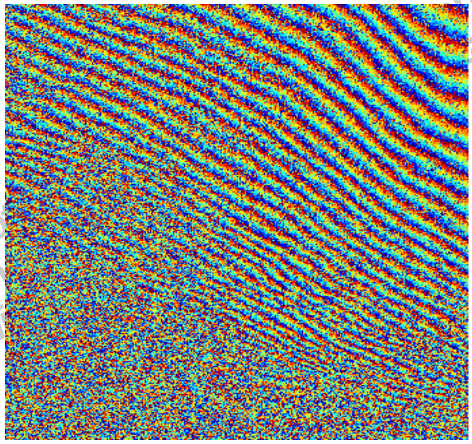
Details

Topography is *not* estimated from data

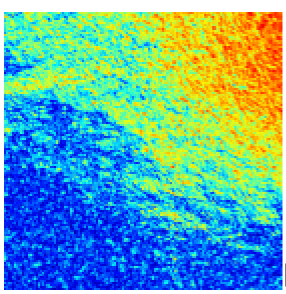
Non Biased Coherence Estimation

Mt. Etna (Italy) X-band interferogram acquired by the airborne ESAR system (DE)

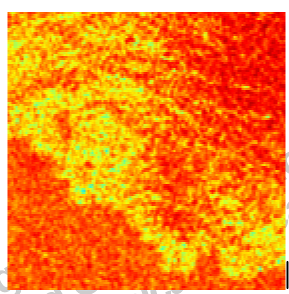
Dimensions: 512x512 pixels



Non Biased Coherence Estimation

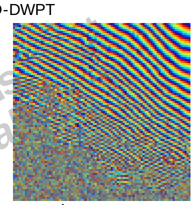


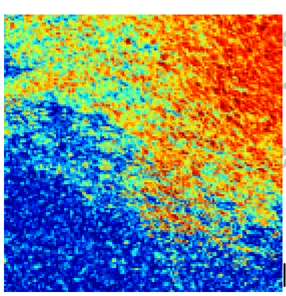
$|\hat{\gamma}_{MLT}|$



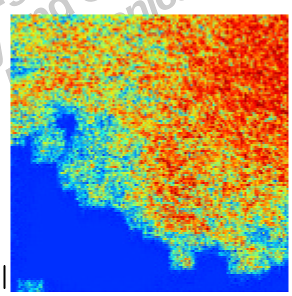
$|\hat{\gamma}_{MLT, comp.}|$

High spatial resolution InSAR phase estimation based on the 2D-DWPT

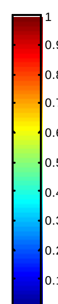


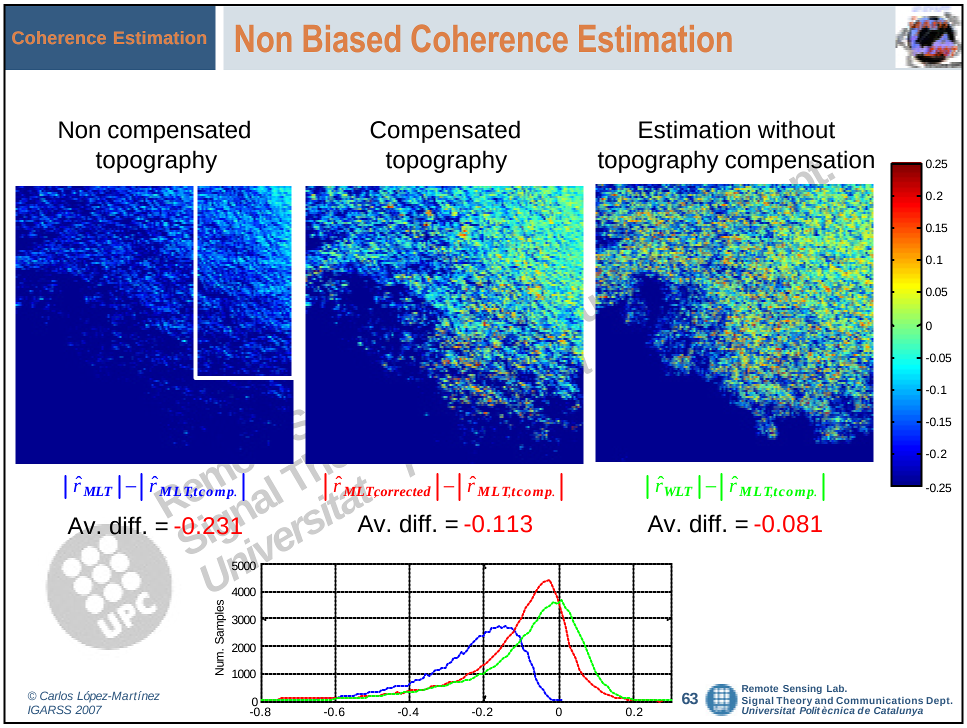


$|\hat{\gamma}_{MLT corrected}|$



$|\hat{\gamma}_{WLT}|$




Coherence Estimation
Non Biased Coherence Estimation
Non compensated topography
Compensated topography
Estimation without topography compensation
$|\hat{r}_{MLT}|-|\hat{r}_{MLT,tcomp.}|$
Av. diff. = -0.231
$|\hat{r}_{MLTcorrected}|-|\hat{r}_{MLT,tcomp.}|$
Av. diff. = -0.113
$|\hat{r}_{WLT}|-|\hat{r}_{MLT,tcomp.}|$
Av. diff. = -0.081
Num. Samples
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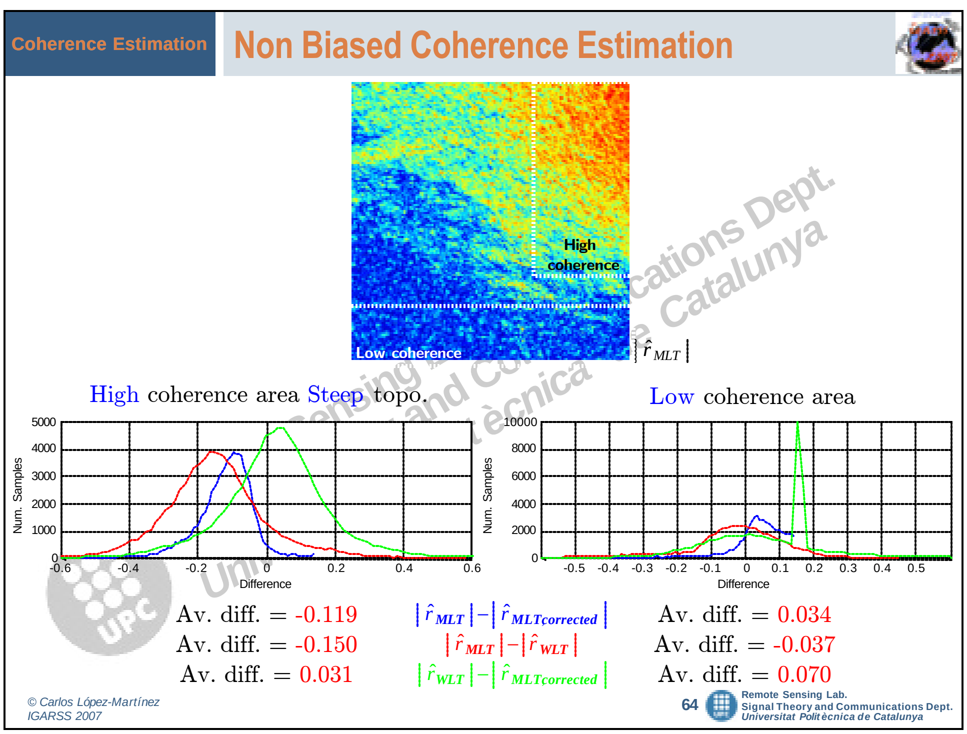
Coherence Estimation
Non Biased Coherence Estimation
High coherence
Low coherence
$|\hat{r}_{MLT}|$
High coherence area Steep topo.
Low coherence area
Num. Samples
Difference
Av. diff. = -0.119
Av. diff. = -0.150
Av. diff. = 0.031
$|\hat{r}_{MLT}|-|\hat{r}_{MLTcorrected}|$
$|\hat{r}_{MLT}|-|\hat{r}_{WLT}|$
$|\hat{r}_{WLT}|-|\hat{r}_{MLTcorrected}|$
Av. diff. = 0.034
Av. diff. = -0.037
Av. diff. = 0.070
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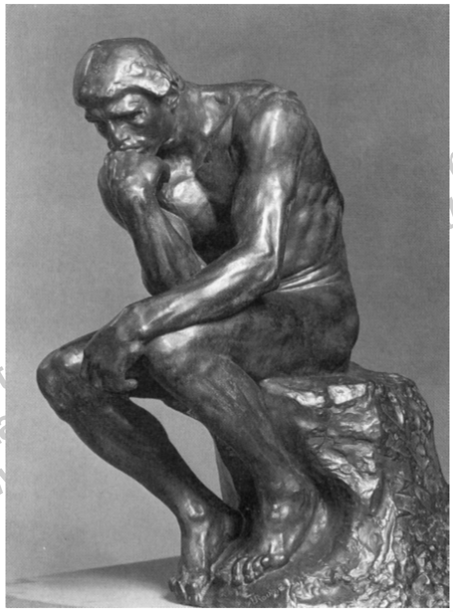


Polarimetric Information Estimation

Incoherent Target Decomposition Theorems

In SAR polarimetry, incoherent target decomposition theorems, allow the physical interpretation of the averaged scattering mechanism in distributed scatterers

- Decomposition theorems represent a way to perform quantitative remote sensing

$$E\{\mathbf{k}\mathbf{k}^H\} = \begin{bmatrix} E\{S_{hh}S_{hh}^H\} & E\{\sqrt{2}S_{hh}S_{hv}^H\} & E\{S_{hh}S_{vv}^H\} \\ E\{\sqrt{2}S_{hv}S_{hh}^H\} & E\{2S_{hv}S_{hv}^H\} & E\{\sqrt{2}S_{hv}S_{vv}^H\} \\ E\{S_{vv}S_{hh}^H\} & E\{\sqrt{2}S_{vv}S_{hv}^H\} & E\{S_{vv}S_{vv}^H\} \end{bmatrix}$$

➡ Theoretical average mechanism

$$\langle \mathbf{k}\mathbf{k}^H \rangle = \begin{bmatrix} \langle S_{hh}S_{hh}^H \rangle & \langle \sqrt{2}S_{hh}S_{hv}^H \rangle & \langle S_{hh}S_{vv}^H \rangle \\ \langle \sqrt{2}S_{hv}S_{hh}^H \rangle & \langle 2S_{hv}S_{hv}^H \rangle & \langle \sqrt{2}S_{hv}S_{vv}^H \rangle \\ \langle S_{vv}S_{hh}^H \rangle & \langle \sqrt{2}S_{vv}S_{hv}^H \rangle & \langle S_{vv}S_{vv}^H \rangle \end{bmatrix}$$

➡ Estimated average mechanism

Depends on the speckle filtering process

H/A/a Decomposition

- Monostatic Pauli scattering vector $\mathbf{k} = \begin{bmatrix} S_{hh} & \sqrt{2}S_{hv} & S_{vv} \end{bmatrix}^T$
- Local estimate of the coherency matrix $\mathbf{Z}_n = \frac{1}{n}\sum_{i=1}^{n}\mathbf{k}_i \cdot \mathbf{k}_i^H = \frac{1}{n}\sum_{i=1}^{n}\mathbf{Z}_{i,n}$
- Eigenvectors/Eigenvalues analysis of the coherency matrix

$$\mathbf{Z}_n = \mathbf{U}_3\mathbf{S}\mathbf{U}_3^{-1} = \begin{bmatrix} \mathbf{u}_1 & \mathbf{u}_2 & \mathbf{u}_3 \end{bmatrix} \begin{bmatrix} l_1 & 0 & 0 \\ 0 & l_2 & 0 \\ 0 & 0 & l_3 \end{bmatrix} \begin{bmatrix} \mathbf{u}_1 & \mathbf{u}_2 & \mathbf{u}_3 \end{bmatrix}^H$$

- Eigenvectors are orthonormal
- Eigenvalues are real $?_1 > ?_2 > ?_3$

$$P_i = \frac{l_i}{\sum_{k=1}^{3} l_k}$$

H/A/a Decomposition

Parametrization of the SU(3) unitary matrix

$$\mathbf{U}_3 = \begin{bmatrix} \cos(a_1) & \cos(a_2) & \cos(a_3) \\ \sin(a_1)\cos(b_1)e^{jd_1} & \sin(a_2)\cos(b_2)e^{jd_2} & \sin(a_3)\cos(b_3)e^{jd_3} \\ \sin(a_1)\sin(b_1)e^{jg_1} & \sin(a_2)\sin(b_2)e^{jg_2} & \sin(a_3)\sin(b_3)e^{jg_3} \end{bmatrix}$$

Target 1 | Target 2 | Target 3

Decomposition basis

l_1 | l_2 | l_3

Decomposition coefficients

H/A/a Decomposition

Eigenvalues l_1 l_2 l_3 : Roll Invariant

Probabilities P_1 P_2 P_3 : Roll Invariant

ENTROPY

(Degree of Randomness
Statistical Disorder)

$$H = -\sum_{i=1}^{3} P_i \log_3(P_i)$$

Pure Target
l_1=SPAN l_2=0 l_3=0
$H = 0$

Distributed Target
$l_1 = l_2 = l_3 = SPAN/3$
$H = 1$

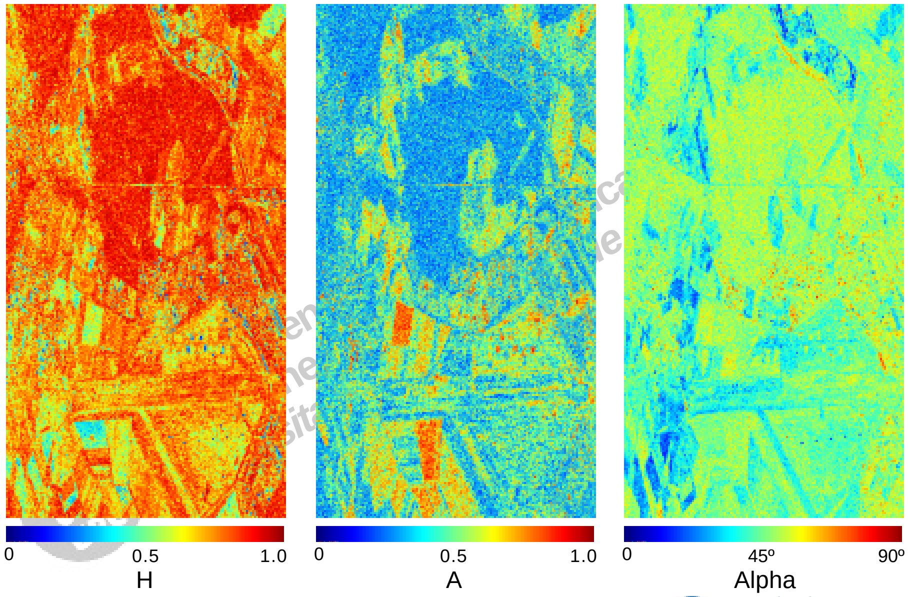
ANISOTROPY

(Eigenvalues spectrum)

$$A = \frac{l_2 - l_3}{l_2 + l_3}$$

Polarimetric Information Estimation

H/A/α Decomposition



Polarimetric Information Estimation

Multidimensional Speckle Noise

Interest of Multidimensional SAR imagery ➡ QUANTITATIVE REMOTE SENSING

⬇

Determined by the capacity to reduce Speckle noise, i.e., the capacity to estimate correctly the Bio- and Geophysical parameters

⬇ ⬇

Speckle noise reduction | **Speckle noise effects**

Low averaging level
- Problem not considered

High averaging level
- Brute force method
- Information mixture
- Spatial resolution losses

?

Multidimensional/PolSAR Data Model

Covariance matrix formulism

- Gaussian scattering model for homogeneous areas

$$\mathbf{k} = [S_1, S_2, \ldots, S_m]^T \qquad S_k = \mathcal{N}_{C^2}\left(0, s^2/2\right) \quad \Rightarrow \quad \mathbf{Z}_n = \frac{1}{n}\sum_{i=1}^{n} \mathbf{k}_i \mathbf{k}_i^H \qquad \mathbf{Z}_n \sim \mathcal{W}(n, \mathbf{C})$$

Wishart PDF

$$p_{\mathbf{Z}_n}(\mathbf{Z}_n) = \frac{n^{mn} |\mathbf{Z}_n|^{n-m}}{|\mathbf{C}|^n \tilde{\Gamma}_m(n)} \operatorname{etr}\left(-n\mathbf{C}^{-1}\mathbf{Z}_n\right)$$

Limitations $\begin{cases} \mathbf{Z}_n \;\; \mathbf{C} \text{ Positive definite} \\ n \geq m \end{cases}$

Two eigenvalue decomposition

- Physical information retrieved via the H/A/a decomposition

$$\mathbf{S} = \mathbf{Q}'^H \mathbf{C}\mathbf{Q}' \quad \longleftrightarrow \quad ? = \mathbf{Q}^H \mathbf{Z}_n \mathbf{Q}$$

?

$$\mathbf{S} = \begin{bmatrix} l_1 & 0 & \cdots & 0 \\ 0 & l_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & l_m \end{bmatrix} \quad \longleftrightarrow \quad ? = \begin{bmatrix} l_1 & 0 & \cdots & 0 \\ 0 & l_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & l_m \end{bmatrix}$$

True eigenvalues | Sample eigenvalues

Joint Eigenvalues Distribution

Eigen decomposition transformation

The eigen decomposition can be regarded as a transformation of independent parameters

$$\mathbf{Z}_n = \mathbf{Q}?\mathbf{Q}^H$$

Eigenvectors
Eigenvalues

Jacobian of the transformation found by means of the exterior product of differential forms (skew symmetric product)

$$(d\mathbf{Z}_n) = \prod_{i<j}^{m} (l_i - l_j)^2 (\mathbf{Q}^H d\mathbf{Q})(d?) \qquad \infty > l_1 \geq l_2 \geq \cdots \geq l_m \geq 0$$

Joint PDF for the eigenvalues/eigenvectors of the matrix $\mathbf{Z}_n$

$$p_{\mathbf{Q}?\mathbf{Q}^H}(\mathbf{Q}?\mathbf{Q}^H) = \frac{n^{mn} \prod_{i=1}^{m} l_i^{n-m} \prod_{i<j}^{m} (l_i - l_j)^2}{\tilde{\Gamma}_m(n) \prod_{i=1}^{m} l_i^n} \operatorname{etr}\left(-n\mathbf{C}^{-1}\mathbf{Q}?\mathbf{Q}^H\right)(\mathbf{Q}^H d\mathbf{Q})$$

Joint eigenvalues PDF: The dependence on Q must be eliminated

Polarimetric Information Estimation

Joint Eigenvalues Distribution

$$p_{?}(?) = \frac{p^{m(m-1)} n^{mn} \prod_{i=1}^{m} l_i^{n-m} \prod_{i<j}^{m} (l_i - l_j)^2}{\tilde{\Gamma}_m(n)\tilde{\Gamma}_m(m) \prod_{i=1}^{m} l_i^n} \int_{U(m)} \mathrm{etr}\left(-n\mathbf{C}^{-1}\mathbf{Q}?\mathbf{Q}^H\right)(d\mathbf{Q})$$

Joint eigenvalues PDF: Integral expression over U(m)

Group representation theory

Fourier-like analysis of functions in the space *U(m)* provided by the *Group Representation* theory

$$p_{?}(?) = \frac{n^{mn} p^{m(m-1)} \prod_{i=1}^{m} l_i^{n-m} \prod_{i<j}^{m} (l_i - l_j)^2}{\tilde{\Gamma}_m(n)\tilde{\Gamma}_m(m) \prod_{i=1}^{m} l_i^n} \, {}_0\tilde{F}_0\left(-n\mathbf{S}^{-1}, ?\right)$$

Complex hypergeometric function of double matrix argument

Joint eigenvalues PDF: Infinite series expression

Polarimetric Information Estimation

Joint Eigenvalues Distribution

Soliton theory and τ-functions

τ functions appear as a sort of potential which give rise to the system of N.L.P.D.E. which solution is the soliton.

$$t\left(n,\mathbf{t},\mathbf{t}^*\right) = \sum_{\kappa,\lambda} K_{\kappa\lambda} s_\kappa(\mathbf{t}) s_\lambda\left(\mathbf{t}^*\right) \Rightarrow t_r\left(n,\mathbf{t},\mathbf{t}^*\right) = \sum_{\kappa} r_\kappa(n) s_\kappa(\mathbf{t}) s_\kappa\left(\mathbf{t}^*\right)$$

τ functions of hypergeometric type

$$t_r(M,\mathbf{X},\mathbf{Y}) = {}_p\tilde{F}_q\left(M+a_1, M+a_2, \ldots, M+a_p; M+b_1, M+b_2, \ldots, M+b_q; \mathbf{X},\mathbf{Y}\right)$$

τ functions of hypergeometric type and matrix argument

Determinant expression for τ functions of hypergeometric type and matrix argument

$$t_r(M,\mathbf{X},\mathbf{Y}) = c_m(M) \frac{\left| t_r\left(M-m+1, x_i, y_j\right) \right|_{i,j=1}^{m}}{\Delta(\mathbf{x})\Delta(\mathbf{y})}$$

$$c_1 = 1, \quad c_m(M) = \prod_{k=1}^{m-1} \left(r(M-m+k)\right)^{k-m}, \quad m > 1$$

$$t_r\left(M-m+1, x_i, y_j\right) = 1 + r(M-m+1)x_i y_j + r(M-m+1)r(M-m+2)x_i^2 y_j^2 + \ldots$$

Joint sample eigenvalues PDF: Determinant expression

$$p_{?}(?) = \frac{p^{m(m-1)} n^{\frac{m}{2}(2n-m+1)}}{\tilde{\Gamma}_m(m)\tilde{\Gamma}_m(n)} \prod_{k=1}^{m-1} \left(\frac{1}{k}\right)^{k-m} \frac{\prod_{i=1}^{m} l_i^{n-m} \prod_{i<j}^{m} (l_i - l_j)}{\prod_{i=1}^{m} l_i^n \prod_{i<j}^{m} \left(l_j^{-1} - l_i^{-1}\right)} \left| \exp\left(-n\frac{l_j}{l_i}\right) \right|_{i,j=1}^{m}$$

Sorted sample eigenvalues $\infty > l_1 \geq l_2 \geq \cdots \geq l_m \geq 0$

Joint Eigenvalues Distribution

Joint sample eigenvalues PDF: Simplified expression

$$p_{?}(?) = K(m,n,l_1,\ldots,l_m)\prod_{i=1}^{m} l_i^{n-m}\prod_{i<j}^{m}(l_i - l_j)\sum_{p\in S_m}\operatorname{sgn}(p)\prod_{i=1}^{m}\exp\left(-n\frac{l_i}{l_{p_i}}\right)$$

$$K(m,n,l_1,\ldots l_m) = \frac{p^{m(m-1)} n^{\frac{m}{2}(2n-m+1)}}{\tilde{\Gamma}_m(m)\tilde{\Gamma}_m(n)}\frac{\prod_{k=1}^{m-1} k^{m-k}}{\prod_{i=1}^{m} l_i^{n}\prod_{i<j}^{m}(l_j^{-1}-l_i^{-1})}$$

Sorted sample eigenvalues $\infty > l_1 \geq l_2 \geq \cdots \geq l_m \geq 0$

Dependences and analysis of the joint eigenvalues PDF

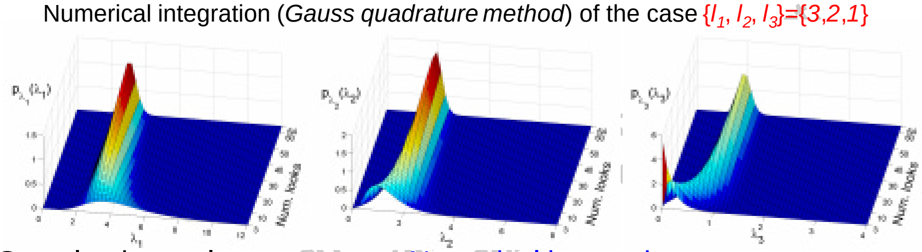
- n: Number of looks. It indicates the effects of the filtering strength
- m: Number of channels. Equal to 3 for PolSAR data
- **C**, S : True information to be retrieved. Important effect over signal estimation
- Sample eigenvalues PDF not separable ➡ Sample eigenvalues not independent

Joint Eigenvalues Distribution

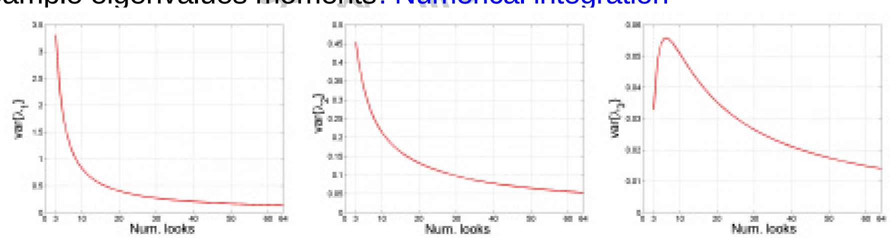
Sample eigenvalues PDFs: Numerical integration

Analytical integration too complex due to the condition $\infty > l_1 \geq l_2 \geq \cdots \geq l_m \geq 0$

Numerical integration (*Gauss quadrature method*) of the case $\{l_1, l_2, l_3\}=\{3,2,1\}$



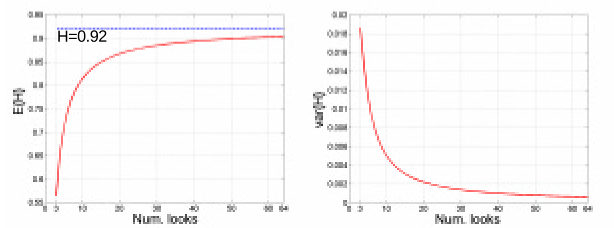
Sample eigenvalues moments: Numerical integration



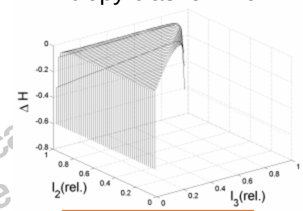
Overestimated | Over/Underestimated | Underestimated

Joint Eigenvalues Distribution

Entropy (H) moments: Numerical integration

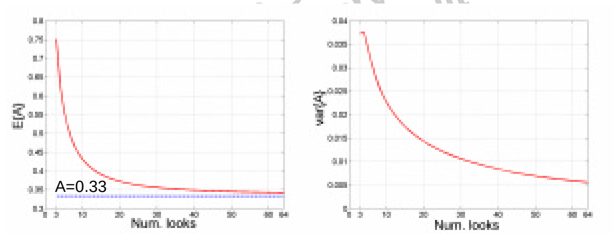


Entropy bias for n=64

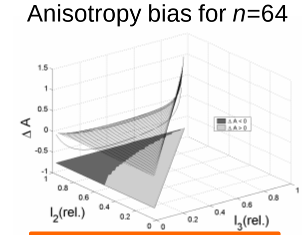


Underestimated

Anisotropy (A) moments: Numerical integration



Anisotropy bias for n=64



Over/Underestimated

AQ-MLE Eigenvalues Estimation

Alternative eigenvalues estimation

Classical estimation approach: Maximum Likelihood Approach (MLE)

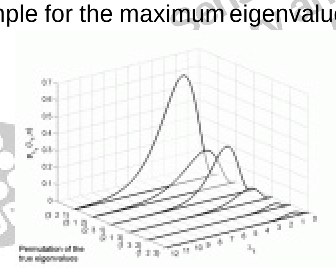
Direct application not possible due to the necessity to maximize a determinant

Asymptotic approach

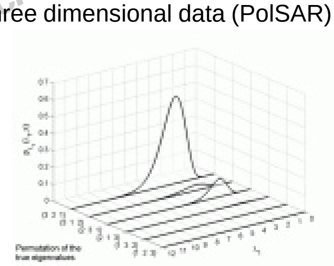
Asymptotic analysis of the sample eigenvalues with respect to the number of looks provided two main results: asymptotic *expressions* and asymptotic *estimation*

$$\left|\exp\left(-n\frac{l_i}{l_j}\right)\right|_{i,j=1}^{m} = \sum_{p\in S_m} \operatorname{sgn}(p)\prod_{i=1}^{m}\exp\left(-n\frac{l_i}{l_{p_i}}\right) \xrightarrow[n\to\infty]{} \prod_{i=1}^{m}\exp\left(-n\frac{l_i}{l_i}\right)$$

Example for the maximum eigenvalue in three dimensional data (PolSAR)



n=3



n=8

AQ-MLE Eigenvalues Estimation

Asymptotic MLE approach

Results in a non-invertible equations system

$$l_i = l_i + \frac{l_i}{n}\sum_{j\neq i}^{m}\frac{l_j}{l_i - l_j} + O\left(n^{-1}\right) \qquad i = 1,2,\ldots,m$$

Sample eigenvalues are asymptotic estimators of the true eigenvalues

Speckle noise introduces an asymptotic bias on the sample eigenvalues

Asymptotic quasi MLE (AQ-MLE) approach

Necessity to simplify algebraic expressions in order to find an approximate solution for the equations system

AQ-MLE $$\hat{l}_i = l_i - \frac{l_i}{n}\sum_{j\neq i}^{m}\frac{l_j}{l_i - l_j} - O\left(n^{-1}\right) \qquad i = 1,2,\ldots,m$$

Drawback: Error in the same order as the eigenvalues correction !!!

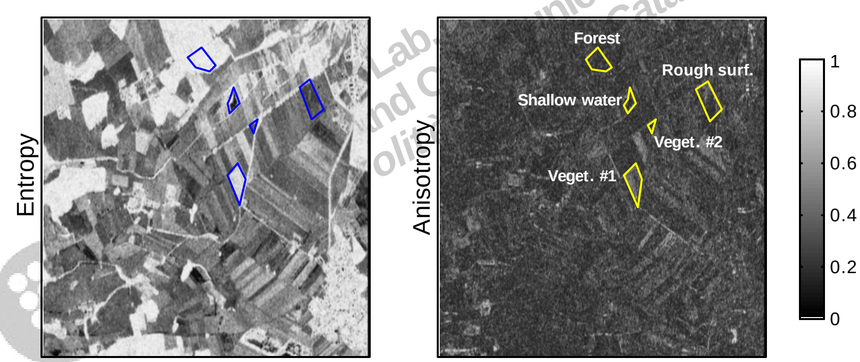
Results

Fully polarimetric L-band dataset acquired with the E-SAR system

Data correspond to the ALLING test-site

Ground truth data available

5 homogeneous areas selected to cover all the Entropy (H) range

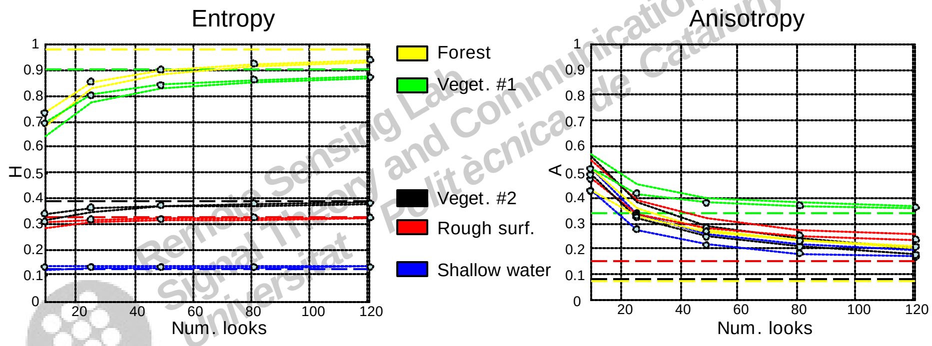


Results

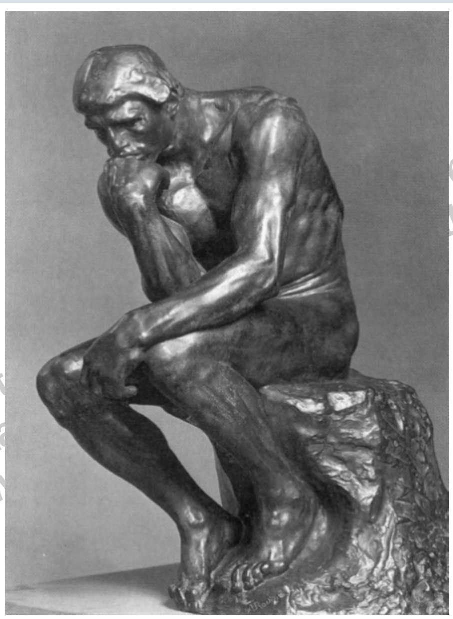
H/A estimated values

Dependence on the number of looks: $n \times n$ averaging windows

Dependence on the *true* values of H/A: Average over all the homogeneous area



A minimum number of looks are necessary to retrieve unbiased physical parameters



Multidimensional SAR Data Estimation

Multilook Estimation

Covariance matrix estimation based on a blind spatial averaging

- Multilooking
- BoxCar filter

$$\mathbf{Z}_n = \frac{1}{n}\sum_{k=1}^{n} \mathbf{k}\mathbf{k}^H = \begin{bmatrix} \frac{1}{n}\sum_{k=1}^{n} S_1(k)\,S_1^*(k) & \frac{1}{n}\sum_{k=1}^{n} S_1(k)\,S_2^*(k) & \cdots & \frac{1}{n}\sum_{k=1}^{n} S_1(k)\,S_m^*(k) \\ \frac{1}{n}\sum_{k=1}^{n} S_2(k)\,S_1^*(k) & \frac{1}{n}\sum_{k=1}^{n} S_2(k)\,S_2^*(k) & \cdots & \frac{1}{n}\sum_{k=1}^{n} S_2(k)\,S_m^*(k) \\ \vdots & \vdots & \ddots & \vdots \\ \frac{1}{n}\sum_{k=1}^{n} S_m(k)\,S_1^*(k) & \frac{1}{n}\sum_{k=1}^{n} S_m(k)\,S_2^*(k) & \cdots & \frac{1}{n}\sum_{k=1}^{n} S_m(k)\,S_m^*(k) \end{bmatrix}$$

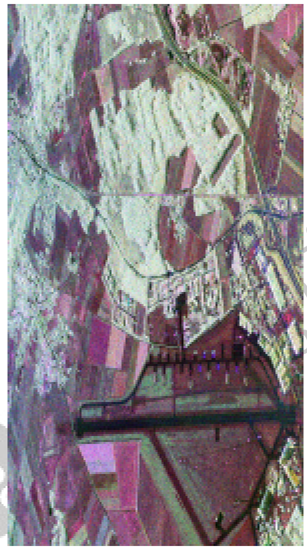
This filter does not take into account the signal morphology neither speckle noise properties

- Good estimation capabilities = Good speckle noise reduction
- Does not consider neither multiplicative nor additive speckle properties
- Spatial Resolution Loss, blurring edges, erasing thin lines, loss of linear or point features

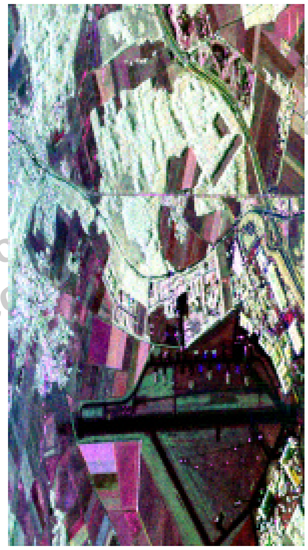
Multidimensional SAR Data Estimation

Multilook Estimation

Original RGB



7x7 MLT RGB



|Shh|
|Shv|
|Svv|

L-band (1.3 GHz) fully PolSAR data
E-SAR system. Oberpfaffenhofen test area (D)

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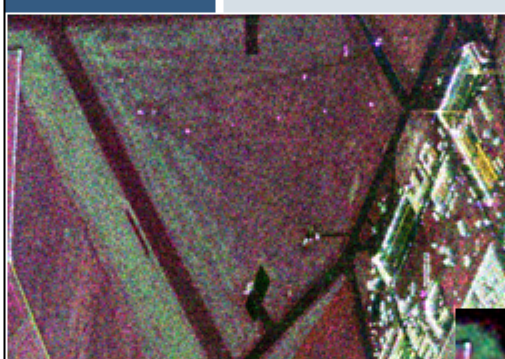
Multidimensional SAR Data Estimation

Multilook Estimation

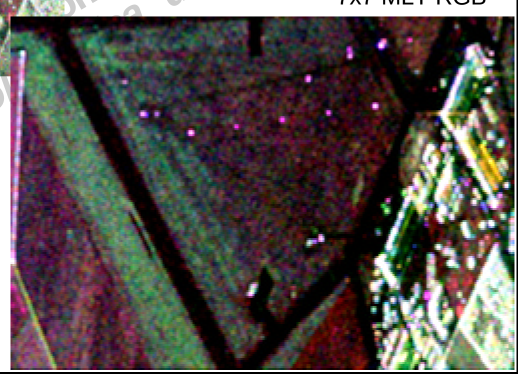


Original RGB

L-band (1.3 GHz) fully PolSAR data
E-SAR system. Oberpfaffenhofen test area (D)

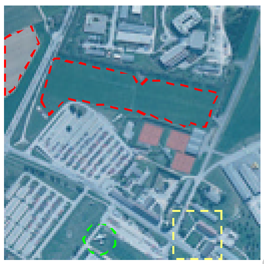
|Shh| |Shv| |Svv|

7x7 MLT RGB



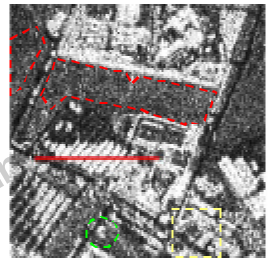
Multilook Estimation

Considerations about speckle noise reduction



Optical image DLR OP

SAR images reflex the Nature's complexity



SAR image DLR OP

Homogeneous areas	Image details	Heterogeneous areas
Maintain useful information (s)	Maintain spatial details (Shape and value)	Maintain both
RADIOMETRIC RESOLUTION	SPATIAL RESOLUTION	LOCAL ANALYSIS

Image data: S_{hh} amplitude. E-SAR L-band system

Local Statistics Linear Filter

Local statistics linear filter (Lee filter)

Filter form $\hat{I}(x,r) = a\,E\{I(x,r)\} + b\,I(x,r)$

Signal noise model $I(x,r) = s(x,r)\,n(x,r)$

Minimization criteria (MMSE) $\min_{(a,b)} J = E\left\{\left(\hat{I}(x,r) - I(x,r)\right)\right\}$

MMSE gives $a = \frac{1}{E\{n\}} - b$ $\quad b = E\{n\}\frac{\mathrm{var}(s)}{\mathrm{var}(I)}$

$$\hat{I}(x,r) = \frac{E\{I(x,r)\}}{E\{n\}} + b\left(I(x,r) - E\{I(x,r)\}\right)$$

Statistics need to be derived from noisy data

$a = \frac{1}{E\{n\}} - b = 1 - b$ ← $E\{n\} = 1$

$b = E\{n\}\frac{\mathrm{var}(s)}{\mathrm{var}(I)} = \frac{\mathrm{var}(I) - E^2\{I\}s_n^2}{\mathrm{var}(I)\left(1 + s_n^2\right)}$ ← Information estimated from data, $E\{n\} = 1$

$$\hat{I}(x,r) = E\{I(x,r)\} + b\left(I(x,r) - E\{I(x,r)\}\right)$$

Local statistics $E^2\{I(x,r)\}$ $\quad \mathrm{var}\{I\}$

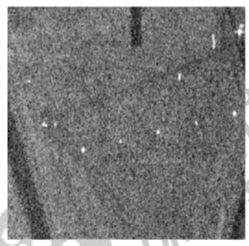
A priori information $s_n^2 = \mathrm{var}(n) = \frac{1}{N}$

Local Statistics Linear Filter

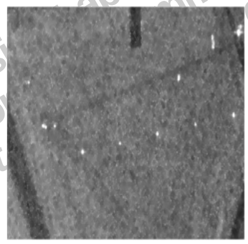
$$\hat{I}(x,r)=E\{I(x,r)\}+b\left(I(x,r)-E\{I(x,r)\}\right)$$

$\mathrm{var}(I) >> E^2\{I\} \Rightarrow b \rightarrow 1$ Multiplicative noise model can not explain data variability

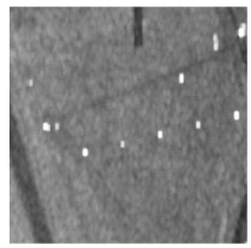
$\mathrm{var}(I) \approx E^2\{I\}s_n^2 \Rightarrow b \rightarrow 0$ Multiplicative noise model can explain data variability



Original SAR intensity image



Filtered SAR intensity image
Lee Filter



Filtered SAR intensity image
Boxcar Filter

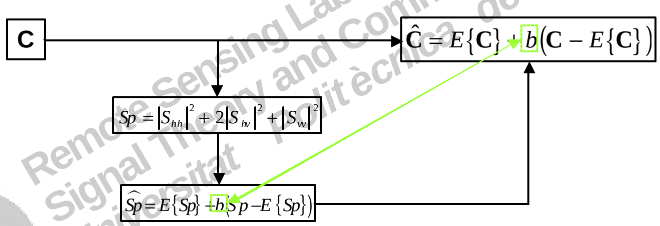
Local Statistics Linear Filter

Polarimetric Lee filter

Nowadays is the most employed polarimetric filtering solution

Extension of the linear scalar Lee filter for SAR images by considering a multiplicative speckle noise model over all the covariance matrix entries

Working principles

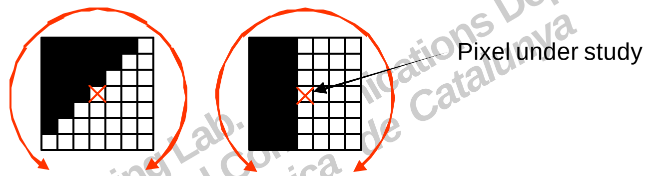


Linear scalar
Lee filter

Local Statistics Linear Filter

Refined Lee filter

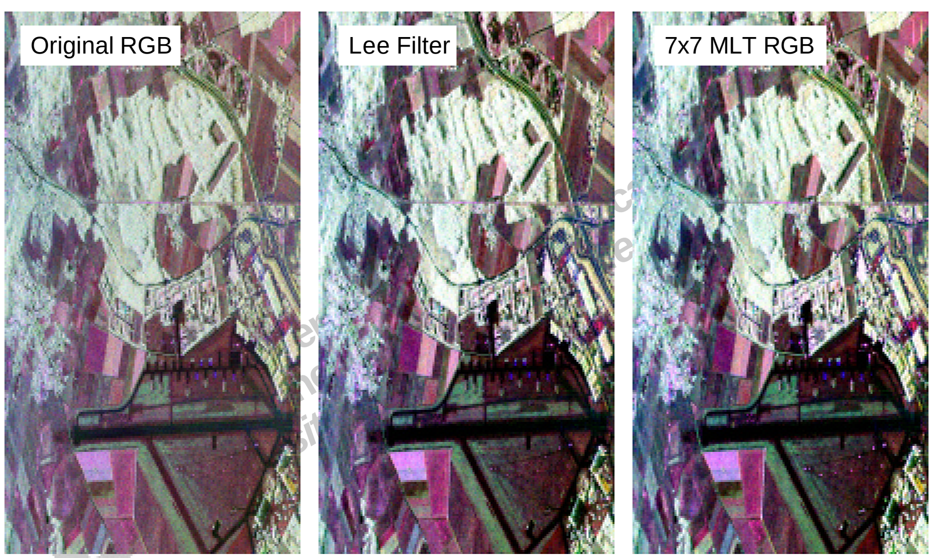
Statistics estimation in windows selected according to the signal morphology in order to retain edges, spatial feature and point targets



The extension of the scalar linear Lee filter presents limitations

- Not based on the multiplicative-additive speckle noise model. This limits the capacity to reduce noise in those images areas characterized by low correlation ➡ The elements of the covariance matrix can be processed differently, but according to the right speckle noise model
- The a priori information in the span image s_n^2 is no longer a constant as the noise content in span depends on the data's correlation structure

Local Statistics Linear Filter

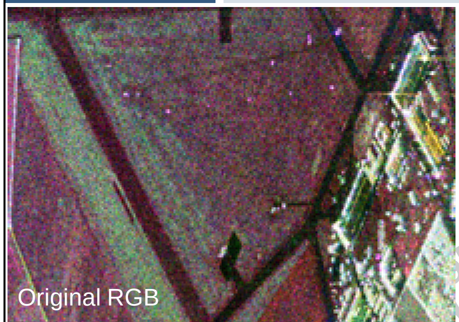


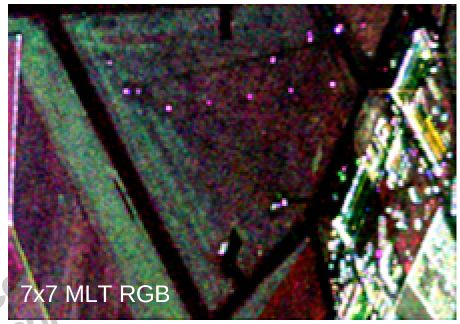
|Shh| |Shv| |Svv|

L-band (1.3 GHz) fully PolSAR data
E-SAR system. Oberpfaffenhofen test area (D)

Multidimensional SAR Data Estimation

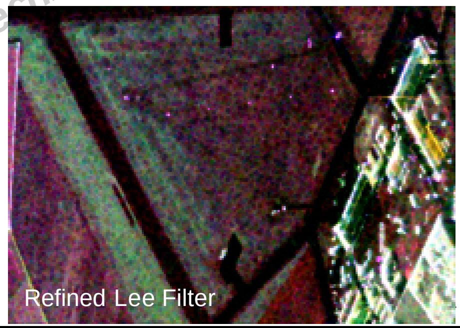
Local Statistics Linear Filter





|Shh| |Shv| |Svv|

L-band (1.3 GHz) fully PolSAR data
E-SAR system. Oberpfaffenhofen test area (D)



Multidimensional SAR Data Estimation

Single-look Multidimensional Speckle Noise Model

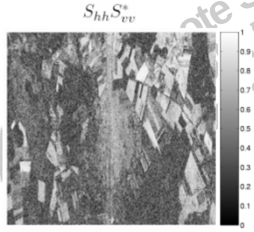
Hermitian product speckle noise model: $S_i S_j^* = \underbrace{y\,\bar{z}_n n_m N_c e^{j\phi_x}}_{\text{Multiplicative term}} + \underbrace{y\,(|r| - N_c \bar{z}_n)\, e^{j\phi_x} + y\,(n_{ar} + j n_{ai})}_{\text{Additive term}}$

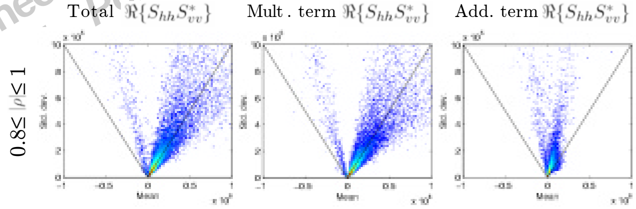
C. López-Martínez and X. Fàbregas, "Polarimetric SAR Speckle Noise Model"
IEEE TGRS, vol. 41, no. 10, pp. 2232 – 2242, Oct. 2003

Multiplicative speckle noise component: n_m ➡ Important for high coherence areas

Additive speckle noise component: $n_{ar} + j n_{ai}$ ➡ Important for low coherence areas

Combination controlled by complex coherence





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Multidimensional SAR Data Estimation

Multilook Multidimensional Speckle Noise Model

Hermitian product speckle noise model: $\left\langle S_i S_j^* \right\rangle_n = \underbrace{y n_m \exp(jf_x)}_{Multiplicative term} + \underbrace{y\left(|r| - N_c \bar{z}_n\right)\exp(jf_x) + y\left(n_{ar} + jn_{ai}\right)}_{Additive term}$

C. López-Martínez and E. Pottier, "Extended multidimensional speckle noise model and its implicatio ns on the estimation of physical information," *IGARSS 06*, Denver (CO) USA, July 2006

Multiplicative speckle noise component

- Dominant for high coherences
- Modulated by phase information

$E\{n_m\} = N_c \bar{z}_n$ $\qquad s_{n_m}^2 = N_c^2 \dfrac{\left(1+|r|^2\right)}{2n}$

Additive speckle noise component

- Dominant for low coherences
- Not affected by phase information

$E\{n_{ar}\} = E\{n_{ai}\} = 0$ $\qquad s_{n_{ar}}^2 = s_{n_{ai}}^2 \simeq \dfrac{1}{2n}\left(1-|r|^2\right)^{1.32\sqrt{n}}$

Effect of the approximations

- Mean value IS NOT approximated ➡ No loss of information

$$\lim_{n\to\infty}\left\{y n_m \exp(jf_x) + y\left(|r| - N_c \bar{z}_n\right)\exp(jf_x) + y\left(n_{ar} + jn_{ai}\right)\right\} = y|r|\exp(jf_x)$$

- Std. Dev. ARE approximated

Multidimensional SAR Data Estimation

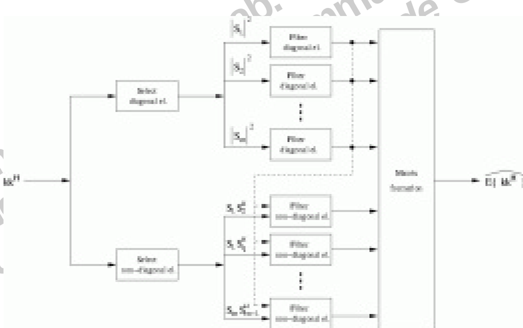
Multidimensional Speckle Noise Filtering

Define a multidimensional SAR data filtering strategy based on the multidimensional speckle noise model

Element to consider: Covariance matrix

↳ Diagonal element: Multiplicative noise source

↳ Non-diagonal element: Multiplicative and additive noise sources combined according to the complex correlation coefficient



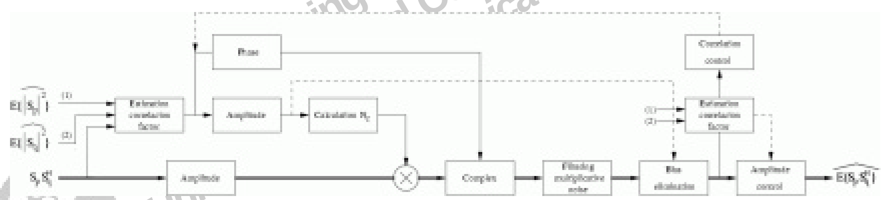
Multidimensional Speckle Noise Filtering

Diagonal element processing

- Any alternative to filter multiplicative noise can be considered
- Non-iterative scheme

Off-diagonal element processing

The filter uses the Hermitian product speckle model: $S_i S_j^* = \underbrace{y\,\bar{z}_n n_m N_c e^{j\phi_x}}_{Multiplicative\,term} + \underbrace{y\,(|\rho| - N_c \bar{z}_n)e^{j\phi_x} + y\,(n_{ar} + j n_{ai})}_{Additive\,term}$



- Iterative scheme to take benefit of the improved coherence estimation
- This strategy filters differently the covariance matrix elements

Results: Simulated Multidimensional SAR Data

Quantitative evaluation of the filter difficult with experimental SAR data due to speckle

⬇

Necessity to consider an evaluation with simulated multidimensional SAR data

The application considered in this paper is: PolSAR data
Nevertheless results and conclusions may be extended to any multidimensional SAR

PolSAR data simulated according to the covariance matrix

$$\mathbf{C} = E\{\mathbf{k}\mathbf{k}^H\} = \begin{bmatrix} 1 & 0 & |\rho|e^{j\phi_x} \\ 0 & 0.75 & 0 \\ |\rho|e^{-j\phi_x} & 0 & 1 \end{bmatrix}$$

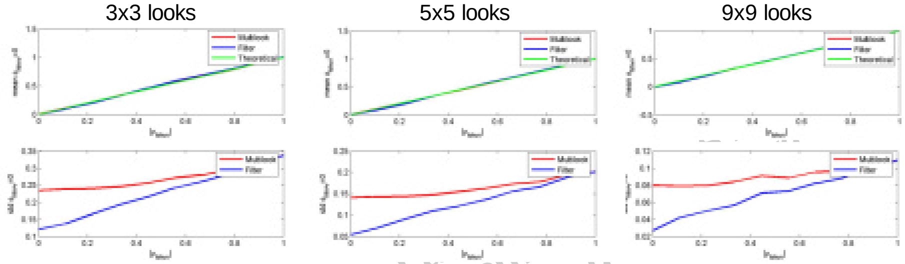
Matrix parameterized by the co-polar complex correlation coefficient

Performed tests:

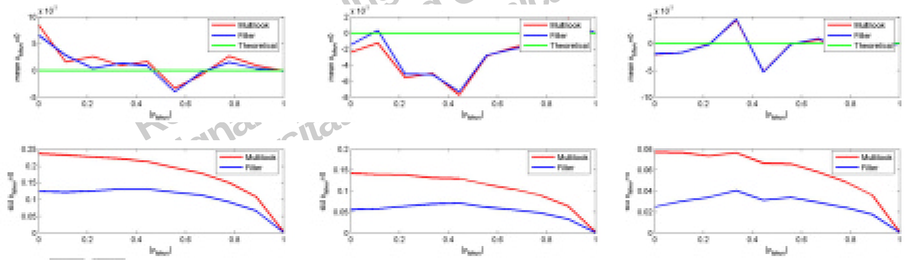
- Covariance matrix elements
  - Analysis of: Real and imaginary parts, amplitude, phase, correlation
- Covariance matrix
  - Analysis of: Eigendecomposition, polarimetric signatures

Results: Simulated Multidimensional SAR Data

3x3 looks | 5x5 looks | 9x9 looks



$\Re\{S_{hh}S_{vv}^{*}\} \quad f_x = 0$



$\Im\{S_{hh}S_{vv}^{*}\} \quad f_x = 0$

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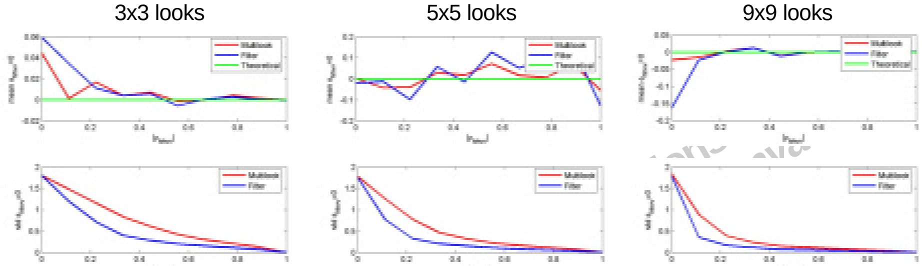
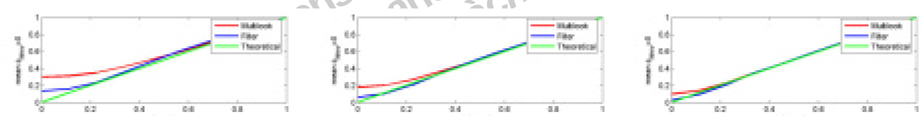
Results: Simulated Multidimensional SAR Data

3x3 looks | 5x5 looks | 9x9 looks



f_{xhhvv}



$|r_{hhvv}|$

⬇

The multidimensional speckle noise model improves the estimation of covariance matrix components, but what happens with the whole covariance matrix?

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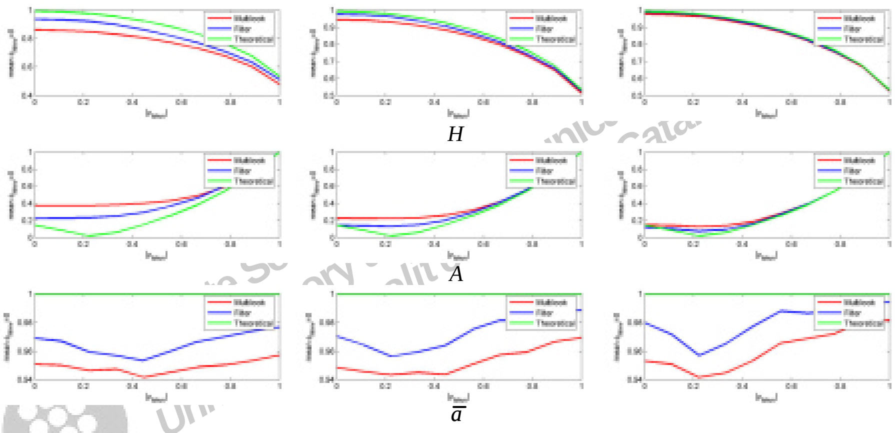
Multidimensional SAR Data Estimation

Results: Simulated Multidimensional SAR Data

Eigendecomposition applied to the estimated covariance matrices

3x3 looks **5x5 looks** **9x9 looks**

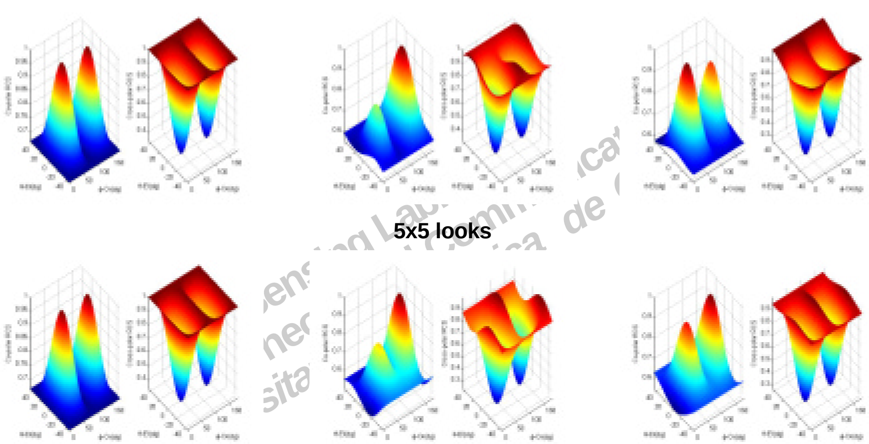


To filter covariance elements differently does not damages information

Multidimensional SAR Data Estimation

Results: Simulated Multidimensional SAR Data

Theoretical PS **Multilook PS** **Filter PS**



11x11 looks

What happens in this particular case?

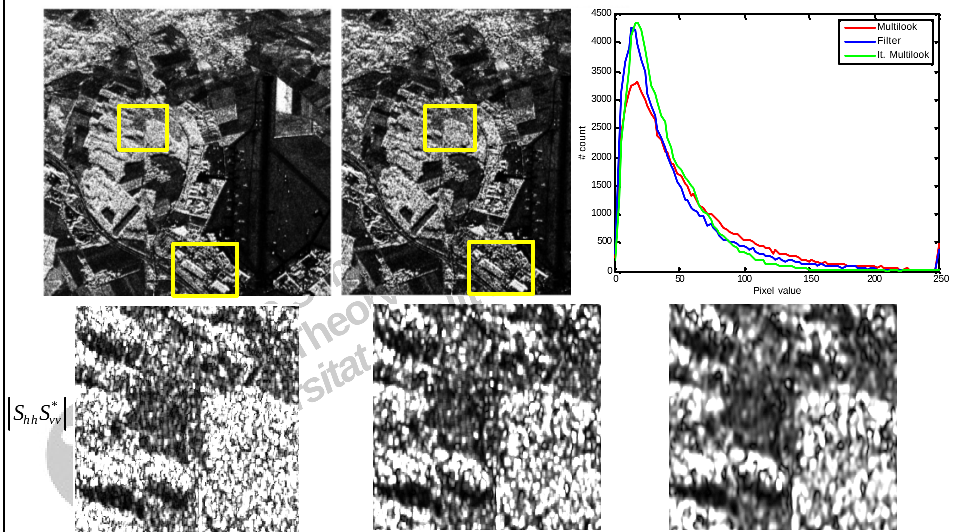
Multidimensional SAR Data Estimation

Results: Experimental PolSAR Data

Full-polar ESAR L-Band SAR data in Oberpfaffenhofen (DE)

5x5 Multilook | **Filter** | **5x5 It. multilook**

$|S_{hh}S_{vv}^{*}|$



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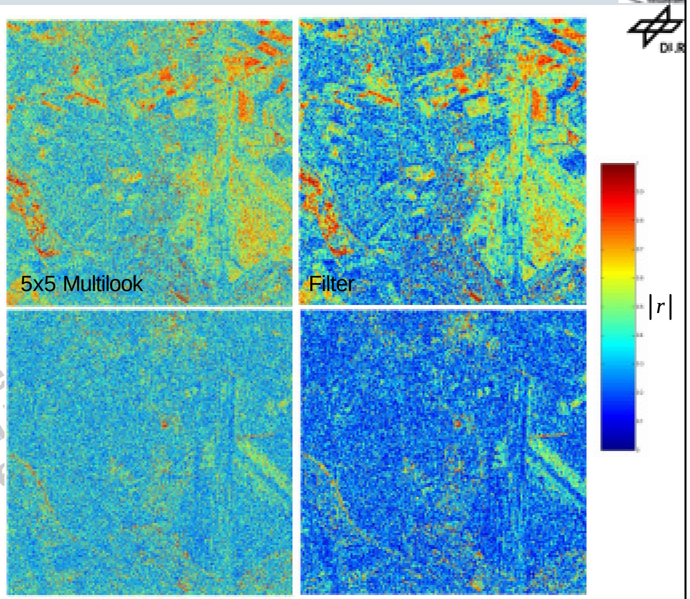
Multidimensional SAR Data Estimation

Results: Experimental PolSAR Data

Co-polar correlation
$|r_{hhvv}|$

Cross-polar correlation
$|r_{hhhv}|$



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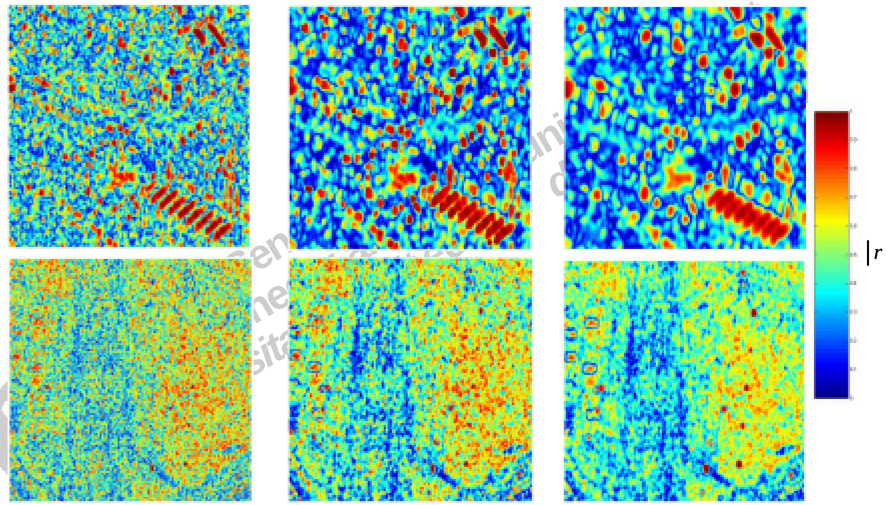
Results: Experimental PolSAR Data

Co-polar correlation $|r_{hhvv}|$ Details analysis

5x5 Multilook — Filter — 5x5 It. multilook



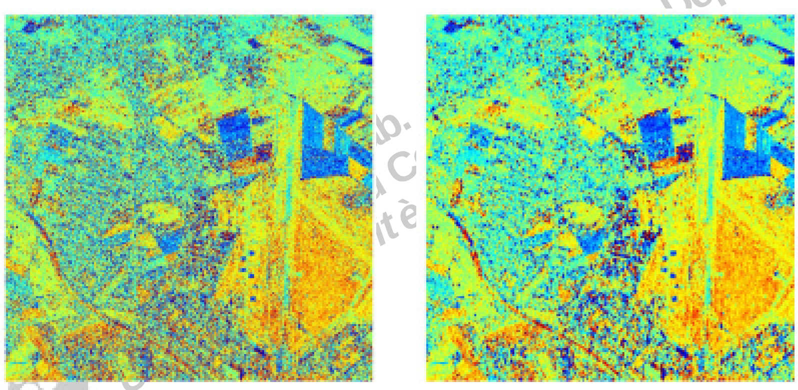
Multidimensional SAR Data Estimation

Results: Experimental PolSAR Data

Co-polar correlation phase

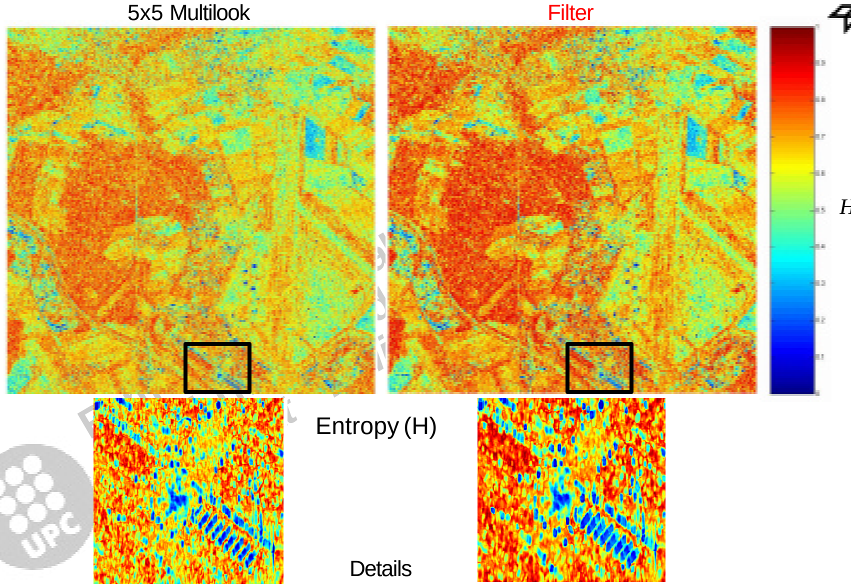
$|r_{hhvv}|$

5x5 Multilook — Filter



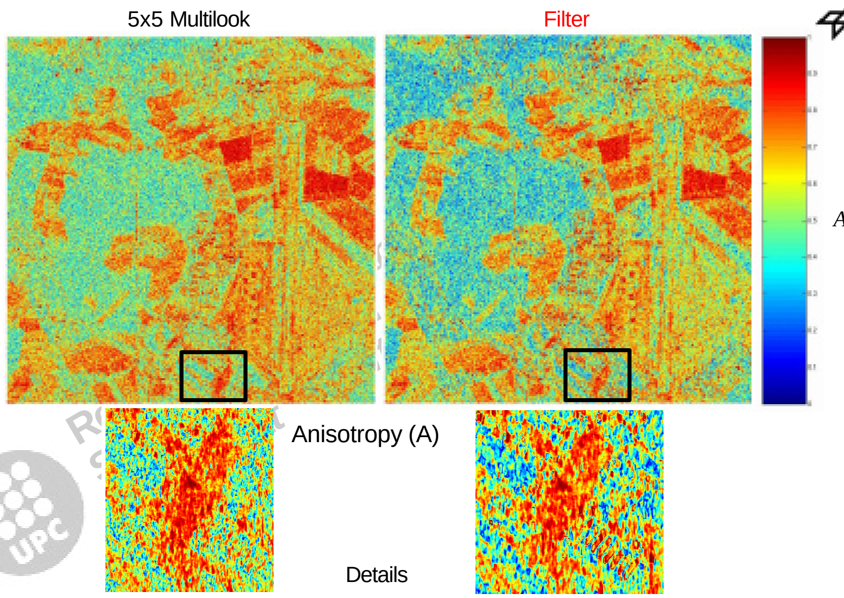
Multidimensional SAR Data Estimation

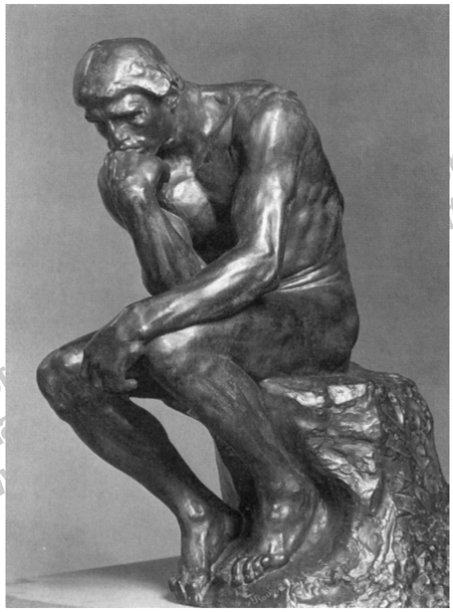
Results: Experimental PolSAR Data



Multidimensional SAR Data Estimation

Results: Experimental PolSAR Data





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PolInSAR Data Estimation

PolInSAR Data Estimation

Combined use of *Polarimetry* and *Interferometry* to study the scattering centers vertical distribution

$$\mathbf{k}_{p,1} = \left(1/\sqrt{2}\right)\left[S_{HH+VV,1}, S_{HH-VV,1}, 2S_{HV,1}\right]^T$$

$$\mathbf{k}_{p,2} = \left(1/\sqrt{2}\right)\left[S_{HH+VV,2}, S_{HH-VV,2}, 2S_{HV,2}\right]^T$$

$$\Rightarrow \mathbf{k}_6 = \begin{bmatrix}\mathbf{k}_{p,1} \\ \mathbf{k}_{p,2}\end{bmatrix} \Rightarrow \mathbf{T}_6 = \frac{1}{n}\sum_{i=1}^{n}\mathbf{k}_6\mathbf{k}_6^H = \begin{bmatrix}\mathbf{T}_{11} & \mathbf{T}_{12} \\ \mathbf{T}_{12}^H & \mathbf{T}_{22}\end{bmatrix}$$

Estimation process

The capability to explore the *polarizations space* allows to consider the idea to *optimize* interferometric coherences

$$r(\mathbf{w}_1, \mathbf{w}_2) = \frac{\mathbf{w}_1^H \mathbf{T}_{12} \mathbf{w}_2}{\sqrt{\mathbf{w}_1^H \mathbf{T}_{11} \mathbf{w}_1 \, \mathbf{w}_2^H \mathbf{T}_{22} \mathbf{w}_2}}$$

Optimization process

$$L = \mathbf{w}_1^H \mathbf{T}_{12} \mathbf{w}_2 + l_1\left(\mathbf{w}_1^H \mathbf{T}_{11} \mathbf{w}_1\right) + l_2\left(\mathbf{w}_2^H \mathbf{T}_{22} \mathbf{w}_2\right)$$

$$\mathbf{T}_{22}^{-1}\mathbf{T}_{12}^H\mathbf{T}_{11}^{-1}\mathbf{T}_{12}\mathbf{w}_2 = u\,\mathbf{w}_2$$

$$\mathbf{T}_{11}^{-1}\mathbf{T}_{12}\mathbf{T}_{22}^{-1}\mathbf{T}_{12}^H\mathbf{w}_1 = u\,\mathbf{w}_1$$

$$u = l_1 l_2^*$$

Optimum coherences $|r|_{opt} = \sqrt{u}$

Optimum eigenvectors $\mathbf{w}_{1,opt}$ $\mathbf{w}_{2,opt}$

Speckle Noise Effects

How does *speckle* affect *optimum* coherences and *optimum* eigenvectors?

R. T. Fomena and S. R. Cloude , "On the role of coherence optimization in polarimetric SAR interferometry," CEOS SAR workshop, Adelaide, Australia, Sept. 2005.

Optimum parameters are affected by the processes

- *Estimation* process

$$\mathbf{Z}_n = \mathbf{C} + \Delta\mathbf{Z} = \mathbf{C} + \mathbf{N}_M + \mathbf{N}_A$$

- *Optimization* process

$$\hat{l}_i = l_i - \frac{l_i}{n}\sum_{j \neq i}^{m}\frac{l_j}{l_i - l_j} - O\left(n^{-1}\right) \qquad i = 1,2,\ldots,m$$

$$\mathbf{T}_{22}^{-1}\mathbf{T}_{12}^H\mathbf{T}_{11}^{-1}\mathbf{T}_{12}\mathbf{w}_2 = u\,\mathbf{w}_2$$

$$\mathbf{T}_{11}^{-1}\mathbf{T}_{12}\mathbf{T}_{22}^{-1}\mathbf{T}_{12}^H\mathbf{w}_1 = u\,\mathbf{w}_1$$

Degrees of freedom/dependence

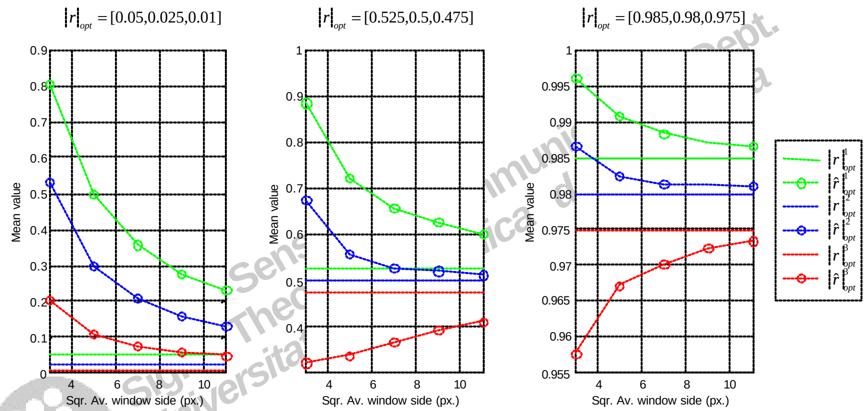
- $\mathbf{T}_{11}$ $\mathbf{T}_{22}$ PolSAR information
- $\mathbf{T}_{12}$ PolInSAR information

Analysis and study based on simulated data
Under the *Gaussian* hypothesis

PolInSAR Data Estimation

Optimum Eigenvalues Estimation

Optimum coherence ***mean value*** vs. *optimum coherence* ***exact value***

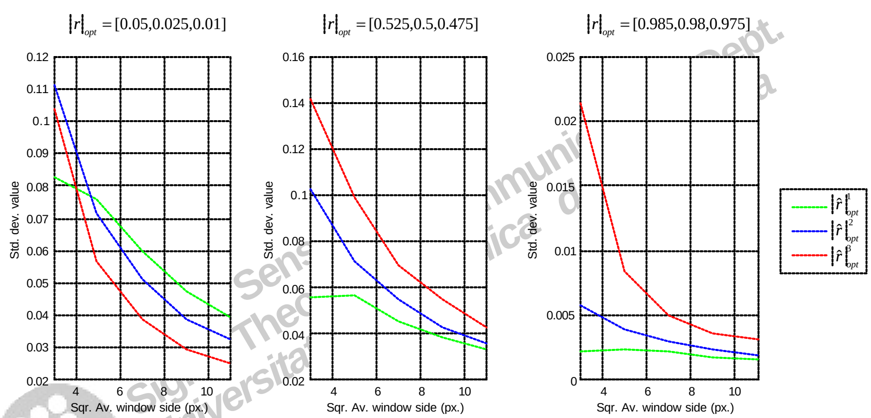


Estimated optimum coherences

- *Asymptotically non-biased*
- *Over- or underestimation*
- Depend on the corresponding exact values, but also on the other optimum coherences

PolInSAR Data Estimation

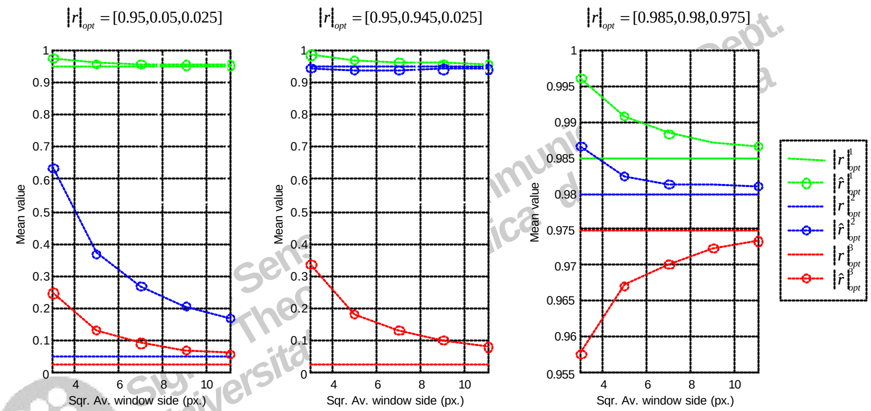
Optimum Eigenvalues Estimation



- *Std. Dev.* decreases with the number of averaged samples
- Not clear behaviour of the Std. Dev. Value
  - Higher values for medium optimum coherences
  - It appears that Std. Dev. is inversely proportional to the optimum coherence value

Optimum Eigenvalues Estimation

Optimum coherence ***mean value*** vs. number of optimum coherences presenting high values



1 high optimum coherence — *2 high* optimum coherences — *3 high* optimum coherences

- *High optimum coherences* are robust
  - Different behaviour respect to the sample eigenvalues in PolSAR due to an *amplitude limitation*
- Over- or underestimation depending of the optimum coherences spectra

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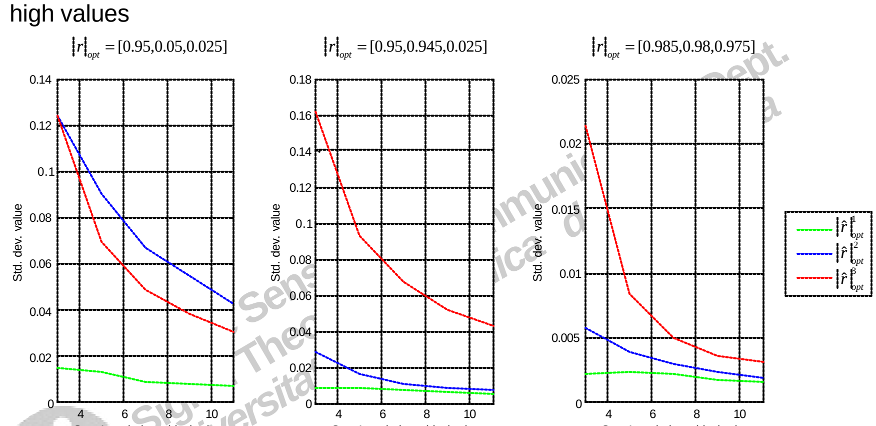
Optimum Eigenvalues Estimation

Optimum coherence ***Std. Dev.*** vs. number of optimum coherences presenting high values



1 high optimum coherence — *2 high* optimum coherences — *3 high* optimum coherences

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PolInSAR Data Estimation

Optimum Eigenvectors Estimation

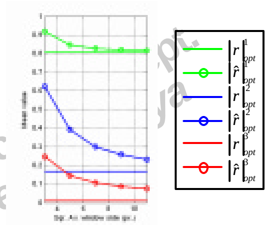
Optimum eigenvectors vs. PolSAR datasets structure

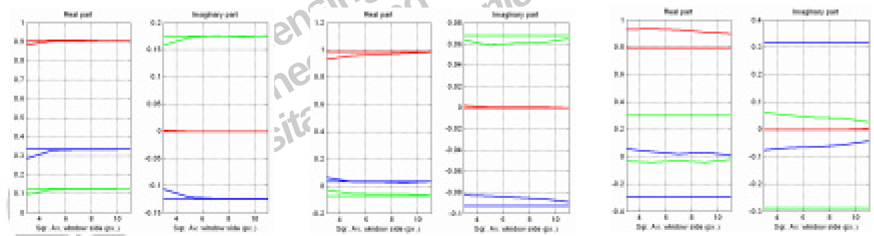
Simulated configuration

$$\mathbf{C}=\begin{bmatrix} 2 & \sqrt{0.2}\,0.1\angle 0.1 & \sqrt{6}\,0.1\angle 1.0 & \sqrt{4}\,0.4\angle 1.0 & 0 & 0 \\ \times & 0.1 & \sqrt{0.3}\,0.7\angle 1.0 & \times & \sqrt{0.02}\,0.1\angle 1.0 & 0 \\ \times & \times & 3 & \times & \times & \sqrt{3}\,0.03\angle 0.2 \\ \times & \times & \times & 2 & \sqrt{0.4}\,0.5\angle 1.0 & \sqrt{2}\,0.7\angle 1.0 \\ \times & \times & \times & \times & 0.2 & \sqrt{0.2}\,0.5\angle 1.0 \\ \times & \times & \times & \times & \times & 1 \end{bmatrix}$$

($\mathbf{C}_{11}$, $\mathbf{C}_{12}$, $\mathbf{C}_{22}$)

$$|r|_{opt}=[0.8073,0.1668,0.0115] \qquad \left[\mathbf{w}^1_{1,opt},\mathbf{w}^2_{1,opt},\mathbf{w}^3_{1,opt}\right]=\begin{bmatrix} 0.9089 & 0.9903 & 0.7992 \\ 0.3341-0.1252i & 0.0345-0.0929i & -0.2910+0.3182i \\ 0.1267+0.1747i & -0.0697+0.0681i & 0.3028-0.2892i \end{bmatrix}$$





$\mathbf{w}^1_{1,opt}$ $\mathbf{w}^2_{1,opt}$ $\mathbf{w}^3_{1,opt}$

PolInSAR Data Estimation

Optimum Eigenvectors Estimation

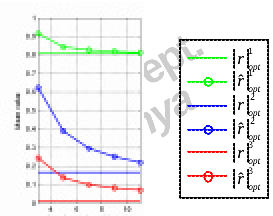
Optimum eigenvectors vs. PolSAR datasets structure

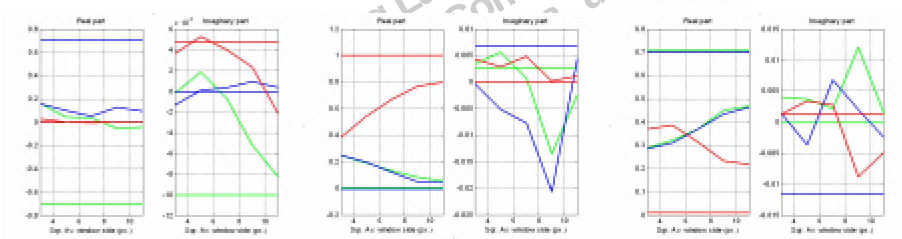
Simulated configuration

$$\mathbf{C}=\begin{bmatrix} 1 & 0 & 0 & \sqrt{4}\,0.4\angle 1.0 & 0 & 0 \\ \times & 1 & 0 & \times & \sqrt{0.02}\,0.1\angle 1.0 & 0 \\ \times & \times & 1 & \times & \times & \sqrt{3}\,0.03\angle 0.2 \\ \times & \times & \times & 1 & 0 & 0 \\ \times & \times & \times & \times & 1 & 0 \\ \times & \times & \times & \times & \times & 1 \end{bmatrix}$$

($\mathbf{C}_{11}$, $\mathbf{C}_{12}$, $\mathbf{C}_{22}$)

$$|r|_{opt}=[0.8073,0.1668,0.0115] \qquad \left[\mathbf{w}^1_{1,opt},\mathbf{w}^2_{1,opt},\mathbf{w}^3_{1,opt}\right]=\begin{bmatrix} 0.7071 & 1 & 0 \\ -0.7071 & 0 & -0.7071 \\ 0 & 0 & -0.7071 \end{bmatrix}$$





$\mathbf{w}^1_{1,opt}$ $\mathbf{w}^2_{1,opt}$ $\mathbf{w}^3_{1,opt}$

- *Optimum eigenvectors* affected by the internal structure of the PolSAR datasets
  - Possible influence of the speckle additive noise component (important for low coherences)