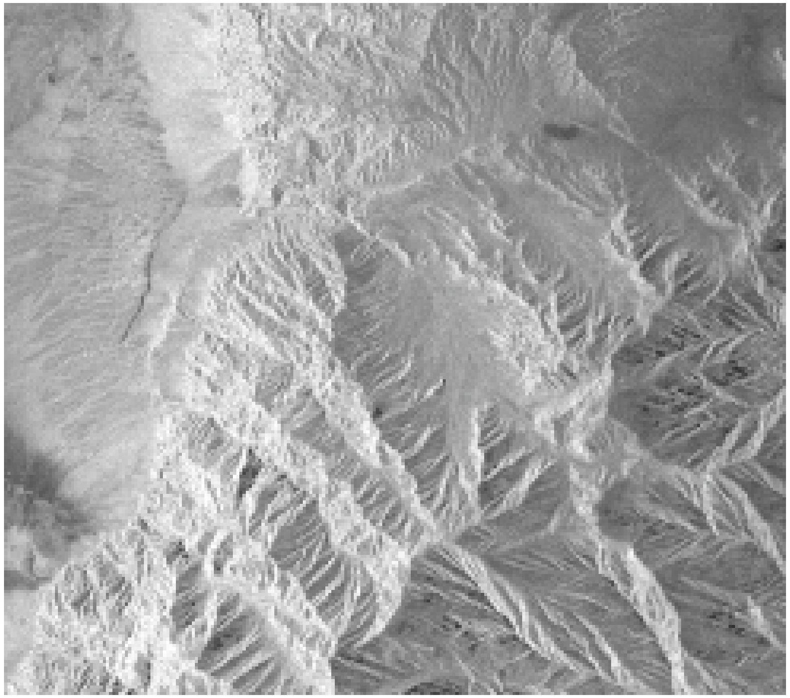
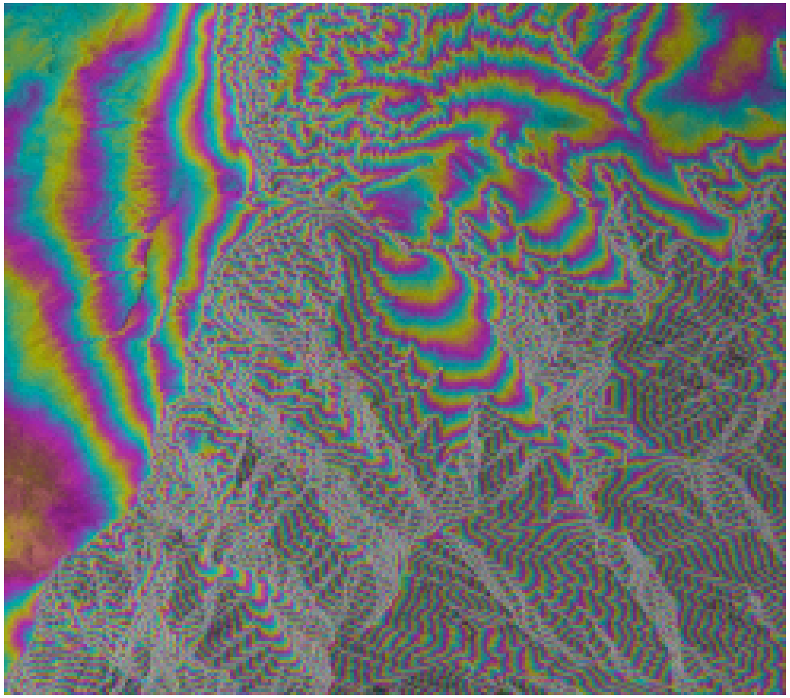
Radar Interferometry Example



Standard Radar Image



Interference fringes follow the topography

One cycle of color represents 1/2 wavelength of path difference

Interferometry Applications

- Mapping/Cartography
  - Radar Interferometry from airborne platforms is routinely used to produce topographic maps as digital elevation models (DEMs).
    - 2-5 meter circular position accuracy
    - 5-10 m post spacing and resolution
    - 10 km by 80 km DEMs produced in 1 hr on mini-supercomputer
  - Radar imagery is automatically geocoded, becoming easily combined with other (multispectral) data sets.
  - Applications of topography enabled by interferometric rapid mapping
    - Land use management, classification, hazard assessment, intelligence, urban planning, short and long time scale geology, hydrology
- Deformation Mapping and Change Detection
  - Repeat Pass Radar Interferometry from spaceborne platforms is routinely used to produce topographic *change* maps as digital displacement models (DDMs).

Interferometry for Topography

Measured phase difference:

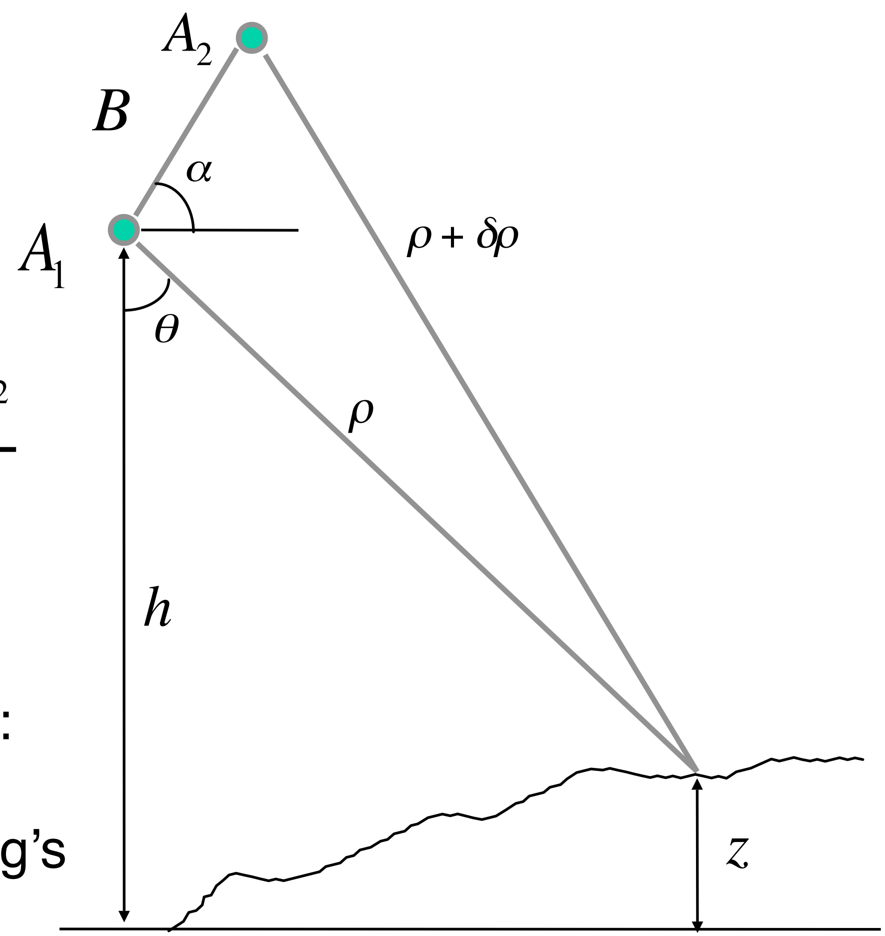
$$\phi = -\frac{2\pi}{\lambda}\delta\rho$$

Triangulation:

$$\sin(\theta - \alpha) = \frac{(\rho + \delta\rho)^2 - \rho^2 - B^2}{2\rho B}$$

$$z = h - \rho\cos\theta$$

Critical Interferometer Knowledge:
- Baseline, (B, α), to mm's
- System phase differences, to deg's



Height Reconstruction

- Interferometric height reconstruction is the determination of a target's position vector from known platform ephemeris information, baseline information, and the interferometric phase.

$\vec{P} =$ platform position vector

$\rho =$ range to target

$\hat{\ell} =$ unit vector pointing from platform to target

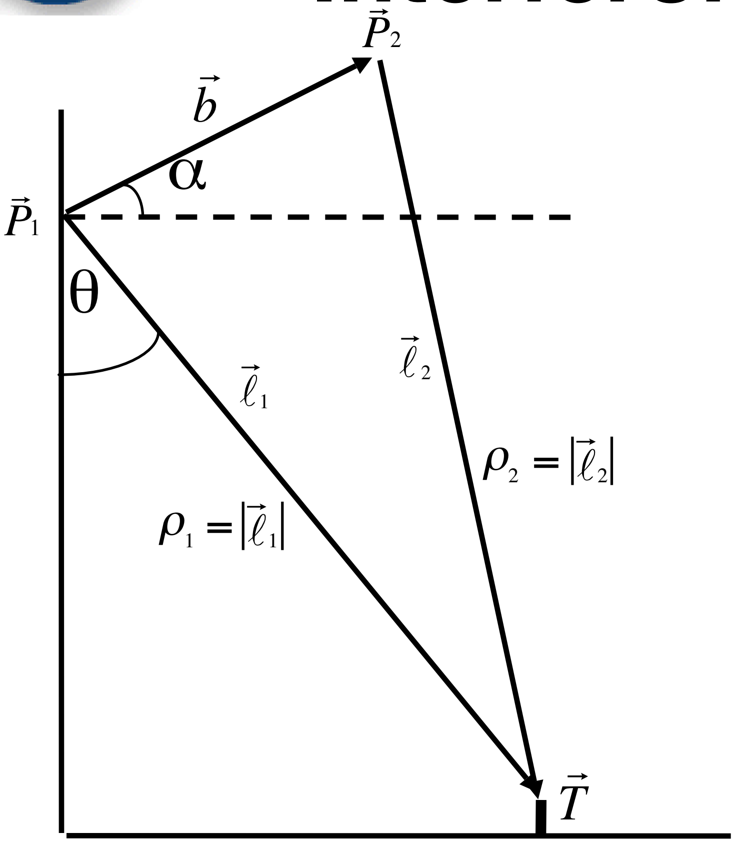
$\vec{T} =$ target location vector

BASIC EQUATION

$$\vec{T} = \vec{P} + \rho\,\hat{\ell}$$

- Interferometry provides a means of determining $\hat{\ell}$.

Interferometric Geometry



$$\vec{b} = \vec{P}_2 - \vec{P}_1 = \vec{\ell}_1 - \vec{\ell}_2$$

$$\vec{\ell}_2 = \vec{\ell}_1 - \vec{b}$$

$$b = |\vec{b}| = \langle \vec{b}, \vec{b} \rangle^{\frac{1}{2}} \quad , \quad \hat{\ell}_1 = \frac{\vec{\ell}_1}{|\vec{\ell}_1|} = \frac{\vec{\ell}_1}{\rho_1}$$

$$\phi = \frac{2\pi p}{\lambda}(\rho_2 - \rho_1) = \frac{2\pi p}{\lambda}\left(|\vec{\ell}_2| - |\vec{\ell}_1|\right)$$

$$= \frac{2\pi p}{\lambda}\left(\langle \vec{\ell}_2, \vec{\ell}_2 \rangle^{\frac{1}{2}} - \rho_1\right)$$

$$= \frac{2\pi p}{\lambda}\left(\langle \vec{\ell}_1 - \vec{b}, \vec{\ell}_1 - \vec{b} \rangle^{\frac{1}{2}} - \rho_1\right)$$

$$= \frac{2\pi p}{\lambda}\left(\left(\rho^2{}_1 - 2\langle \vec{\ell}_1, \vec{b} \rangle + b^2\right)^{\frac{1}{2}} - \rho_1\right)$$

$$= \frac{2\pi p}{\lambda}\rho_1\left(\left(1 - \frac{2\langle \hat{\ell}_1, \vec{b} \rangle}{\rho_1} + \left(\frac{b}{\rho_1}\right)^2\right)^{\frac{1}{2}} - 1\right)$$

- p equals 1 or 2 depending on system

2-D Height Reconstruction - Flat Earth

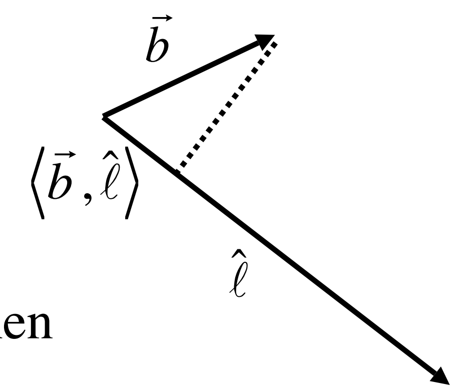
- Before considering the general 3-D height reconstruction it is instructive to first solve the two dimensional problem.

Assume that b << ρ and let $\hat{\ell}_1 = \frac{\vec{\ell}_1}{|\vec{\ell}_1|} = \frac{\vec{\ell}_1}{\rho}$. Taking a first order Taylor's expansion of

$$\phi = \frac{2\pi p}{\lambda}\rho_1\left(\left(1 - \frac{2\langle\hat{\ell}_1,\vec{b}\rangle}{\rho_1} + \left(\frac{b}{\rho_1}\right)^2\right)^{\frac{1}{2}} - 1\right)$$

the interferometric phase can be approximated as

$$\phi \approx -\frac{2\pi p}{\lambda}\langle\hat{\ell}_1,\vec{b}\rangle$$



With $\vec{b}$ = (bcos(α),bsin(α)) and $\hat{\ell}$ = (sin(θ),-cos(θ)) then

$$\phi = -\frac{2\pi p}{\lambda}b\sin(\theta - \alpha)$$

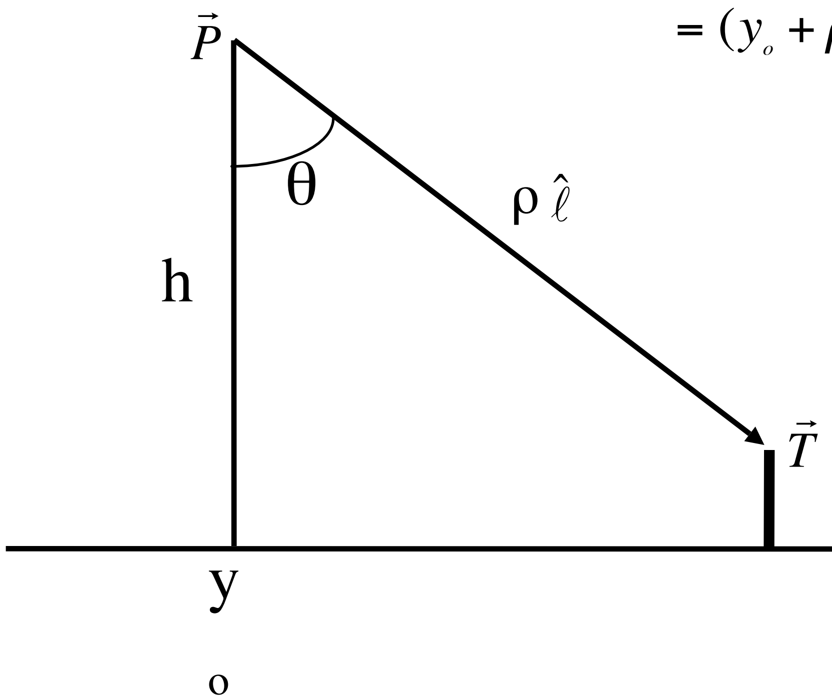
2-D Height Reconstruction - Flat Earth II

- Let $\vec{P}$ = (y_o , h) be the platform position vector, then

$$\vec{T} = \vec{P} + \rho\,\hat{\ell}$$

$$= (y_o, h) \;+\; \rho\,(\sin(\theta), -\cos(\theta))$$

$$= (y_o + \rho\,\sin(\theta), h - \rho\,\cos(\theta))$$



- Solving for θ in terms of the interferometric phase, ϕ, yields

$$\theta = \sin^{-1}\left(\frac{-\lambda\phi}{2\pi\,p\,b}\right) + \alpha$$

3-D Height Reconstruction

- The full three dimensional height reconstruction is based on the observation that the target location is the intersection locus of three surfaces

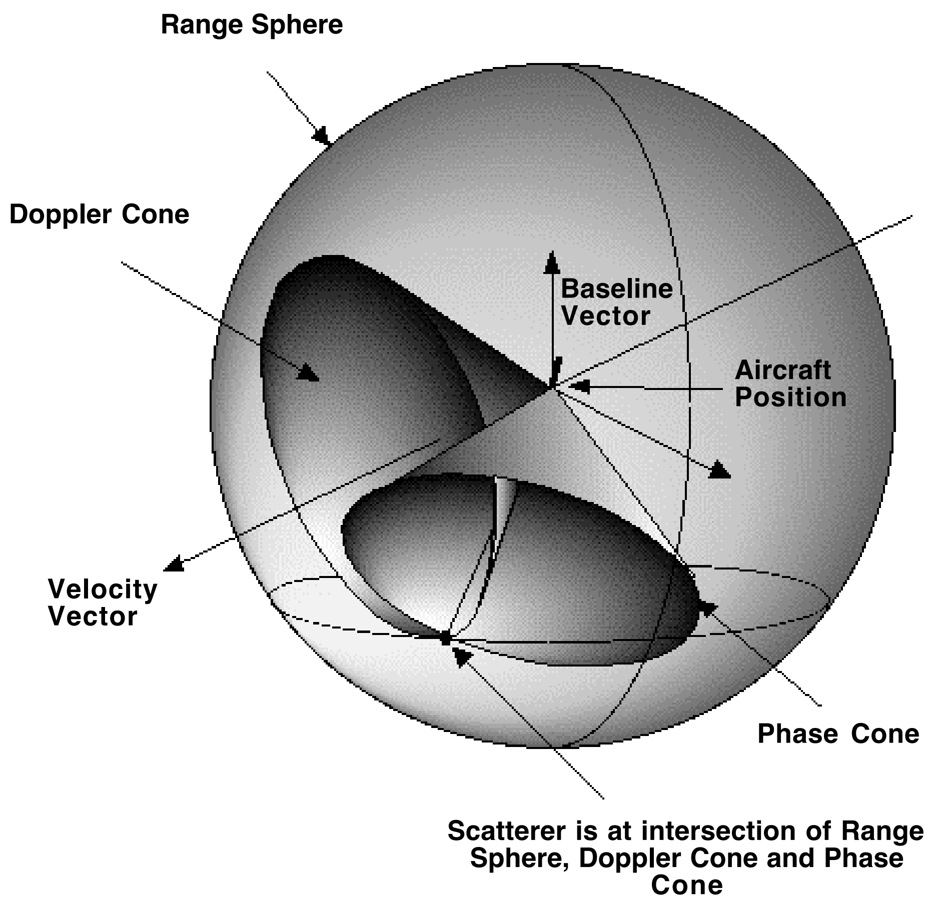
  - range sphere $\left|\vec{P}-\vec{T}\right| = \rho$
  - Doppler cone $f = \frac{2}{\lambda}\left\langle \vec{v}, \hat{\ell} \right\rangle$
  - phase cone* $\phi = -\frac{2\pi p}{\lambda}\left\langle \vec{b}, \hat{\ell} \right\rangle$

  Doppler and phase cones give two angles defining spherical coordinate system

- The cone angles are defined relative to the generating axes determined by
  - velocity vector — Doppler cone
  - baseline vector — phase cone

* Actually the phase surface is a hyperboloid, however for most applications where the phase equation above is valid, the hyperboloid degenerates to a cone.

Height Reconstruction Geometry



Sensitivity of Height with Respect to Phase

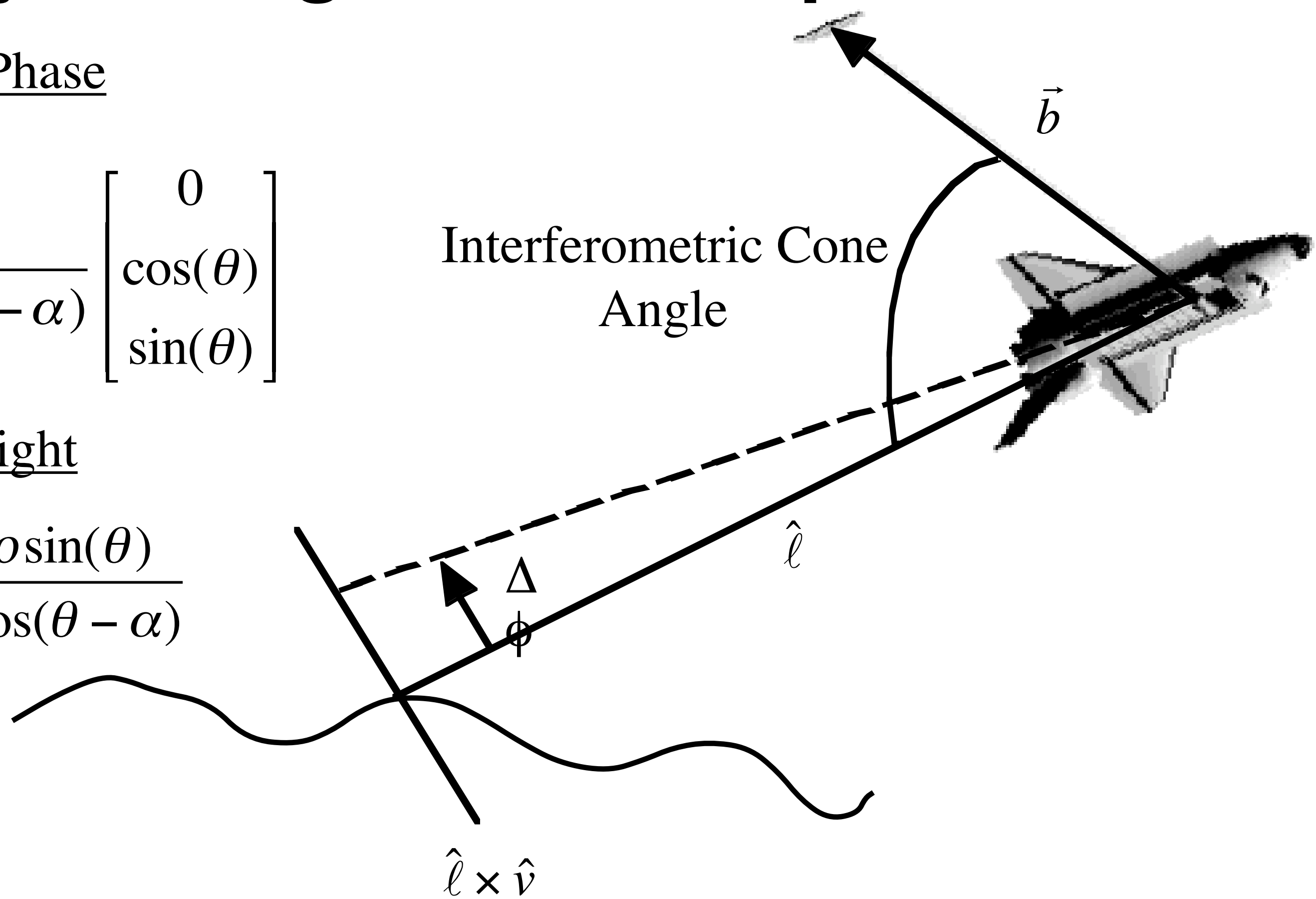
Sensitivity to Phase

$$\frac{\partial \vec{T}}{\partial \phi} = \frac{-\lambda\rho}{2\pi p b \cos(\theta - \alpha)} \begin{bmatrix} 0 \\ \cos(\theta) \\ \sin(\theta) \end{bmatrix}$$

Ambiguity Height

$$h_a = 2\pi \frac{\partial \vec{T}_z}{\partial \phi} = \frac{-\lambda\rho \sin(\theta)}{p b \cos(\theta - \alpha)}$$

p=1,2



- Observe that $\frac{\partial \vec{T}}{\partial \phi}$ is parallel to $\hat{\ell} \times \hat{v}$.