A NEW SPATIAL SPARSITY-BASED METHOD FOR EXTRACTING ENDMEMBER SPECTRA FROM HYPERSPECTRAL DATA WITH SOME PURE PIXELS

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Hyperspectral Remote sensing has become an unavoidable tool for better managing our environment. It consists of simultaneous acquisition of images in contiguous narrow bands (up to several hundred bands) in the electromagnetic spectrum.

It provides a spectrum for each pixel of the image (the number of samples in the spectrum is the number of spectral bands).
Analysis of the spectra allows us to identify the surface components, and to quantify and map their spatial distribution.

Analysis of the hyperspectral remote sensing images generally leads to maps of land cover involving classification techniques.

The traditional classification algorithms for hyperspectral images ASSIGN ONE AND ONLY ONE CLASS OF LAND TO EACH PIXEL.
The hyperspectral vector associated with each observed pixel may be a mixture (linear and instantaneous mixture in our work) of contributions from pure materials (endmembers) contained in the observed area.
### Problem statement

Each spectral vector associated with a pixel in a hyperspectral image is a **linear mixture** of the pure material spectra within the pixel.

The non-negative reflectance \( x_i(p) \) at wavelength \( \lambda_i \) from pixel \( p \) of the hyperspectral image is given by

\[
x_i(p) = \sum_{j=1}^{L} a_{ij} \cdot s_j(p), \quad \forall \ i = 1 \ldots N \text{ and } \forall \ p = 1 \ldots K,
\]

- \( a_{ij} \): the non-negative reflectance of pure material \( j \) at wavelength \( \lambda_i \),
- \( s_j(p) \): the non-negative abundance fraction of pure material \( j \) at pixel \( p \),
- \( L \): the number of pure materials,
- \( N \): the number of spectral bands,
- \( K \): the number of pixels.

The abundance fractions are subject to the **abundance sum-to-one constraint**:

\[
\sum_{j=1}^{L} s_j(p) = 1, \quad \forall \ p = 1 \ldots K.
\]
In such situations, classification techniques are not acceptable for many applications.

*Linear spectral unmixing techniques* are attractive in this framework.

The first step of this process is called *linear endmember spectra extraction*.

The first reported techniques are based on the pure pixel assumption.

This assumption is generally not valid.

Other techniques have been designed.

*Higher spatial resolution* of future hyperspectral sensors will *allow the existence of some pure pixels*. 

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In this work, we propose a new unsupervised method for endmember spectra extraction from hyperspectral data with some pure pixels.
The proposed method is related to the **Blind Mixture Identification (BMI)** problem. Using the **Blind Source Separation (BSS)** terminology, the *endmember spectra* $a_j$ are *the mixing coefficients*.

The proposed method is based on **Sparse Component Analysis (SCA)**. It extracts the endmember spectra by using a **spatial variance-based SCA method**, which detects a few pure-pixel zones.
The proposed method is based on some definitions and assumptions concerning the abundance fractions of pure materials in the spatial domain.

We divide this domain into small zones, called analysis zones and denoted $\Omega$.

The spatial domain is explored using these analysis zones, and we associate with each zone the variance values $\text{var}_{p \in \Omega}[x_i(p)]$ of the observed signals in all bands.
### Definitions and assumptions

**Definition 1** A pure material is said to be “isolated” in an analysis zone if only this pure material is present in this zone, i.e., yields a non-zero vector with elements $s_j(p)$ in this zone. This zone is then called a pure zone. The abundance fraction of this pure material is then equal to one in this zone.

**Definition 2** A pure material is said to be “accessible” in the spatial domain if there exist at least one analysis zone where it is isolated.

**Assumption 1** Each pure material is accessible in the spatial domain.

This sparsity assumption is equivalent to the existence of at least one small pure zone for each pure material in hyperspectral data, which is quite realistic for future hyperspectral sensors with high spatial resolution.

**Assumption 2** When several pure materials are present in a given analysis zone, their abundance fractions take significantly different values in different pixels of this analysis zone, so that at least one variance of $x_i(\Omega)$ with $i = 1, \ldots, N$ is non-negligible.
The proposed method operates in three stages.

1. **The detection stage** consists in *automatically detecting* the pure zones, which is performed using the following property:

   **Property 1** A *necessary and sufficient* condition for a pure material to be isolated in a zone $\Omega$ is

   $$\var{p \in \Omega}{x_i(p)} = 0, \quad \forall \quad i = 1 \ldots N.$$  

   Property 1 is used as follows in our method. For each analysis zone we compute the following *parameter*

   $$\max_i \left( \var{p \in \Omega}{x_i(p)} \right), \quad \forall \quad i = 1 \ldots N.$$  

   If this parameter is below a threshold value (positive and close to 0), we consider that this analysis zone is a pure zone.
2. The estimation stage consists in computing estimates of endmember spectra. Every pure zone \( \Omega \) yields an estimate of one endmember spectrum \( \hat{\alpha}_j \) (column vector), using the following formula

\[
\hat{\alpha}_j = \text{median} \left\{ \begin{array}{c}
x_1(\Omega) \\
\cdot \\
\cdot \\
\cdot \\
x_N(\Omega) 
\end{array} \right\}
\]

In this stage, we also associate with each estimated endmember spectrum the computed parameter (which can be used as a confidence degree in the next stage).
3. The extraction stage consists in *extracting the endmember spectra*.

Each of the pure zones yields one tentative endmember spectrum.

After extracting the first endmember spectrum $\hat{a}_1$, each estimated endmember spectrum $\hat{a}_r$ is compared with all the previously extracted endmember spectra $\hat{a}_j$ by using the Spectral Angle Mapper (SAM) criterion.

$$\text{SAM}(\hat{a}_j, \hat{a}_r) = \arccos \left( \frac{\langle \hat{a}_j, \hat{a}_r \rangle}{\|\hat{a}_j\| \|\hat{a}_r\|} \right), \quad \forall \ j \neq r.$$ 

If all computed angles exceed a threshold, we keep this estimated endmember spectrum as a new extracted endmember spectrum.

Otherwise, i.e. if $\hat{a}_r$ is similar to one of the previously extracted $\hat{a}_j$, we choose the endmember spectrum with the lowest computed parameter as the extracted endmember spectrum (i.e. we replace $\hat{a}_j$ by $\hat{a}_r$ if $\hat{a}_r$ yields a lower computed parameter than $\hat{a}_j$).
The above-defined stages constitute our proposed method, called “2D-VM” for “2 Dimensions - Variance of Mixtures”, which aims at endmember spectra extraction from hyperspectral data with some pure pixels.
Performance evaluation criterion

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**Performance evaluation criteria**

The SAM between the original and estimated endmember spectra is the **first** criterion used to evaluate the performance of the tested methods.

The **second** performance evaluation criterion used in our tests is the Normalized Mean Square Error (NMSE). This criterion is defined as follows

\[
NMSE_j = \frac{\left\| a_j - \hat{a}_j \right\|_{\text{Frobenius}}^2}{\left\| a_j \right\|_{\text{Frobenius}}^2}, \quad \forall \ j = 1 \ldots L,
\]
Two sets of eight endmember spectra are selected from a spectral library compiled by the USGS and measured from 0.4 to 2.5 μm.

The first set contains eight randomly selected endmember spectra.

The second set contains eight endmember spectra highly correlated in pairs.
These two sets of spectra are then independently used to linearly generate two 400x400-pixel synthetic but realistic hyperspectral images. The eight abundance fraction maps used are derived from a real classification of land cover with eight classes. Each of these eight abundance fraction maps contains at least 1.5% of pure pixels.
## Results and discussion

The **proposed method (2D-VM)** and **five methods (VCA, SMACC, SISAL, MVSA, and MVC-NMF)** from the literature are applied to the generated images for comparison.

The first set:  
**Original** and **extracted endmember spectra no. 3**.

The second set:  
**Original** and **extracted endmember spectra no. 3**.
Results and discussion

The means over all endmember spectra of the SAM (in degrees) and NMSE (%) criteria are given in the following Table.

<table>
<thead>
<tr>
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<th>2D-VM</th>
<th>VCA</th>
<th>SMACC</th>
<th>SISAL</th>
<th>MVSA</th>
<th>MVC-NMF</th>
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<tr>
<td><strong>The first set</strong></td>
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<tr>
<td>SAM</td>
<td>0.24</td>
<td>0.72</td>
<td>5.69</td>
<td>3.71</td>
<td>4.09</td>
<td>20.10</td>
</tr>
<tr>
<td>NMSE</td>
<td>0.55</td>
<td>2.82</td>
<td>10.86</td>
<td>9.22</td>
<td>10.25</td>
<td>49.23</td>
</tr>
<tr>
<td><strong>The second set</strong></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SAM</td>
<td>0.40</td>
<td>0.54</td>
<td>2.19</td>
<td>37.48</td>
<td>38.01</td>
<td>8.42</td>
</tr>
<tr>
<td>NMSE</td>
<td>1.03</td>
<td>4.49</td>
<td>32.01</td>
<td>154.78</td>
<td>107.35</td>
<td>100.15</td>
</tr>
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Globally, we show that the proposed approach (2D-VM) yields much better performance than methods from the literature.

For the first set, the mean NMSE is 0.55% for our 2D-VM method, i.e. about 5 to 90 times lower than with the used literature methods. The mean SAM is 0.24° for our method, i.e. about 3 to 84 times lower than with the literature methods.

Similarly, for the second set, the mean NMSE and SAM of our 2D-VM method are 1.03% and 0.40°, i.e. respectively about 4 to 97 times and 1.4 to 21 times lower than with the literature methods.
In this paper, a new unsupervised method, called 2D-VM, was proposed for extracting endmember spectra from remote sensing hyperspectral data with some pure pixels.

This method relies on a spatial variance-based SCA approach.

Compared to five methods from the literature, and according to the results obtained in these investigations, the proposed method is very attractive for endmember spectra extraction from future high spatial resolution hyperspectral data.

Future extensions of this work may consist in extracting the non-negative abundance fraction maps after endmember spectra extraction, by means of methods using nonnegativity constraints.
THANK YOU FOR YOUR ATTENTION