SEGMENTATION OF POLARIMETRIC SAR DATA WITH A MULTI-TEXTURE PRODUCT MODEL

Anthony P. Doulgeris,
S. N. Anfinsen, and T. Eltoft

IGARSS 2012
Background: IGARSS 2011

Presented a multi-texture model for PolSAR data

- Probability Density Function for Scalar and Dual-texture models
- Log-cumulant expressions for all multi-texture models
- Hypothesis tests to determine most appropriate multi-texture model

Showed evidence for multi-texture from manually chosen box-window estimates
Objective 2012

Place multi-texture models into an advanced segmentation algorithm

- Hypothesis tests to choose between Scalar or Dual-texture models
- $U$-distribution for flexible texture modelling
- Log-cumulants for parameter estimation
- Goodness-of-fit testing for number of clusters
- Markov Random Fields for contextual smoothing

Show real multi-texture segmentation results.
Scalar-texture

Scattering vector:
\[ \mathbf{s} = [s_{hh}; s_{hv}; s_{vh}; s_{vv}]^t \]

Scalar product model:
\[ \mathbf{s} = \sqrt{t} \mathbf{x} \]

where

texture variable \( t \sim \Gamma(1, \alpha), \Gamma^{-1}(1, \lambda), \text{ or } \mathcal{F}(1, \alpha, \lambda) \)

speckle variable \( \mathbf{x} \sim \mathcal{N}_d^c(0, \Sigma) \)

1. Scalar \( t \) modulates all channels equally.
2. Speculation: scattering mechanisms impact specific channels and may lead to different textural characteristics per channel.
Multi-texture

Scattering vector:
\[ \mathbf{s} = [s_{hh}, s_{hv}, s_{vh}, s_{vv}]^t \]

Multi-texture product model:
\[ \mathbf{s} = \mathbf{T}^{1/2}\mathbf{x} \]

where
\[ \mathbf{T} = \text{diag}\{t_{hh}, t_{hv}, t_{vh}, t_{vv}\} \]

Special cases:

Scalar-texture \( t = t_{hh} = t_{hv} = t_{vh} = t_{vv} \)

Dual-texture \( t_{co} = t_{hh} = t_{vv} \) and \( t_{cross} = t_{hv} = t_{vh} \)
Multi-texture PDF

Given

\[ s = T^{1/2}x \]

\[ C = \frac{1}{L} \sum_{i=1}^{L} s_i s_i^H = T^{1/2} C_x T^{1/2} \]

\[ f_{C|T}(C|T; L, \Sigma_x) = \frac{L^{Ld|C|L-d} \text{etr}(-L\Sigma_x^{-1}T^{-1/2}CT^{-1/2})}{\Gamma_d(L)|T|^L|\Sigma_x|^L} \]

Then

\[ f_C(C; L, \Sigma_x) = \int f_{C|T}(C|T; L, \Sigma_x) f_T(T) dT \]
Dual-texture Case

- Reciprocal and reflection symmetric assumptions

\[ f_C(C; L, \Sigma_x) = \frac{L^3 L}{\Gamma_3(L)} \frac{|C|^{L-3}}{|\Sigma_x|^L} \]

\[ \times \int \frac{1}{t^2 L} \exp \left( -L \left( \frac{q_{11}c_{11} + q_{14}c_{41} + q_{41}c_{14} + q_{44}c_{44}}{t_{co}} \right) \right) f_{t_{co}}(t_{co}) \, dt_{co} \]

\[ \times \int \frac{1}{t^2 L} \exp \left( -L \left( \frac{q_{22}c_{22} + q_{23}c_{32} + q_{32}c_{23} + q_{33}c_{33}}{t_{cross}} \right) \right) f_{t_{cross}}(t_{cross}) \, dt_{cross} \]

where

- \( q_{ij} \) denotes the \( ij \)th elements of \( \Sigma_s^{-1} \)
- \( f_{t_{co}}(t_{co}) \) and \( f_{t_{cross}}(t_{cross}) \) denotes the PDFs of \( t_{co} \) and \( t_{cross} \), respectively.
Multi-texture Log-Cumulants

Scalar: $\kappa_\nu\{C\} = \kappa_\nu\{W\} + d^\nu \kappa_\nu\{T\}$

Dual: $\kappa_\nu\{C\} = \kappa_\nu\{W\} + d^\nu_{co} \kappa_\nu\{T_{co}\} + d^\nu_{cross} \kappa_\nu\{T_{cross}\}$

(Scaled) Wishart-distribution $(L, \Sigma)$

$\kappa_\nu\{W\} = \begin{cases} 
\psi_d^{(0)}(L) + \ln |\Sigma| - d \ln(L) & , \nu = 1 \\
\psi_d^{(\nu-1)}(L) & , \nu > 1 
\end{cases}$

$\mathcal{F}$-distribution $(\alpha, \lambda)$

$\kappa_\nu\{T\} = \begin{cases} 
\psi^{(0)}(\alpha) - \psi^{(0)}(\lambda) + \ln(\frac{\lambda-1}{\alpha}) & , \nu = 1 \\
\psi^{(\nu-1)}(\alpha) + (-1)^\nu \psi^{(\nu-1)}(\lambda) & , \nu > 1 
\end{cases}$

Texture Parameters:
Scalar $(\alpha, \lambda)$
Dual $(\alpha_{co}, \lambda_{co})$ & $(\alpha_{cross}, \lambda_{cross})$
Multi-texture Hypothesis Test

Scalar: \( \kappa_\nu \{ C \} = \kappa_\nu \{ W \} + d_\nu \kappa_\nu \{ T \} \)

Dual: \( \kappa_\nu \{ C \} = \kappa_\nu \{ W \} + d_{co} \kappa_\nu \{ T_{co} \} + d_{cross} \kappa_\nu \{ T_{cross} \} \)

Estimate texture parameters for \( T, T_{co} \) and \( T_{cross} \)

Choose from smallest of \( D_{\text{scalar}} \) or \( D_{\text{dual}} \)
Segmentation Algorithm

Iterative expectation maximisation algorithm with a few modifications, scalar-texture version detailed in Doulgeris *et al.* TGRS-EUSAR (2011) and Doulgeris *et al.* EUSAR (2012).

The key features are:

- $U$-distribution for flexible texture modelling
- Log-cumulants for parameter estimation
- Goodness-of-fit testing for number of clusters
- Markov Random Fields for contextual smoothing
Segmentation Algorithm → Multi-texture

Iterative expectation maximisation algorithm with a few modifications, scalar-texture version detailed in Doulgeris et al. TGRS-EUSAR (2011) and Doulgeris et al. EUSAR (2012).

The key features are:

• $\mathcal{U}$-distribution for flexible texture modelling → Multi-texture

• Log-cumulants for parameter estimation → Multi-texture

• Goodness-of-fit testing for number of clusters

• Markov Random Fields for contextual smoothing

• Hypothesis tests to choose Scalar or Dual-texture model
## Simulated 8-look Data

<table>
<thead>
<tr>
<th>Class 1</th>
<th>K-Wishart $\alpha = 15$</th>
<th>U-distribution $\alpha = 16.5, \lambda = 4220$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Class 2</td>
<td>Co-pol</td>
<td></td>
</tr>
<tr>
<td></td>
<td>K-Wishart $\alpha = 10$</td>
<td>$\alpha = 10.4, \lambda = 217$</td>
</tr>
<tr>
<td></td>
<td>Cross-pol $\mathcal{G}^0$ $\lambda = 30$</td>
<td>$\alpha = 4220, \lambda = 28.8$</td>
</tr>
</tbody>
</table>

Lexicographic RGB
Simulated Results

(a) Lexicographic RGB

(b) Class segmentation

(c) Class histograms

(d) Class log-cumulants

(e) Dual-texture log-cumulants
Real Data 1: San Francisco City
Radarsat-2 sample image from 9 April, 2008, 25-looks.
Real Data 2: Amazon Rainforest
NO MULTI-TEXTURE!

But previously ...

...
Mixtures Give Multi-texture

Example: small co-pol difference, large cross-pol difference. Texture (skewness) of each mixture are different = Multi-texture.
Conclusions

• Developed a Multi-texture segmentation algorithm

• Tests found only the Scalar-texture case

• Previous window-estimation may have found multi-texture due to mixtures

• This automatic segmentation algorithm will split-up such mixtures

• Less complicated scalar-product model is generally suitable of PolSAR analysis

Wanted: Data-sets that may display multi-texture for testing.