Scattering from Tree Branches Using the Fresnel Double Scattering Approximation

Qianyi Zhao and Roger H. Lang
Department of Electrical and Computer Engineering
The George Washington University
Washington DC 20052
• Objective
• Motivation
• Scattering from one single scatterer
• Scattering from multiple scatterers using the FDS method
• A numerical implementation of the FDS method
• Examples
• Conclusion
To develop a fast and accurate numerical model to compute the scattering amplitude/cross sections of a cluster of two tree branches with the second order multiple scattering (double scattering) interaction taken into account
MOTIVATION

- Study of scattering from tree branches at L band
- Microwave tree scattering models:
  - Single scattering
  - Numerical full wave models
- Accurate and efficient numerical calculation
- Single and double scattering effects between branches
**SCATTERING FROM SINGLE SCATTERER**

- **Vector wave equation**
  \[ \nabla \times \nabla \times \mathbf{E}(\mathbf{r}) - k_0^2 \varepsilon(\mathbf{r}) \mathbf{E}(\mathbf{r}) = -i\omega \mu_0 \mathbf{J} \]

- **Operator notation**
  \[ (\mathcal{L} - \mathcal{V}) \cdot \mathbf{E} = \mathbf{g} \]
  \[ \mathcal{L} = \nabla \times \nabla \times \mathbf{I} - k_0^2 \mathbf{I}; \quad \mathcal{V} = k_0^2 (\varepsilon(\mathbf{r}) - 1) \mathbf{I}; \quad \mathbf{g} = -i\omega \mu_0 \mathbf{J} \]

- **Equivalent source term**
  \[ g_{eq} = \mathcal{V} \cdot \mathbf{E} = \mathcal{T} \cdot \mathbf{E}^{inc} \]

- **Scattered E field**
  \[ \mathbf{E}^{sca} = \mathcal{L}^{-1} \cdot \mathcal{V} \cdot \mathbf{E} = \mathcal{L}^{-1} \cdot \mathcal{T} \cdot \mathbf{E}^{inc} \]
**Scattering from Multiple Scatterers**

- **Incident wave** $E^{inc}$
- **Scatterer**
- **Scattered field** $L^{-1} \cdot J_i \cdot E^{inc}$
- **Induced source due to scattered field** $J_j \cdot L^{-1} \cdot J_i \cdot E^{inc}$
- **Double scattered field** $L^{-1} \cdot J_j \cdot L^{-1} \cdot J_i \cdot E^{inc}$

**Fresnel Double Scattering (FDS) Approximation**

$$E_{FDS}^{sca} = \sum_{i=1}^{N} L^{-1} \cdot J_i \cdot E^{inc} + \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{j \neq i} L^{-1} \cdot J_j \cdot L^{-1} \cdot J_i \cdot E^{inc}$$

- **Single Scattering Contribution**
- **Double Scattering Contribution**
OTHER APPROACHES

• Exact Approach:

\[ \mathbf{E}_{sca}^{\text{exact}} = \mathbf{J}^{-1} \cdot \mathbf{J} \cdot \mathbf{E}^{inc} \]

• Multiple scatterers are treated as a whole scatterer
• *The coupling interaction between scatterers are fully taken into account*

• Coherent Single Scattering Approximation

\[ \mathbf{E}_{CSS}^{sca} = \sum_{i=1}^{N} \mathbf{J}^{-1} \cdot \mathbf{J}_i \cdot \mathbf{E}^{inc} \]

• *The coupling interaction between scatterers are totally neglected*
• The discrete dipole approximation (DDA) method
• The volume integral equation

\[ E^{inc}(\mathbf{r}) = E(\mathbf{r}) - k_0^2 \int_{V} G(\mathbf{r}, \mathbf{r}') \cdot (\varepsilon_r - 1) E(\mathbf{r}') d\mathbf{r}' \quad \mathbf{r}' \in V \]

• Meshing the scatterer into N small cells
• Discretizing the volume integral equation

\[ E^{inc}(\mathbf{r}_n) = E(\mathbf{r}_n) - k_0^2 \sum_{m=1}^{M} G(\mathbf{r}_n, \mathbf{r}_m) \cdot (\varepsilon_r - 1) E(\mathbf{r}_m) \Delta V_m + (\varepsilon_r - 1) \mathbf{L} \cdot \mathbf{E}(\mathbf{r}_m) \]

• Formulating a matrix equation

\[ [E^{inc}(\mathbf{r}_n)] = [A(\mathbf{r}_n, \mathbf{r}_m)] [E(\mathbf{r}_m)] \]

• Finding the scattered field

\[ [E^{sca}] = [k_0^2 (\varepsilon_r - 1) \Delta V_m G(\mathbf{r}_n, \mathbf{r}_m)] [E(\mathbf{r}_m)] \]
Matrix Representation of Operators

- **FDS Approximation**

$$E_{FDS}^{sca} = \sum_{i=1}^{N} \mathcal{L}^{-1} \cdot \mathcal{I}_i \cdot E^{inc} + \sum_{i=1}^{N} \sum_{j=1, j \neq i}^{N} \mathcal{L}^{-1} \cdot \mathcal{I}_j \cdot \mathcal{L}^{-1} \cdot \mathcal{I}_i \cdot E^{inc}$$

$$[E^{inc}(r_n)] = [A(r_n, r_m)] [E(r_m)]$$

$$[E^{sca}] = [k_0^2 (\varepsilon_r - 1) \Delta V_m G(r_n, r_m)] [E(r_m)]$$

$$[\mathcal{L}^{-1}] = [G(r_n, r_m) \Delta V_m]$$

$$[\mathcal{I}_i] = [A_i(r_n, r_m)]^{-1}$$

- **DDA implementation of FDS approximation**

$$[E_{FDS}^{sca}] = \sum_{i=1}^{N} [\mathcal{L}^{-1}] [\mathcal{I}_i] [E^{inc}] + \sum_{i=1}^{N} \sum_{j=1}^{N} [\mathcal{L}^{-1}] [\mathcal{I}_j] [\mathcal{L}^{-1}] [\mathcal{I}_i] [E^{inc}]$$
EXAMPLE 1

• Incident wave
  • Frequency = 1.4GHz
  • Incident Angle
    • $\theta_i = 20^\circ$ to $70^\circ$
    • $\phi_i = 250^\circ$
  • H- and V- polarized

• Branch parameters
  • Two identical cylinders
  • Radius = 4 cm; Length ($D$) = 30 cm
  • Relative dielectric constant = 12+4i

• Distance between centers
  • $s = 0.75D^2/\lambda$;
  • $s = D^2/\lambda$
  • $s >> D^2/\lambda$

• Cubical cells:
  • Dimension $1/20\lambda \times 1/20\lambda \times 1/20\lambda$
BACKSCATTERING CROSS SECTIONS \( s = 0.75D^2/\lambda \)

![Graphs showing backscattering cross sections for HH, VV, HV, and VH configurations with incident angles ranging from 20 to 65 degrees. The graphs compareExact, FDS, and CSS results.](image-url)
**Backscattering Cross Sections $s = D^2/\lambda$**

**HH**

![Graph showing backscattering cross sections for HH polarization.](image)

**VV**

![Graph showing backscattering cross sections for VV polarization.](image)

**HV**

![Graph showing backscattering cross sections for HV polarization.](image)

**VH**

![Graph showing backscattering cross sections for VH polarization.](image)

- **Exact**
- **FDS**
- **CSS**
BACKSCATTERING CROSS SECTIONS $s \gg D^2/\lambda$

**HH**

Incident Angle $\theta_i$ [deg]

$\sigma_{hh}$ [dB]

**VV**

Incident Angle $\theta_i$ [deg]

$\sigma_{vv}$ [dB]

**HV**

Incident Angle $\theta_i$ [deg]

$\sigma_{hv}$ [dB]

**VH**

Incident Angle $\theta_i$ [deg]

$\sigma_{vh}$ [dB]
EXAMPLE 2

- Incident wave
  - Frequency = 1.4GHz
  - Incident Angle
    - $\theta_i = 20^\circ$ to $70^\circ$
    - $\phi_i = 270^\circ$
  - H- and V- polarized

- Branch parameters
  - Branch 1: Radius = 4 cm; Length ($D$) = 40 cm
  - Branch 2: Radius = 4 cm; Length = 30 cm
  - Relative dielectric constant = 12+4i
  - Distance between centers
    - $s = 0.5D^2/\lambda$; $s = D^2/\lambda$; $s >> D^2/\lambda$
  - Cubical cells:
    - Dimension $1/20\lambda \times 1/20\lambda \times 1/20\lambda$
BACKSCATTERING CROSS SECTIONS $s = 0.5D^2/\lambda$

HH

VV

HV

VH

\[ \sigma_{hh} \text{ [dB]} \]

\[ \sigma_{vv} \text{ [dB]} \]

\[ \sigma_{hv} \text{ [dB]} \]

\[ \sigma_{vh} \text{ [dB]} \]

Incident Angle $\theta_i$ [deg]

EXACT ○ FDS --- CSS
**Backscattering Cross Sections $s = D^2/\lambda$**

![Graphs showing backscattering cross sections for HH, VV, HV, and VH polarizations.]

**HH**
- Incident Angle $\theta_i$ [deg]
- $\sigma_{hh}$ [dB]

**VV**
- Incident Angle $\theta_i$ [deg]
- $\sigma_{vv}$ [dB]

**HV**
- Incident Angle $\theta_i$ [deg]
- $\sigma_{hv}$ [dB]

**VH**
- Incident Angle $\theta_i$ [deg]
- $\sigma_{vh}$ [dB]

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- **EXACT**
- **FDS**
- **CSS**
**BACKSCATTERING CROSS SECTIONS s>>D^2/λ**

- **HH**
  - Incident Angle $\theta_i$ [deg]
  - $\sigma_{hh}[dB]$ vs. Incident Angle $\theta_i$ [deg]
  - Exact, FDS, CSS

- **VV**
  - Incident Angle $\theta_i$ [deg]
  - $\sigma_{vv}[dB]$ vs. Incident Angle $\theta_i$ [deg]
  - Exact, FDS, CSS

- **HV**
  - Incident Angle $\theta_i$ [deg]
  - $\sigma_{hv}[dB]$ vs. Incident Angle $\theta_i$ [deg]
  - Exact, FDS, CSS

- **VH**
  - Incident Angle $\theta_i$ [deg]
  - $\sigma_{vh}[dB]$ vs. Incident Angle $\theta_i$ [deg]
  - Exact, FDS, CSS
CONCLUSION

• When tree branches are in the first order Fresnel zone of one another, double scattering dominates the multiple scattering effects.

• Results obtained by the Fresnel Double Scattering (FDS) approximation are very close to the exact results in all cases presented.

• The FDS method includes single and double scatter.

• The FDS approximation is fully polarimetric and bistatic.

• As the distance between scatterers increases the FDS and the coherent single scattering results approaches the exact results.

• The FDS provides flexibilities to break down a large problem into numbers of small problems, and is easy to be implemented.