Coherence Linearity and SKP-Structured Matrices in Multi-Baseline PolInSAR

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IGARSS 2011, Vancouver
Introduction

The availability of Multi-baseline PolInSAR data makes it possible to decompose the signal into ground-only and volume-only contributions.

Properties of the vegetation layer
- Vertical structure
- Polarimetry

• Phase calibration
• Digital Terrain Model
• Ground properties
Polarimetric SAR Interferometry (PolInSAR)

- The coherence locus is assumed to be a straight line in the complex plane
- G/V decomposition is carried out by fitting a straight line in each interferometric pair

Algebraic Synthesis

- The data covariance matrix is assumed to be structured as a Sum of 2 Kronecker Products
- G/V dec is carried out by taking the first 2 terms of the SKP decomposition of the data covariance matrix

Scope of this work:

Compare the two approaches from the algebraic and statistical points of view
Model of the acquisitions

We consider a multi-polarimetric and multi-baseline (MPMB) data

- Monostatic acquisitions: up to 3 independent SLC images per track

\[ y_n(w_i) \Leftrightarrow Track\ n Polarization\ w_i \]

\[ \begin{align*}
\text{Re}\{y_n(w_1)\} & \quad \text{Re}\{y_n(w_2)\} & \quad \text{Re}\{y_n(w_3)\} \\
\text{Im}\{y_n(w_1)\} & \quad \text{Im}\{y_n(w_2)\} & \quad \text{Im}\{y_n(w_3)\}
\end{align*} \]
PolInSAR is based on the variation of the interferometric coherence w.r.t. polarization

\[ \gamma_{nm}(w_i, w_j) = \frac{w_i^H \Sigma_{nm} w_j}{\sqrt{w_i^H \Sigma_{nn} w_i w_j^H \Sigma_{mm} w_j}} \]

\[ \Sigma_{nm} = E[y_n y_m^H] \]

\[ y_n = \begin{bmatrix} y_n(HH + VV) \\ y_n(HH - VV) \end{bmatrix} \]

Coherence linearity (*):

RVoG model \( \Leftrightarrow \) ESM coherences describe a straight line in the complex plane

\[ \gamma(w, w) = (1 - \mu(w)) \cdot \gamma^v + \mu(w) \cdot \gamma^g \]

(*) Papathanassiou and Cloude, “Single Baseline Polarimetric SAR Interferometry”
Coherence linearity

PolInSAR is based on the variation of the interferometric coherence w.r.t. polarization

\[ \gamma_{nm}(w_i, w_j) = \frac{w_i^H \Sigma_{nm} w_j}{\sqrt{w_i^H \Sigma_{nn} w_i w_j^H \Sigma_{mm} w_j}} \]

\[ \Sigma_{nm} = E[y_n y_m^H] \]

\[ y_n = \begin{bmatrix} y_n(HH + VV) \\ y_n(HH - VV) \\ \sqrt{2}y_n(HV) \end{bmatrix} \]

\( w_i \neq w_j \Leftrightarrow \) Multiple Scattering Mechanisms (MSM)

\( w_i = w_j \Leftrightarrow \) Equalized Scattering Mechanisms (ESM)

**Coherence linearity (\(*\)):**

RVoG model \(\Rightarrow\) ESM coherences describe a straight line in the complex plane

Multiple baselines: one line per interferometric pair

\( n = 1 \quad m = 2 \)

\( n = 1 \quad m = 3 \)

\( n = 1 \quad m = 4 \)

\( \text{Volume coherence} \)

\( \text{Ground coherence} \)

\( \bullet \text{ESM coherences} \)

(* Papathanassiou and Cloude, “Single Baseline Polariometric SAR Interferometry
The SKP structure

Without loss of generality, the received signal can be assumed to be contributed by \( K \) distinct Scattering Mechanisms (SMs), representing ground, volume, ground-trunk scattering, or other

\[
y_n(w_i) = \sum_{k=1}^{K} s_k(n; w_i)
\]

\( s_k(n, w_i) \): contribution of the \( k \)-th SM in Track \( n \), Polarization \( w_i \)

Hp: the data covariance is structured as a Sum of Kronecker Products

\[
y_n(w_i) = \sum_{k=1}^{K} s_k(n; w_i) \iff W_K = E[yy^H] = \sum_{k=1}^{K} C_k \otimes R_k
\]

Each SM is represented by a Kronecker Product

- \( R_k \): interferometric coherences of the \( k \)-th SM alone [N\!x\!N]
- \( C_k \): polarimetric correlation of the \( k \)-th SM alone [3\!x\!3]

Note that \( R_k, C_k \) are positive definite
The SKP decomposition

The key to the exploitation of the SKP structure is the existence of a decomposition of any matrix into a SKP

\[ \mathbf{W} \xrightarrow{\text{SKP Dec}} \sum_{p=1}^{P} \mathbf{U}_p \otimes \mathbf{V}_p \]

**Theorem:**

Let \( \mathbf{W} \) be contributed by \( K \) SMs according to H1,H2,H3, i.e.:

\[ \mathbf{W} = \sum_{k=1}^{K} \mathbf{C}_k \otimes \mathbf{R}_k \]

**then,** the matrices \( \mathbf{U}_k, \mathbf{V}_k \) are related to the matrices \( \mathbf{C}_k, \mathbf{R}_k \) via a linear, invertible transformation defined by exactly \( K(K-1) \) real numbers

**Corollary:**

**If** only ground and volume scattering occurs, i.e.:

\[ \mathbf{W} = \mathbf{C}_g \otimes \mathbf{R}_g + \mathbf{C}_v \otimes \mathbf{R}_v \]

**then,** there exist two real numbers \( (a,b) \) such that:

\[ \mathbf{C}_g = (a-b)^{-1}((1-b)\mathbf{U}_1 - b\mathbf{U}_2) \quad \mathbf{R}_g = a\mathbf{V}_1 + (1-a)\mathbf{V}_2 \]

\[ \mathbf{C}_v = (a-b)^{-1}(-(1-a)\mathbf{U}_1 + a\mathbf{U}_2) \quad \mathbf{R}_v = b\mathbf{V}_1 + (1-b)\mathbf{V}_2 \]
Forested areas: how many KPs?

**BIOSAR 2007 – Southern Sweden – P-Band**

- **HH**
- **HV**

**LIDAR Terrain Height**
**LIDAR Forest Height**

**BIOSAR 2008 – Northern Sweden – P-Band and L-Band**

- **P-Band - HV**
- **L-Band - HV**

**TROPISAR – French Guyana – P-Band**

*Courtesy of ONERA*

- **HH**
- **HV**

**Slant range [m]**

**Ground range [m]**

**Azimuth**
Forested areas: how many KPs?

2 KPs account for about 90% of the information carried by the data in all investigated cases.

⇒ 2 Layered-models (Ground + Volume) are well suited for forestry investigations.
Forested areas: how many KPs?

Overview talk:

P-Band penetration in tropical and boreal forests: Tomographical results

Friday – 14:40

Room 1
Coherence linearity and 2KPs: Algebraic connections

- Polarimetric Stationarity (PS):
  - Introduced by Ferro-Famil et al. to formalize the widely considered – RVoG consistent – case where the scene polarimetric properties are invariant to the choice of the passage
    \[
    \Sigma_{nn} = E[y_n y_n^H] = \Sigma_{mm} = E[y_m y_m^H]
    \]
  - Always valid after whitening the polarimetric information of each image in such a way as:
    \[
    \Sigma_{nn} = I_{3x3} \quad \forall n
    \]
    \[\Rightarrow\] Always retained in the remainder

- Under the PS condition the ESM coherence can be decomposed into a weighted sum:
  \[
  W = \sum_{k=1}^{K} C_k \otimes R_k \quad \xrightarrow{\text{(PS)}} \quad \Sigma_{nm} = E[y_n y_m^H] = \sum_{k=1}^{K} C_k \cdot \gamma_{nm}^{(k)} \quad \{R_k\}_{nm} = \gamma_{nm}^{(k)}
  \]
  \[
  \gamma_{nm}(w, w) = \sum_{k=1}^{K} \mu_{k}(w) \cdot \gamma_{nm}^{(k)} \quad \mu_{k}(w) = \frac{w^H C_k w}{w^H \left( \sum_{k=1}^{K} C_k \right) w}
  \]
Algebraic connections

### SKP decomposition

\[ W = \sum_{k=1}^{K} C_k \otimes R_k \quad \leftrightarrow \quad (PS) \quad \gamma_{nm}(w, w) = \sum_{k=1}^{K} \mu_k(w) \cdot \gamma_{nm}^{(k)} \quad \text{ESM coherence decomposition} \]

- **2 KPs \implies Coherence Linearity**

\[
W = C_g \otimes R_g + C_v \otimes R_v \quad \leftrightarrow \quad \gamma_{nm}(w, w) = \mu_v(w) \cdot \gamma_{nm}^v + \mu_g(w) \cdot \gamma_{nm}^g
\]

\[
\mu_{g,v}(w) = \frac{w^H C_{g,v} w}{w^H (C_g + C_v) w}
\]

- **Coherence Linearity \implies N(N-1)/2 +1 KPs**

\[
\text{Im}(\gamma_{nm}(w, w)) = a_{nm} \text{Re}(\gamma_{nm}(w, w)) + b_{nm} \quad \forall w \quad \leftrightarrow \quad \text{Im}(\gamma_{nm}^{(k)}) = a_{nm} \text{Re}(\gamma_{nm}^{(k)}) + b_{nm}
\]

\[
\Rightarrow \quad \text{Each of the matrices } R_k \text{ is fully specified by the real parts (N(N-1)/2) plus one constant that multiplies the affine term } b_{nm}
\]

\[
\Rightarrow \quad \text{There are at most N(N-1)/2 +1 linearly independent KPs}
\]

**Poor physical interpretation:**

- N(N-1)/2 +1 Scattering Mechanisms whose interferometric coherences are constrained to belong to the same line
Algebraic connections

- Single baseline \((N=2)\): perfect equivalence
  
  \[ 2\text{KPs} \iff \text{Coherence Linearity} \]

- Multi-baseline \((N>2)\): assuming 2KPs entails more algebraic constraints than assuming coherence linearity:
  
  \[ 2\text{KPs} \implies \text{Coherence Linearity} \]

Poor physical interpretation:

\[ N(N-1)/2 +1 \text{ Scattering Mechanisms whose interferometric coherences are constrained to belong to the same line} \]
Algebraic connections

⇒ In the multi-baseline case assuming 2KPs bring two advantages over coherence linearity:

1. Determination of physically valid solutions:

\[ W = C_g \otimes R_g + C_v \otimes R_v \quad \text{with } C_g, C_v, R_g, R_v \text{ positive definite} \]

Assuming coherence linearity:
Imposing pair-wise positive definitiveness results in physically valid ground and volume coherences to be \( \leq 1 \) in magnitude

Assuming 2KPs:
The positive definitiveness constraint results in the regions of physical validity to shrink from the outer boundaries towards the true ground and volume coherences
⇒ The higher the number of tracks, the easier it is to pick the correct solution
Algebraic connections

⇒ In the multi-baseline case assuming 2KPs bring two advantages over coherence linearity:

1. **Determination of physically valid solutions:**

\[ W = C_g \otimes R_g + C_v \otimes R_v \]

with \( C_g, C_v, R_g, R_v \) positive definite

Assuming coherence linearity:
   Imposing pair-wise positive definitiveness results in physically valid ground and volume coherences to be \( \leq 1 \) in magnitude

Assuming 2KPs:
   The positive definitiveness constraint results in the regions of physical validity to shrink from the outer boundaries towards the true ground and volume coherences
   ⇒ The higher the number of tracks, the easier it is to pick the correct solution

2. **Coherence identification:**

Assuming coherence linearity:
   Independent identification in each interferometric pair
   \( \Leftrightarrow 2^{N(N-1)/2} \) possibilities

Assuming coherence 2KPs:
   Joint identification on all interferometric pairs
   \( \Leftrightarrow 2 \) possibilities
Simulated scenario:

- 2 KPs: $W = C_g \otimes R_g + C_v \otimes R_v$
- Number of tracks : $N = 4$
- Number of independent looks: $L = \{9 – 169\}$

- Case 1: High ground coherences

- Case 2: Low ground coherences

2KP Estimators:

L2 norm minimization $\Leftrightarrow$ fast but NOT optimal

1. Pair-Wise Estimator
   Each pair is processed independently
   $\Leftrightarrow$ Equivalent to assuming coherence linearity

2. Joint Estimator
   All pairs are processed jointly

3. Preconditioned Joint Estimator
   All pairs are processed jointly
   $\Leftrightarrow$ As above, but the retrieved coherence matrices are allowed to be slightly negative

Criteria for coherence retrieval:

- Volume: existence of a ground-free polarization
- Ground: coherence maximization

Note: coherence are assigned to ground or volume basing on knowledge of the true values
$\Leftrightarrow$ coherence identification is NOT considered
Estimation from sample data

Case 1: High ground coherences

- $L = 16$

Remarks:

**Pair wise:**

Ground coherence is algebraically bounded to belong to the unitary circle

$\Rightarrow$ Good accuracy when the true ground coherence is close to 1

$\Rightarrow$ Systematic bias for low ground coherences

**Joint:**

Ground coherence is NOT bounded to belong to the unitary circle

$\Rightarrow$ High coherence may be underestimated

Improved accuracy over the Pair Wise approach for lower volume coherences

**Preconditioned Joint:**

Ground coherence underestimation is partly recovered

Legend:

- $\bigcirc$ True ground
- $\bullet$ Estimated ground
- $\bigcirc$ True volume
- $\bullet$ Estimated volume
- $\bigcirc$ Estimated volume after ground phase compensation
Case 1: High ground coherences

- $L = 49$

Remarks:

**Pair wise:**

Ground coherence is algebraically bounded to belong to the unitary circle

⇒ Good accuracy when the true ground coherence is close to 1

⇒ Systematic bias for low ground coherences

**Joint:**

Ground coherence underestimation is mitigated by increasing the number of looks

⇔ Not a systematic bias

Improved accuracy over the Pair Wise approach for lower volume coherences

**Preconditioned Joint:**

Underestimation of ground coherence is recovered

**True ground**  **Estimated ground**  **Estimated volume**  **Estimated volume after ground phase compensation**
Estimation from sample data

Case 1: High ground coherences

- $L = 100$

Remarks:

**Pair wise:**

Ground coherence is algebraically bounded to belong to the unitary circle

⇒ Good accuracy when the true ground coherence is close to 1

⇒ Systematic bias for low ground coherences

**Joint:**

Ground coherence underestimation is mitigated by increasing the number of looks

⇔ Not a systematic bias

Improved accuracy over the Pair Wise approach for lower volume coherences

**Preconditioned Joint:**

Underestimation of ground coherence is recovered

![Graphs showing real and imaginary parts for different cases](image)
Estimation from sample data

Case 1: High ground coherences

Error on volume coherence

Error on ground coherence

Error on volume coherence after ground phase compensation

- Pair-wise
- Joint
- Preconditioned Joint
Estimation from sample data

Case 2: Low ground coherences

Error on volume coherence

Error on ground coherence

Error on volume coherence after ground phase compensation

- Pair-wise
- Joint
- Preconditioned Joint
Conclusions

Single-baseline case: assuming Coherence Linearity is equivalent to assuming 2 KPs

Multi-baseline: assuming 2KPs entails more algebraic constraints than assuming coherence linearity

- More accurate estimation of low-valued ground and volume coherence
- Simplifies the coherence identification problem to a single choice
- High ground coherence are underestimated if few looks (say < 50) are employed
- Underestimation is mitigated by pre-conditioning the problem

Estimators operating through L2 norm minimization $\Leftrightarrow$ fast but not optimal

The need for a pre-conditioning operator suggests that significant improvements could be achieved from the investigation of a statistically optimal multi-baseline estimator for the 2KP model