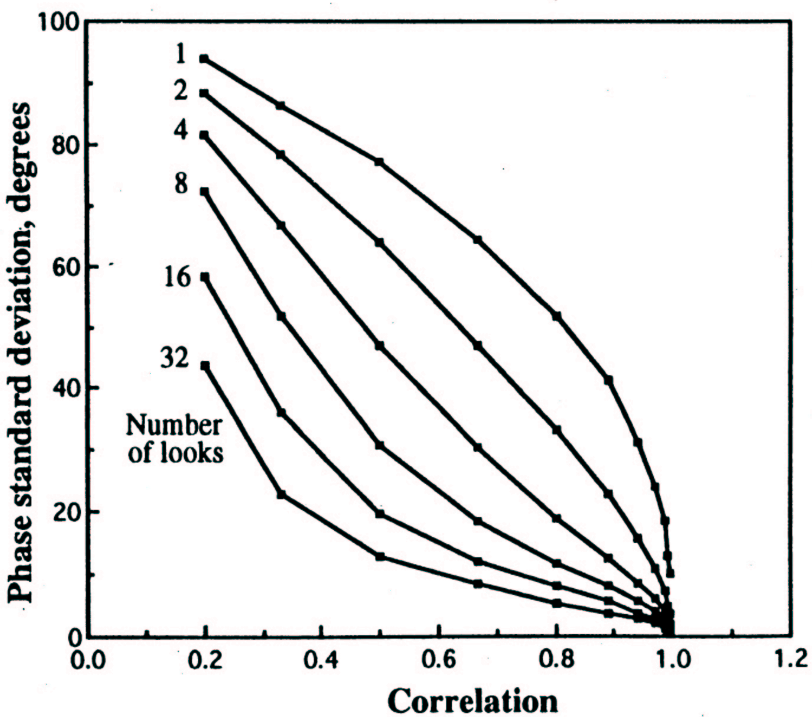
Relationship of Phase Noise and Decorrelation

- **Increased decorrelation is associated with an increase in the interferometric phase noise variance**
- **If averaging *N* > 4 samples, the phase standard deviation approaches the Cramer-Rao bound on the phase estimator:**



$$\sigma_\phi = \frac{1}{\sqrt{2N}} \frac{\sqrt{1-\gamma^2}}{\gamma}$$

Phase noise contributes to height errors in interferometry as demonstrated in the sensitivity equations

Thermal Noise Decorrelation

Radar receiver electronics will add thermally generated noise to the image observations.

$$s_1^{'} = A_b e^{j\phi_b} e^{-j\frac{4\pi}{\lambda}\rho_1} + n_1 \; ; s_2^{'} = A_b e^{j\phi_b} e^{-j\frac{4\pi}{\lambda}\rho_2} + n_2$$

The added noise contributes randomly to the interferometric phase from pixel to pixel, causing thermal noise decorrelation. Assuming uncorrelated noise, the correlation between s_1 and s_2 is

$$\gamma = \frac{\left|\left\langle (s_1 + n_1)(s_2 + n_2)^* \right\rangle\right|}{\sqrt{\left\langle (s_1 + n_1)(s_1 + n_1)^* \right\rangle \left\langle (s_2 + n_2)(s_2 + n_2)^* \right\rangle}}$$

$$= \frac{|s_1 s_2|}{\sqrt{\left\langle (s_1^2 + n_1^2) \right\rangle \left\langle (s_2^2 + n_2^2) \right\rangle}}$$

$$= \frac{\sqrt{P_1}\sqrt{P_2}}{\sqrt{P_1 + N_1}\sqrt{P_{21} + N_2}} = \frac{1}{\sqrt{1 + N_1 / P_1}} \frac{1}{\sqrt{1 + N_2 / P_2}}$$

Thermal Noise Decorrelation

The correlation is related in a simple way to the reciprocal of the Signal-to-Noise Ratio (SNR). For observations with identical backscatter and equal noise power,

$$\gamma = \frac{1}{1 + N/P} = \frac{1}{1 + \mathrm{SNR}^{-1}}$$

Decorrelation is defined as

$$\delta = 1 - \gamma$$

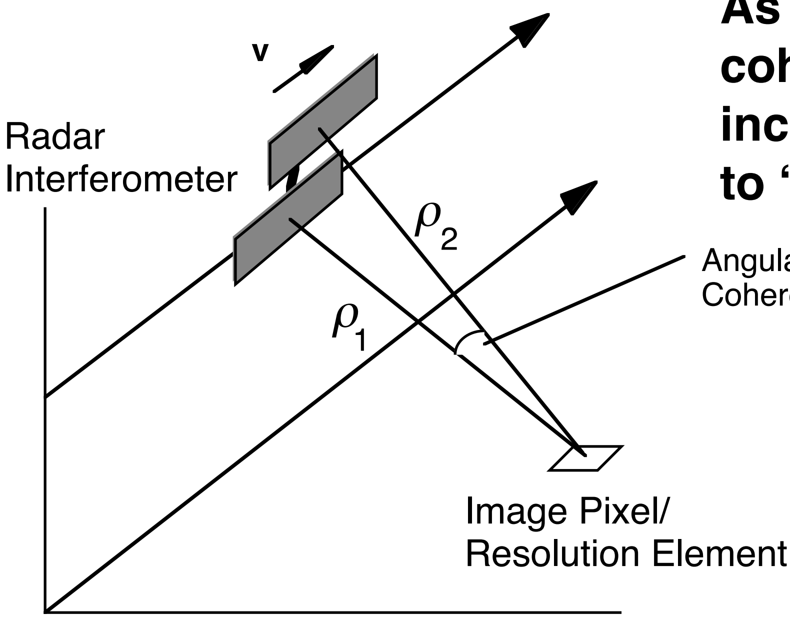
The decorrelation due to thermal noise can vary greatly in a scene, not from thermal noise variations, but from variations in backscatter brightness. Extreme cases are:

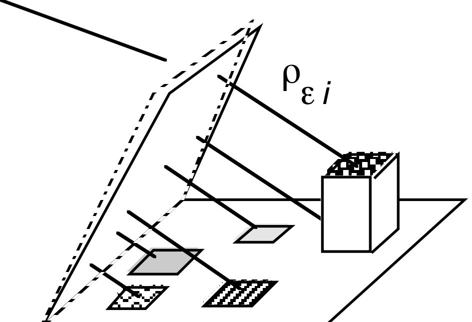
•Radar shadow, where no signal is returns; correlation is zero.
•Bright specular target, where signal dominates return; correlation is 1

Baseline Decorrelation

Pixels in two radar images observed from nearby vantage points have *nearly* the same complex phasor representation of the coherent backscatter from a resolution element on the ground

As interferometric baseline increases, the coherent backscatter phase becomes increasingly different randomly, leading to “baseline” or “speckle” decorrelation.

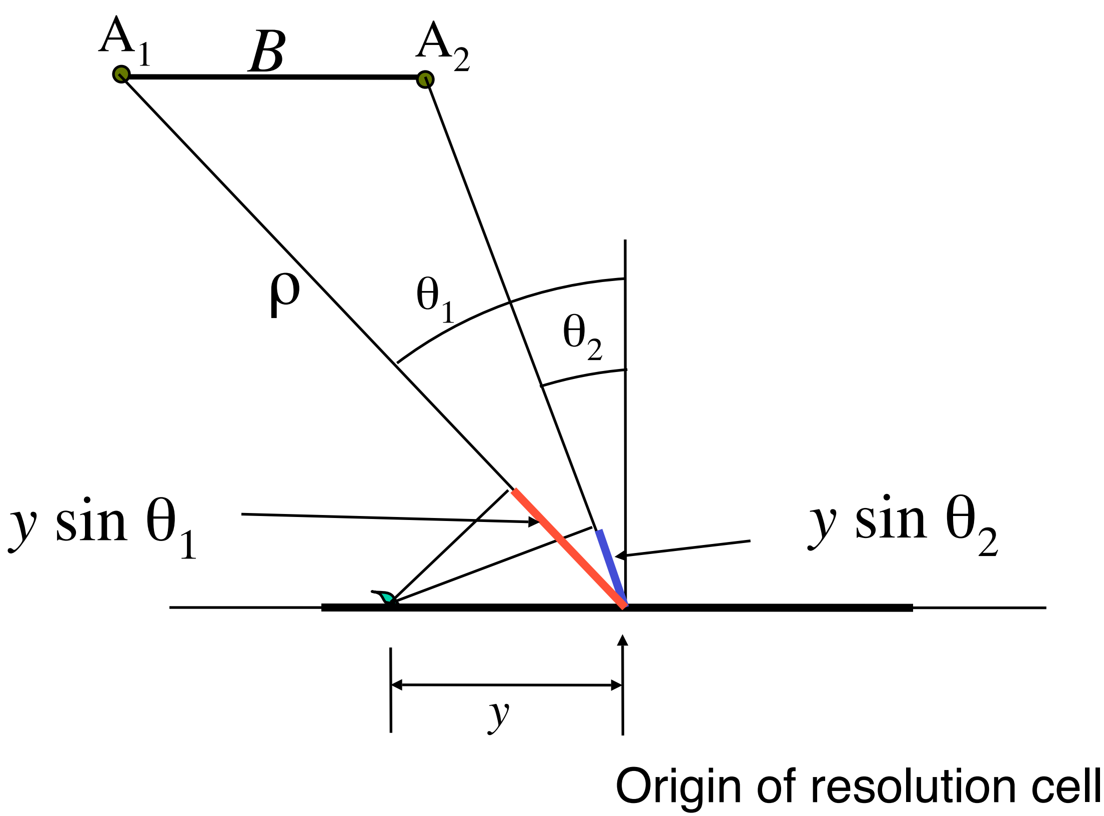




$$s_1 = A_{b1} e^{j\phi_{b1}} e^{-j\frac{4\pi}{\lambda}\rho_1} \qquad s_2 = A_{b2} e^{j\phi_{b2}} e^{-j\frac{4\pi}{\lambda}\rho_2}$$

Baseline Decorrelation: Resolution Cell

Interferometric geometry for a horizontal baseline in the cross-track plane



Baseline Correlation Formulation

Consider signals from the two antennas displaced in angle:

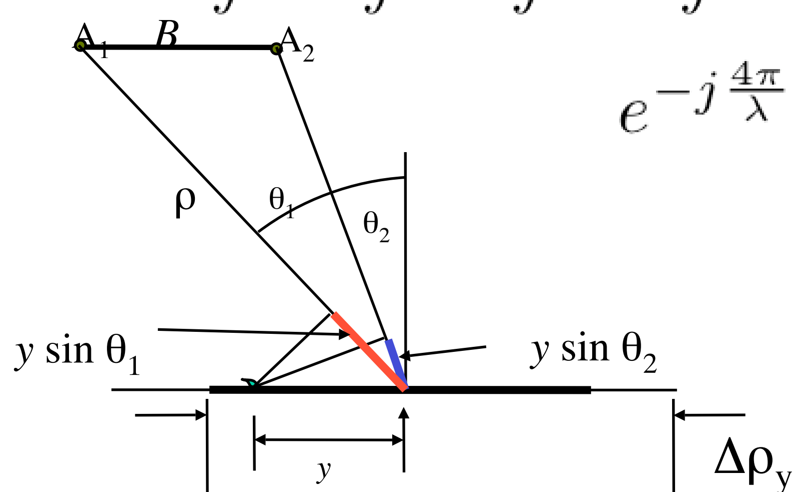
$$s_1(x_0, y_0) = \int dx \int dy f(x - x_0, y - y_0) e^{-j\frac{4\pi}{\lambda}(\rho + y\sin\theta_1)} W(x, y) + n_1(x_0, y_0)$$

$$s_2(x_0, y_0) = \int dx \int dy f(x - x_0, y - y_0) e^{-j\frac{4\pi}{\lambda}(\rho + y\sin\theta_2)} W(x, y) + n_2(x_0, y_0)$$

Cross-correlation yields:

$$s_1 s_2^\star = \int dx \int dy \int dx' \int dy' f(x - x_0, y - y_0) f^\star(x' - x_0, y' - y_0)$$

$$e^{-j\frac{4\pi}{\lambda}(y\sin\theta_1 - y'\sin\theta_2)} W(x, y) W^\star(x', y')$$



plus noise cross-products

$f(x,y)$ = the surface reflectivity function
$W(x,y)$ = the imaging point spread function

Surface Assumption for Correlation Formulation

If the surface is modeled as *uniformly distributed, uncorrelated* scattering centers, that is:

$$\langle f(x,y) f^{\star}(x',y')\rangle = \sigma_0 \delta(x-x', y-y')$$

where σ_0 is the mean backscatter cross section per unit area from the surface

Under this assumption, the correlation expression reduces to:

$$\langle s_1 s_2^{\star}\rangle = \sigma_0 \int dx \int dy \, |W(x,y)|^2 \, e^{-j\frac{4\pi}{\lambda} y \cos\theta \Delta\theta}$$

The shape of the correlation function as a function of angle is the Fourier Transform of the Image Point Spread Function

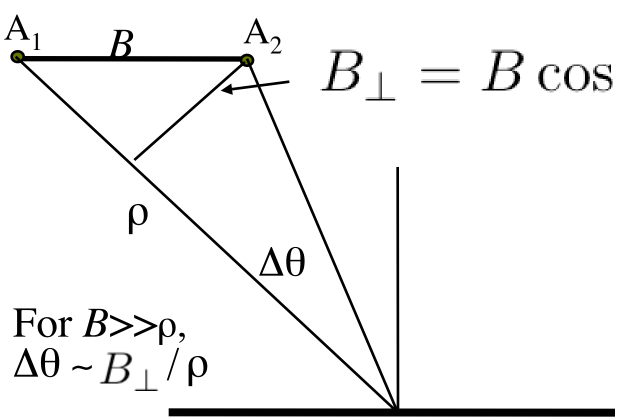
Form of Baseline Correlation Function

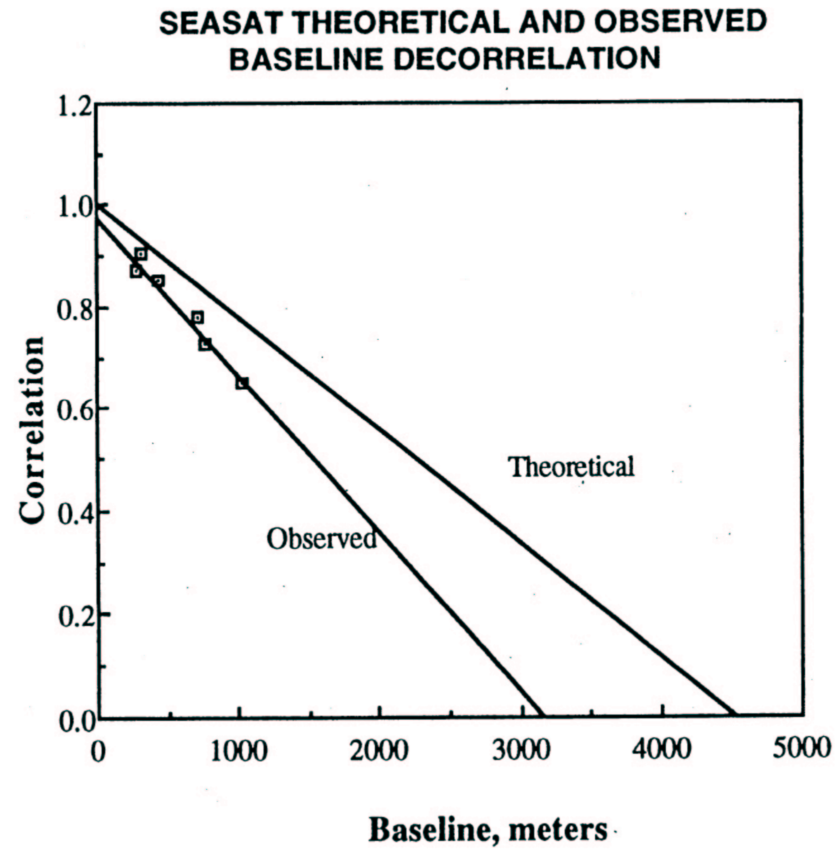
If $W(x,y)$ is assumed to be a sinc function, then the integral can be done in closed form (for ping-pong mode)

$$\gamma_B = 1 - \frac{2(B\cos\theta)(\Delta\rho_y\cos\theta)}{\lambda\rho}$$

$$\boxed{= 1 - \frac{2B_\perp\Delta\rho_\perp}{\lambda\rho}}$$

$$B_\perp = B\cos\theta$$

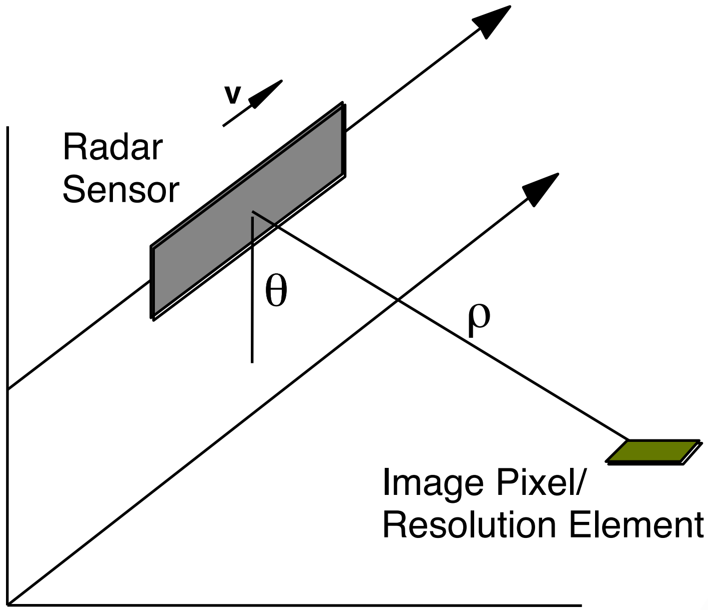




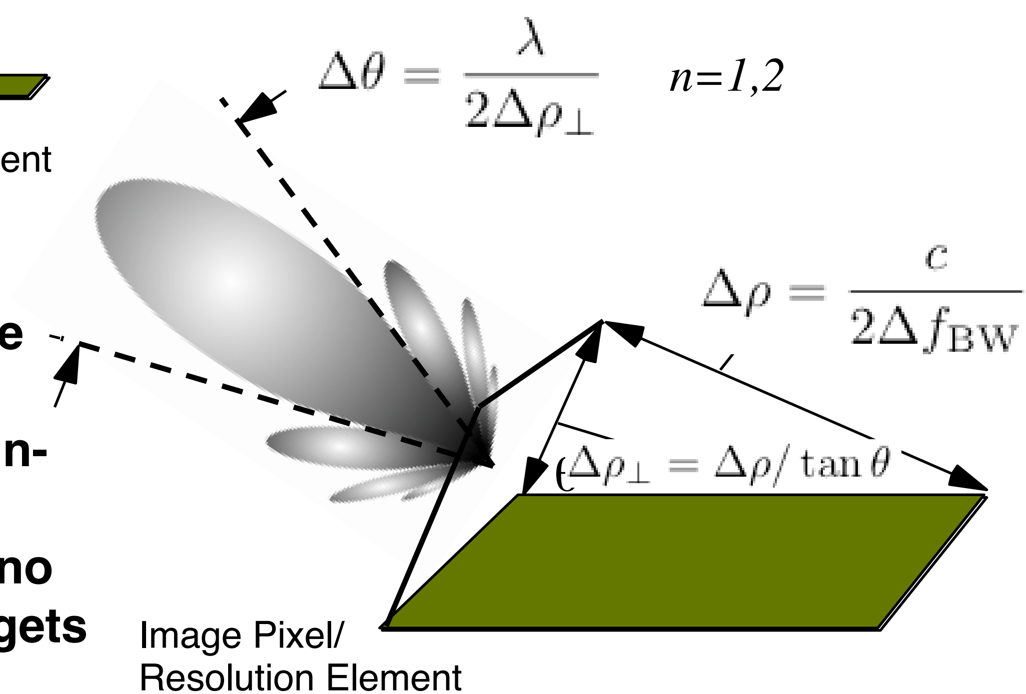
This function goes to zero at the *critical baseline*

$$\boxed{B_{\perp,\mathrm{crit}} = \frac{\lambda\rho}{n\Delta\rho_\perp}} \qquad n = 1, 2$$

The "Pixel Antenna" View of Baseline Decorrelation

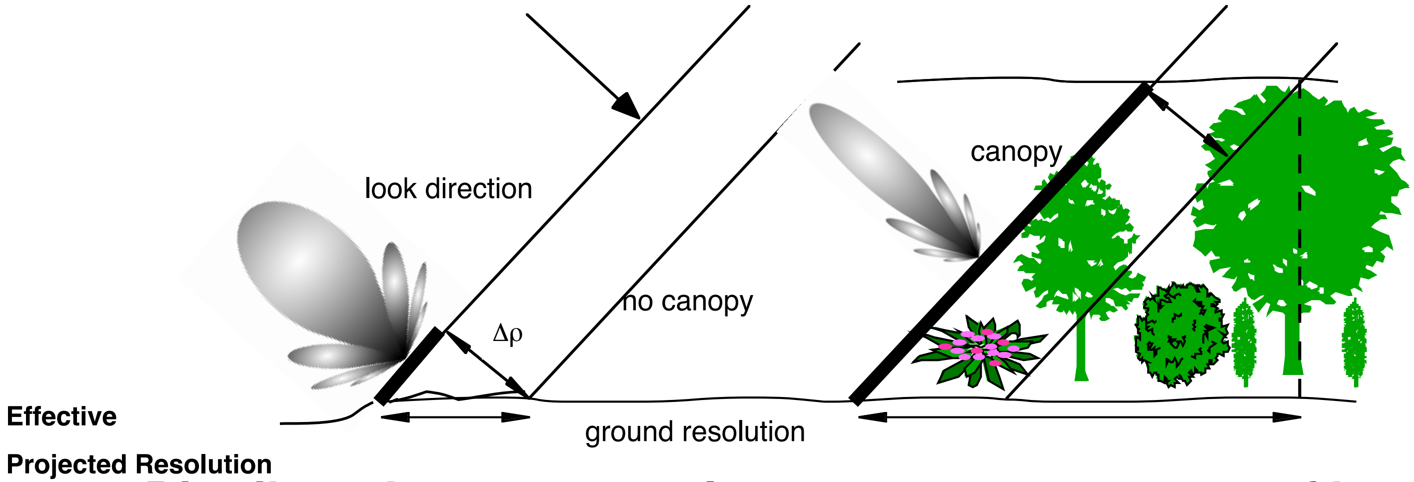


Each resolution element can be considered a radiating antenna with beamwidth of $\Delta\theta_{\Delta\rho_\perp}$, which depends on the range and local angle of incidence.

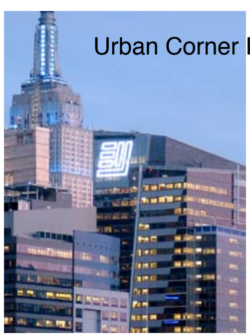


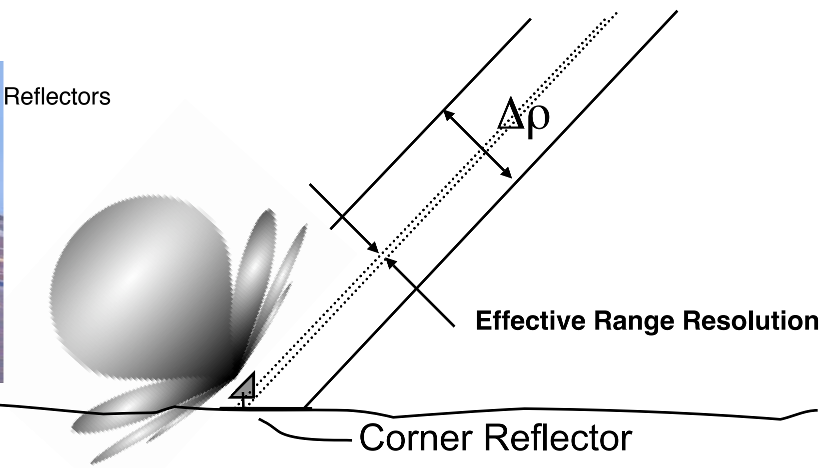
When the two apertures of the interferometer are within this beamwidth, coherence is maintained. Beyond this critical baseline separation, there is no coherence for distributed targets

Overcoming Baseline Decorrelation



Distributed targets correlate over a narrow range of baselines





JPL Corner Reflector

Pixels dominated by single scatterers generally behave though imaged at much finer resolution