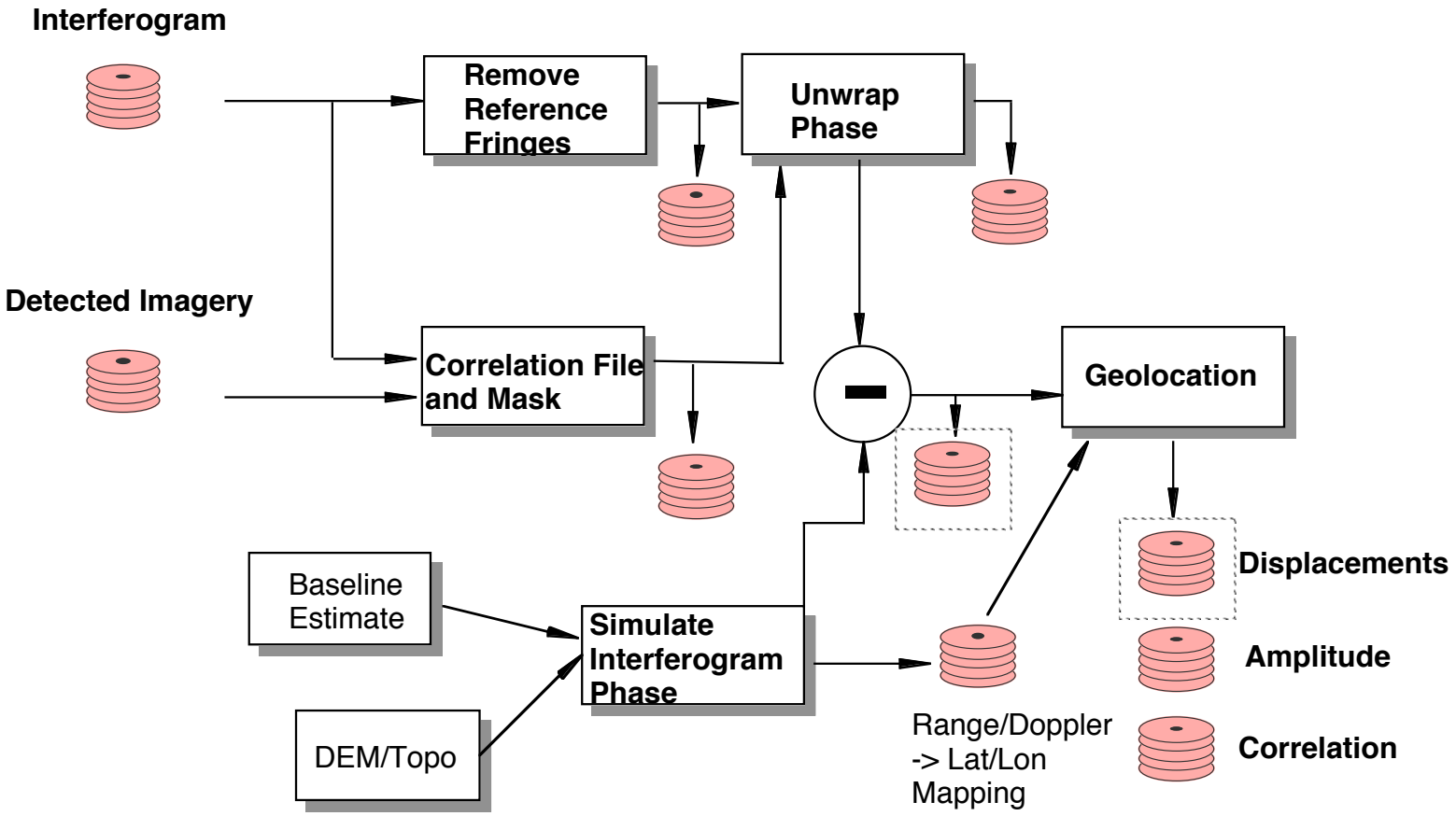
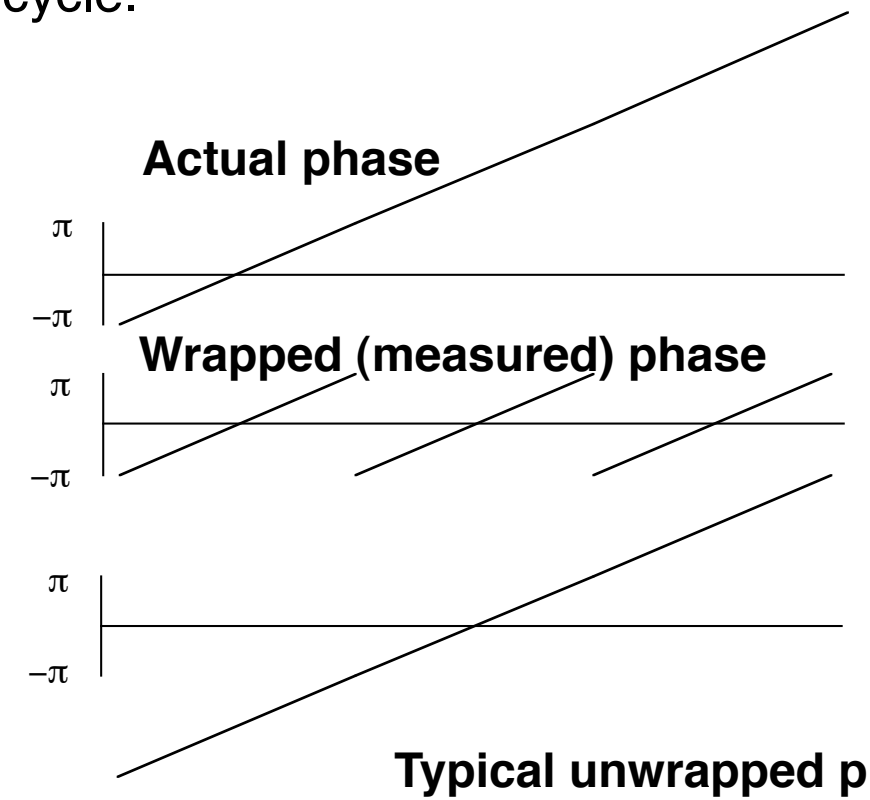
2-Pass Differential Processing



Phase Unwrapping

- Elements of the phase unwrapping problem: From the measured, wrapped phase, unwrap the phase from some arbitrary starting location, then determine the proper ambiguity cycle.



$$\phi = -\frac{4\pi}{\lambda}\left(\rho_1 - \rho_2\right) \cong -\frac{4\pi}{\lambda}\vec{B}\cdot\hat{n}_{\mathrm{los}}$$

$$\phi_{\mathrm{meas}} = \mathrm{mod}\left(\phi, 2\pi\right)$$

$$\phi_{\mathrm{unw}} = \bigcup\left(\phi_{\mathrm{meas}}\right) = \phi + 2\pi k$$

where k is an integer to return the absolute phase

Two-Dimensional Phase Unwrapping

- Two dimensional phase field values below are in units of cycles
- One-dimensional unwrapping criterion of half-cycle proximity is inconsistent in two dimensions

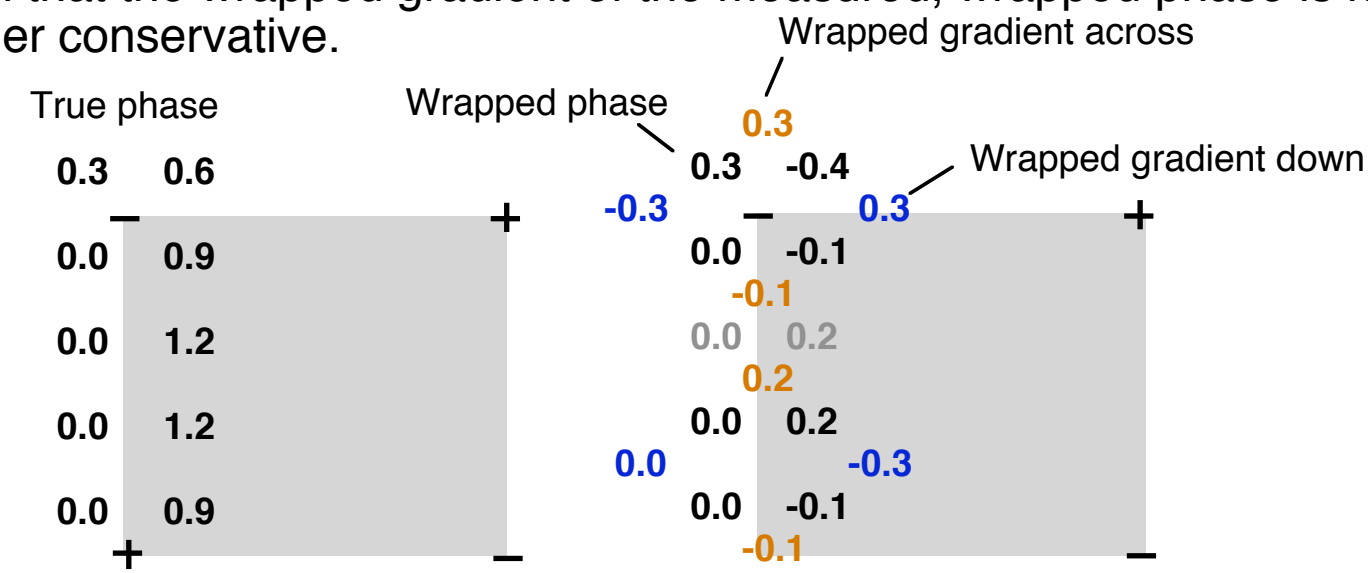
0.0	0.0	0.3	0.0	0.0	0.3	0.0	0.0
0.0	0.3	0.6	0.3	0.3	0.6	0.3	0.0
0.0	0.0	0.9	0.6	0.6	0.9	0.0	0.0
0.0	0.0	1.2	0.9	0.9	1.2	0.0	0.0
0.0	0.0	1.2	0.9	0.9	1.2	0.0	0.0
0.0	0.0	0.9	0.6	0.6	0.9	0.0	0.0
0.0	0.3	0.6	0.3	0.3	0.6	0.3	0.0
0.0	0.0	0.3	0.0	0.0	0.3	0.0	0.0

0.0	0.0	0.3	0.0	0.0	0.3	0.0	0.0
0.0	0.3	0.6	0.3	0.3	0.6	0.3	0.0
0.0	0.0	-0.1	-0.4	-0.4	-0.1	0.0	0.0
0.0	0.0	0.2	-0.1	-0.1	0.2	0.0	0.0
0.0	0.0	0.2	0.9	-0.1	0.2	0.0	0.0
0.0	0.0	-0.1	-0.4	-0.4	-0.1	0.0	0.0
0.0	0.3	0.6	0.3	0.3	0.6	0.3	0.0
0.0	0.0	0.3	0.0	0.0	0.3	0.0	0.0

- **Residues, marked with + and -, define ambiguous boundaries.**

Residues in Phase Unwrapping

- The wrapping operator delivers the true phase modulo 2 π, in the interval $-\pi < \phi < \pi$.
- The true phase gradient is conservative: $\nabla \times \nabla \phi = 0$
- The wrapped gradient of the measured, wrapped phase, however, may not be conservative: $\nabla \times W\{\nabla \phi_w\} \neq 0$
- When this function is non-conservative, its integration becomes path dependent.
- Residues occur at locations of high phase noise and/or phase shear such that the wrapped gradient of the measured, wrapped phase is no longer conservative.



Branch Cuts in Phase Unwrapping

- Branch-cut algorithms (Goldstein, Zebker, and Werner 1986) seek to neutralize these regions of inconsistency by connecting residues of opposite solenoidal sense with cuts, across which integration may not take place.
- Branch cut connections force path independence in the integration of the wrapped phase gradient.
- If done properly, the integrated phase field will be correct. But which is correct?

0.0	0.0	0.3	0.0	0.0	0.3	0.0	0.0
0.0	0.3	0.6	0.3	0.3	0.6	0.3	0.0
0.0	0.0	0.9	0.6	0.6	0.9	0.0	0.0
0.0	0.0	1.2	0.9	0.9	1.2	0.0	0.0
0.0	0.0	1.2	0.9	0.9	1.2	0.0	0.0
0.0	0.0	0.9	0.6	0.6	0.9	0.0	0.0
0.0	0.3	0.6	0.3	0.3	0.6	0.3	0.0
0.0	0.0	0.3	0.0	0.0	0.3	0.0	0.0

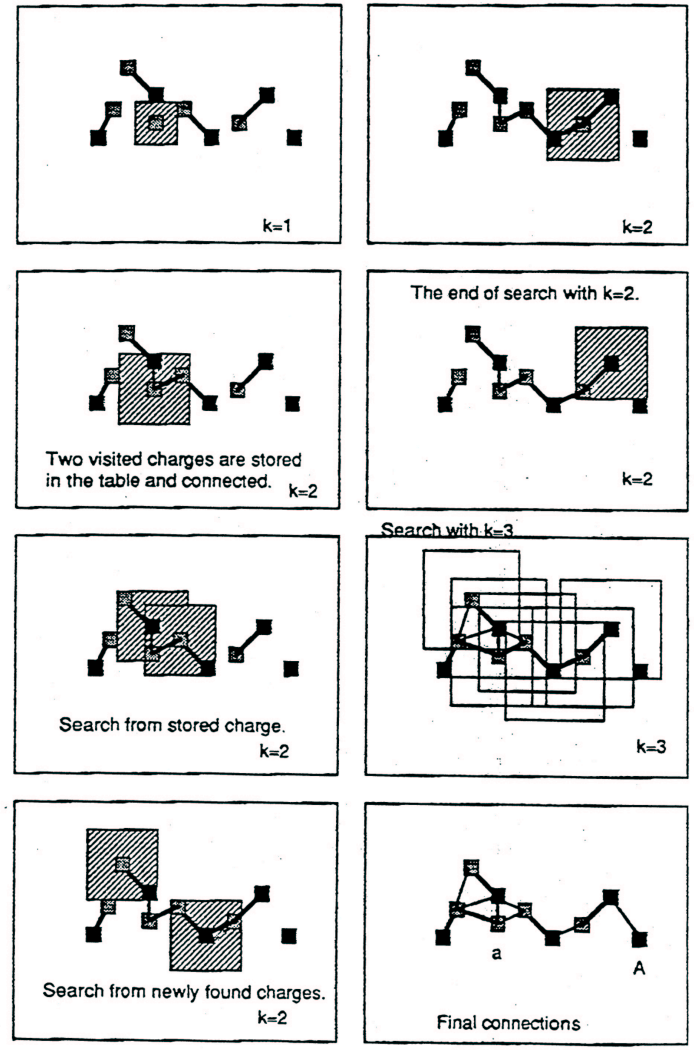
0.0	0.0	0.3	0.0	0.0	0.3	0.0	0.0
0.0	0.3	0.6	0.3	0.3	0.6	0.3	0.0
0.0	0.0	-0.1	-0.4	-0.4	-0.1	0.0	0.0
0.0	0.0	0.2	-0.1	-0.1	0.2	0.0	0.0
0.0	0.0	0.2	0.9	-0.1	0.2	0.0	0.0
0.0	0.0	-0.1	-0.4	-0.4	-0.1	0.0	0.0
0.0	0.3	0.6	0.3	0.3	0.6	0.3	0.0
0.0	0.0	0.3	0.0	0.0	0.3	0.0	0.0

Illustration of Branch Cut Algorithm

Connect all simple pairs

Expand search window and connect where possible

Initiate search from every residue encountered



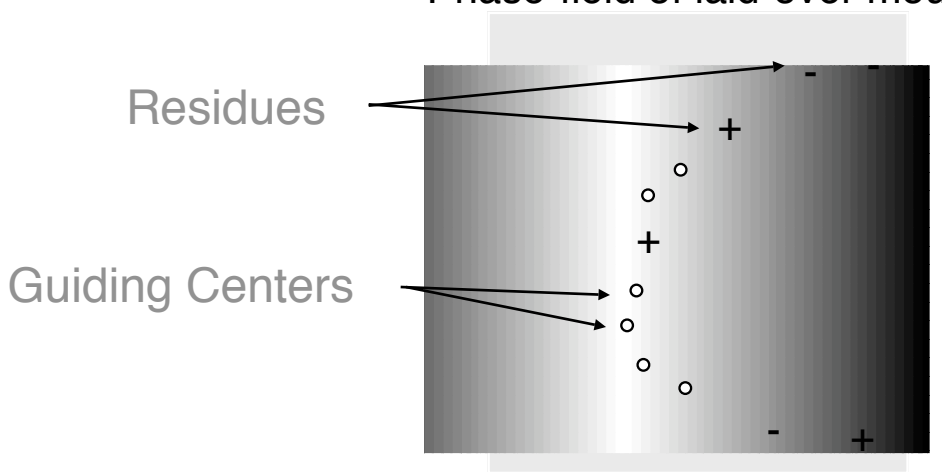
Could not connect last residue with this search window size

Repeat all steps with larger search window size

Branch Cut Strategies

- The standard GZW algorithm is designed to connect residues into a neutral network into the shortest possible connection tree, i.e. to minimize the length of the individual branch cuts comprising a tree
- This will not necessarily create the shortest possible tree, since GZW makes many unnecessary connections in its search for neutrality
- Various criteria have been devised to place guiding centers (unsigned residues) along expected paths to facilitate the right choice in branch cut connection

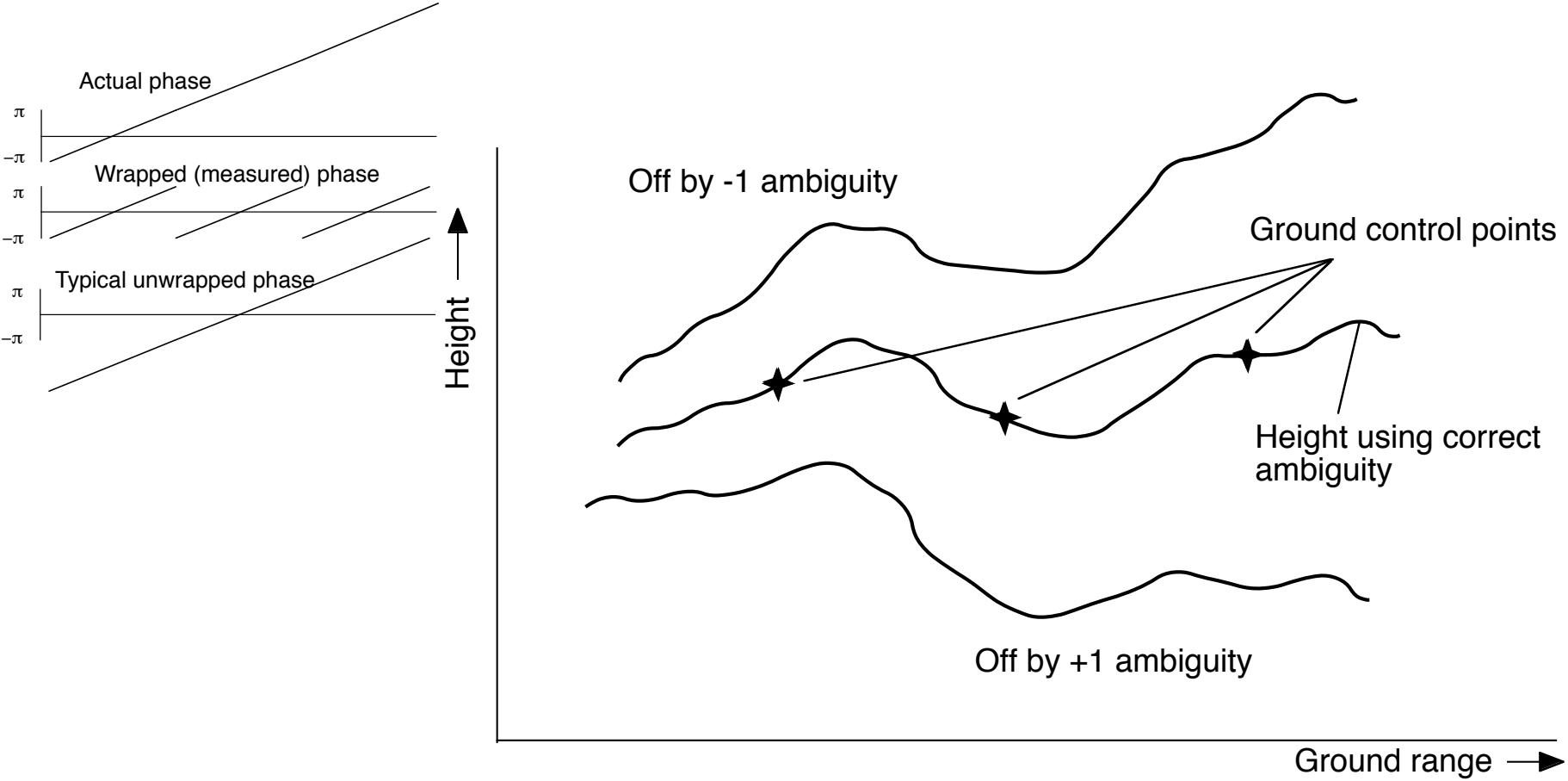
Phase-field of laid-over mountain



Guiding Center Criteria

- A number of criteria have been devised for selecting guiding centers, each more or less tailored to characteristics of SAR data:
  - when phase slope exceeds threshold (implies layover)
  - when derivative of phase slope exceeds threshold
  - when radar brightness exceeds threshold (implies layover)
  - when decorrelation estimator exceeds threshold (implies noise and/or layover
- Some guiding center selections help in some cases
- Difficult to assess performance in a quantitative way

Absolute Phase Determination



Ground control reference points can be used to determine the absolute phase ambiguity